PARAMETER IDENTIFICATION FOR UNDERDETERMINED SYSTEMS ARISING IN OPTION PRICING MODELS AND NEURAL NETWORKS

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In this thesis, we study the convergence behavior of an efficient optimization method used for the identification of parameters for underdetermined systems. The research is motivated by optimization problems arising from the estimation of parameters in neural networks as well as in option pricing models.

In the first application, we are concerned with neural networks used to forecasting stock market indices. Since neural networks are able to describe extremely complex nonlinear structures they are used to improve the modelling of the nonlinear dependencies occurring in the financial markets. Applying neural networks to the forecasting of economic indicators, we are confronted with a nonlinear least squares problem of large dimension. Furthermore, in this application the number of parameters of the neural network to be determined is usually much larger than the number of patterns which are available for the determination of the unknowns. Hence, the residual function of our least squares problem is underdetermined.

In option pricing, an important but usually not known parameter is the volatility of the underlying asset of the option. Assuming that the underlying asset follows a one–factor continuous diffusion model with nonconstant drift and volatility term, the value of an European call option satisfies a parabolic initial value problem with the volatility function appearing in one of the coefficients of the parabolic differential equation. Using this system equation, the estimation of the volatility function is described by a nonlinear least squares problem. Since the adaption of the volatility function is based only on a small number of observed market data these problems are naturally ill–posed.

For the solution of these large–scale underdetermined nonlinear least squares problems we use a fully iterative inexact Gauss–Newton algorithm. We show how the structure of a neural network as well as that of the European call price model can be exploited using iterative methods. Moreover, we present theoretical statements for the convergence of the inexact Gauss–Newton algorithm applied to the less examined case of underdetermined nonlinear least squares problems.

Finally, we present numerical results for the application of neural networks to the forecasting of stock market indices as well as for the construction of the volatility function in European option pricing models. In case of the latter application, we discretize the parabolic differential equation using a finite difference scheme and we elucidate convergence problems of the discrete scheme when the initial condition is not everywhere differentiable.