UNIVERSITÄT TRIER

Large Players, Information and Coordination

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Preface - Vorbemerkungen

Die vorliegende Dissertation ist gemäß den Vorgaben der Promotionsordnung des Fachbereichs IV “Wirtschafts- und Sozialwissenschaften, Mathematik und Informatikwissenschaften” der Universität Trier vom 28. September 2004 angefertigt worden. Kapitel 1 dient der inhaltlichen Zusammenführung der Kapitel 2 bis 4. Eine deutsche Zusammenfassung der Ergebnisse gemäß §5 Abs. 4 der Promotionsordnung befindet sich in Kapitel 5.

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Finally, I thank my family and friends, who have given me support and distraction in times when I needed it the most. I am incredibly grateful to my wonderful wife who supported me all the way, gave me professional advice, helped me focus and gave me hope when it was hard to see the finish line.

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Summary

I know you think you understand what you thought I said, but I'm not sure you realize that what you heard is not what I meant.

- unknown origin, falsely attributed to Alan Greenspan

Though Alan Greenspan probably never said the above quote in this way, it is not surprising that it is often misattributed to him. As chairman of the U.S. Federal Reserve, he was notorious for his masterful use of "Fedspeak", a communication strategy in which statements to the public about monetary policy and economic developments are deliberately vague and ambiguous in order to prevent financial markets from overreacting. The above quote is an apt example of this communication practice and whether it comes from Alan Greenspan or not, it illustrates the weight that smaller market participants attach to his assessments and those of his successors.

As a large player in the financial markets, a central bank primarily uses its statutory powers to influence markets through direct intervention and through open market operations. However, in times of high uncertainty and volatility (such as crises), small market participants are increasingly guided by their forecasts and decisions, so that their actions can have a disproportionate influence on markets and economies. Accordingly, sophisticated communication and obfuscation strategies to manage expectations (such as forward guidance and Fedspeak) are now established tools in the toolbox of central banks. But it is not only central banks that can exert such influence as large players. Governments, international financial institutions, private investment companies and even individuals can act as "centers of gravity" or focal points for small players. An example of such an individual is George Soros, whose short position against the British pound sterling in 1992 encouraged other currency traders to short the pound, allegedly contributing significantly to the UK's withdrawal from the European Exchange Rate Mechanism.

In this dissertation, I analyze in three chapters how large players exert influence on smaller players and how this affects the decisions of the large ones. A particular focus is on how the large players process information in an uncertain environment, form expectations and communicate these to smaller players through their actions. I examine these relationships empirically in the foreign exchange market on the one hand and within a game-theoretical model of an investment project on the other.

In Chapter 2 of this dissertation, I investigate the relationship between the foreign exchange trading activity of large US-based market participants and the volatility of the nominal spot exchange rate. The mixture-of-distributions hypothesis (MDH) predicts that there is a positive relationship between the trading activity of asset market participants and the volatility of asset returns. Previous empirical studies confirm this prediction for the stock and futures markets. However, comparable studies for the foreign exchange market are more scarce due to its decentralized and opaque structure and the associated lack of trading activity data. Using data from the so-called *Foreign Currency Reports* published in the *Treasury Bulletin* (U.S. Department of the Treasury, nd), I use the weekly growth rate of aggregate foreign currency positions of major market participants to proxy trading activity in the foreign exchange market. By estimating the heterogeneous autoregressive model of realized volatility (HAR-RV by Corsi, 2009) extended with exogenous regressors, I find supporting evidence that the positive relationship between trading activity and volatility found in other asset markets also holds in the foreign exchange market. Moreover, I show that the relation is mainly driven by unexpected changes in trading activity and that it is asymmetric for some of the considered currencies. My results extend the empirical findings for the MDH to foreign exchange markets and contribute to understanding the drivers of exchange rate volatility and the role of large players for the flow of information.

In Chapters 3 and 4, I use a game-theoretical model of an investment project to examine how a large creditor influences the decisions of small creditors with its lending decision. I pay particular attention to the timing of the large player's decision, i.e. whether she makes her decision to roll over a credit before or after the small players. Chapter 3 considers this scenario in a sequential global game of regime change. The large creditor is given the choice to roll over an investment early or postpone her decision. Rolling over early sends a positive signal to small creditors, who are thereby induced to roll over as well. At the same time, postponing allows the large creditor to observe the small creditors' aggregate decision and learn from them in order to make a better informed decision. She faces a trade-off between being at the front or at the rear of the crowd of creditors. By being a focal point for small creditor coordination, her actions have a substantial impact on the probability of coordination failure and thus the failure of the investment project. By highlighting the importance of signaling, the findings open up a new perspective on the idea of catalytic finance and the influence of a lender-of-last-resort in self-fulfilling debt crises.

In Chapter 4, I extend these findings by lifting the assumption that the large creditor observes the fraction of small creditors who roll over perfectly, resulting in a game with imperfect learning. I investigate the sensitivity of the equilibrium with respect to information precision. Two central results emerge: 1) I show that the game with perfect learning and the game without the option to wait (both introduced in Chapter 3) are special cases of the game with imperfect learning. Thereby the new parameter for signal precision governs how close the game with imperfect learning is to either of the other two games. 2) I confirm that the qualitative results from Chapter 3 still hold, even if the large creditor is imperfectly informed. Hence, these results are robust towards the assumption that the large creditor observes small creditors perfectly.

Finally, Chapter 5 provides a summary of the findings in this dissertation in German.

2

Large Players' Foreign Currency Positions and Exchange Rate Volatility

Abstract

In this chapter, I investigate the relationship between the foreign exchange (FX) trading activity of large US-based market participants and the volatility of the nominal spot exchange rate. The mixture-of-distributions hypothesis (MDH) predicts that there is a positive relationship between the trading activity of asset market participants and the volatility of asset returns. Previous empirical studies confirm this prediction for the stock and futures markets. However, comparable studies for the FX market are more scarce due to its decentralized and opaque structure and the associated lack of trading activity data. I use the weekly growth rate of large market participant's aggregate foreign currency positions from 1994 to 2020 to proxy trading activity in the FX market. By estimating the heterogeneous autoregressive model of realized volatility (HAR-RV by Corsi, 2009) extended with exogenous regressors, I find supporting evidence that the positive relationship between trading activity and volatility found in other asset markets also holds in the FX market. Moreover, I show that the relation is mainly driven by unexpected changes in trading activity and that it is asymmetric for some of the considered currencies. My results extend the empirical findings for the MDH to FX markets and contribute to understanding the drivers of exchange rate volatility and the role of information flow.

2.1 Introduction

In this chapter, I investigate the relationship between the foreign exchange (FX) trading activity of large US-based market participants and the volatility of the nominal spot exchange rate. For that purpose, I use the weekly growth rate of their aggregate foreign currency positions (FCPs) to proxy trading activity. The so-called mixture-of-distributions hypothesis (MDH) predicts that the rate at which new information arrives in the market is positively related to trading activity or, more specifically, to trading volume of financial assets. At the same time, the arrival of new information leads to a price adjustment, which coincides with an increase in volatility. Hence, trading activity and volatility are positively related by the rate at which new information becomes known to market participants. Previous empirical results show that predictions from the theory, such as a positive volume-volatility correlation, hold for futures and stock markets. I extend this strand of literature by asking whether the positive correlation between trading activity and price volatility also holds in FX markets and if so, whether it is related to a news arrival process as implied by MDH. I use a so far mostly unexploited data set containing weekly FCPs of large US-based market participants' to proxy trading activity and the underlying news arrival process. By estimating the heterogeneous autoregressive model of realized volatility (HAR-RV by Corsi, 2009), I find supporting evidence that the trading activity-volatility relationship also holds in the FX market. In particular, my results show that

a) weekly FCP growth is positively contemporaneously associated with weekly realized volatility of spot exchange rates;

b) there is no convincing evidence for a lagged relation between FCP growth rates and realized volatility, i.e. FCP growth rates have limited predictive power for realized volatility;

c) the positive contemporaneous relation is mainly driven by unexpected innovations in growth rates, rather than expected changes;

d) the relation between FCP growth rates and realized volatility is asymmetric, i.e. positive growth rates are stronger positively associated with realized volatility than negative growth rates.

With an average daily turnover of about $7.5 trillion in April 2022 (Bank for International Settlements, 2022), the FX market is the largest asset market in the world. At the same time, for the year 2022, UNCTAD estimates a total international trade volume of about $32 trillion (UNCTAD, 2022). Hence, there is a large fraction of FX transactions which is not explained by trade flows of goods and services, but rather by capital flows, speculation and investment in foreign currency denominated financial assets. In contrast to equity markets, the FX market is organized in an over-the-counter (OTC) fashion, i.e. there is no trading exchange or central clearing agency to arrange trades, but there are large international dealer banks and other financial institutions that function as market makers and liquidity providers for smaller market participants. Moreover, about 80% of the world's FX trading is concentrated in a few large trading centers, especially London, New York, Singapore, Hong Kong and Tokyo (Bank for International Settlements, 2022, p. 7f.). Its size, decentralized and largely unregulated structure are the reason why the FX market is more opaque from a research perspective than other asset markets. Data, such as turnover, volume and individual trading positions are not collected centrally, but have to be gathered and matched from different sources and are often available only for specific currency pairs or trading locations. For this reason, empirical research on the implications of MDH in FX markets is more scarce than for the more centralized equity and futures markets. In this chapter I use FCP data collected by the Federal Reserve Bank of New York through so called *Treasury Foreign Currency Reports* to infer large US-based market participants' FX trading activities (U.S. Department of the Treasury, nd). To the best of my knowledge, this data has not been used in this context before. Therefore, I aim to provide new insights in the relation between FX trading activity and exchange rate volatility.

Understanding and explaining the drivers of exchange rates and their volatility have been of utmost interest for academics for decades. The now infamous exchange-rate-disconnect and Meese-Rogoff puzzles (Meese and Rogoff, 1983) state that models based on economic and monetary fundamentals are essentially unable to consistently explain and forecast nominal exchange rates out-of-sample. But also the volatility of exchange rates has gained attention from researchers, as well as from policy makers and international business. In particular, the exchange rate risks involved from investing in a foreign currency are closely connected to the volatility of the exchange rate. Foreign currency denominated assets are subject to the risk that the exchange rate moves in an unfavorable direction and reduces the domestic currency value of the asset, if it is not hedged properly. At the same time, a currency mismatch resulting from borrowing in a foreign currency, while lending in domestic currency can lead to instability of banks and the financial sector across borders in general. In recent reports, Borio et al. (2017, 2022) show that at least since the Global Financial Crisis, non-US banks and other financial institutions have started to borrow excessively US dollars through FX markets instead of money and capital markets. In particular, these entities enter FX swap, outright forward and currency swap agreements to satisfy their US dollar hedging and funding demands. Although technically FX and currency swaps or combined spot and forward transactions are equivalent to borrowing a foreign currency while putting domestic currency as collateral, the full notional amounts determined in these agreements usually do not appear in the balance sheets of the institutions involved due to accounting conventions. According to Borio et al. (2022, p. 71) this "missing debt" entails incalculable risks because of the difficulty to assess its scale and geographical distribution. In an attempt to quantify it, the authors state that about 70% of the FX swap turnover is in fact based on short-term (i.e. less than one week to maturity) contracts, giving rise to maturity mismatch concerns, on top of potential currency mismatch issues related to the US dollar funding and domestic currency lending of non-US banks (Borio et al., 2022, p. 69). Consequently, in times of financial market distress and heightened exchange rate volatility, reduced liquidity and increased dollar hedging demands could make the non-US banks involved vulnerable to funding liquidity squeezes and thereby also cause problems for their contract partners abroad. In order to prevent this kind of financial distress, it is not only necessary to measure the exposure of financial systems to these risks and to prepare an appropriate policy response once it sets off, but also to understand the drivers of potential triggers, such as exchange rate volatility.

The rest of the chapter is structured as follows: Section 2.2 summarizes previous theoretical and empirical findings on the relation between FX trading activity and exchange rate volatility. After that, section 2.3 gives an overview of the data used, especially with regard to the measurement of large players' FCPs and the transformations conducted to infer trading activity. Section 2.4 introduces the HAR-RV framework to analyze the trading activity-volume relation and section 2.5 presents estimation results for different specifications of the relation. Sections 2.6 summarizes and concludes the findings and their implications.

2.2 Volatility, trading activity and volume

Theory on volume-volatility relation: There is a vast literature addressing the relation between trading activity and price volatility of financial assets from empirical and theoretical perspectives. Giot et al. (2010) give a comprehensive overview over the different model types that relate trading activity to return volatility. They distinguish three classes of theoretical models, namely competitive microstructure, strategic microstructure and mixture-of-distributions hypothesis (MDH) models.

Representatives of the first class are given by Easley and O'Hara (1987), Shalen (1993) and Harris and Raviv (1993). The latter two studies introduce the dispersion-of-beliefs hypothesis (DOBH). Their models emphasize that the interpretation of information may differ across groups of traders. Accordingly, the positive relation between trading activity and price changes is a result of relative heterogeneity in the interpretation of and reaction to new information in one group of traders, in comparison to another group. While informed traders with relatively homogeneous beliefs trade within a narrow range of prices, uninformed traders with more heterogeneous beliefs tend to overreact to signals, causing exaggerated price movements and persistence in trading activity. Other models that fall under the category of competitive microstructure models focus on the role of noise and chartist traders for short-term asset volatility (i.a. Bauer and Herz, 2004; De Long et al., 1990).

Kyle (1985) and Admati and Pfleiderer (1988) give examples of strategic microstructure models. They emphasize the importance of asymmetric information and market makers for asset price formation. A relation between the number of trades or order imbalances and return volatility results e.g. from market makers' learning from order imbalances about asset values and adjusting prices accordingly.

The mixture-of-distributions hypothesis (MDH) is established by Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983) and Harris (1986). It states that asset returns are drawn from a mixture of distributions with differing variances, where a latent information flow constitutes the mixture variable. In the canonical MDH model by Tauchen and Pitts (1983), the arrival rate of information per period of time is a random variable. Every arrival of new information in the market leads heterogeneous market participants to adjust their reservation prices and re-balance their positions. Consequently, prices adjust and the associated trading activity creates volume. Tauchen and Pitts (1983) show that with the number of traders assumed fixed, trading volume and price change variance are proportional to the number of news arriving per period of time, creating contemporaneous covariance between volume and asset return volatility. In later contributions, the model's assumptions have been modified in various ways to account for other factors that might explain the volume-volatility relation, e.g. Andersen (1996) extends the MDH by introducing a liquidity component.

A common criticism of MDH is that it does not explicitly provide insights into the behavior of economic agents and how it affects the volume-volatility correlation. Rather, it provides a mere statistical explanation of why and how trading activity variables (e.g. volume) and price volatility covary, depending on the information flow process (Jones et al., 1994, p. 645; Chen and Daigler, 2008, p. 964). Hence, alternative and complementary explanations for the relation have been developed. Copeland (1976) is among the first to propose the sequential-arrival-of-information hypothesis (SAIH). The hypothesis asserts that not all market participants learn about new information at the same time, but sequentially, which causes a lead-lag relationship between trading activity and price changes. An early survey of theoretical and empirical approaches relating to MDH and SAIH is given in Karpoff (1987). More recently, Chen and Daigler (2008) summarize the theoretical literature about MDH, SAIH, DOBH and noise trader models, while arguing empirically for the hypotheses to be complementary rather than mutually exclusive. Unlike the other model classes, an empirical analysis of MDH does not require the distinction of trading activity variables between different types of traders. Since the FCP data used in this study does not allow for such a distinction, for the most part of my analysis, MDH constitutes the benchmark to evaluate and interpret my results.

Empirical literature on trading activity-volatility relation: In the empirical literature, the trading activity-volatility relation has been addressed using various methods and for a number of different asset classes. Stocks and futures are the most researched asset classes, because they are exchange traded and trading activity data such as volume, open interest and even individual trading positions is readily available. Generally, there is a consensus that there is a significant positive contemporaneous relation between trading volume and asset return volatility, as predicted by MDH (Schwert, 1989; Lamoureux and Lastrapes, 1990; Gallant et al., 1992; Hiemstra and Jones, 1994; Daigler and Wiley, 1999; Vlastakis and Markellos, 2012). The FX market, however, is less researched in that regard, mainly due to its opacity and decentralized structure and the consequent lack of reliable data for trading activity (cf. Cespa et al., 2022; Schrimpf and Sushko, 2019b).

Nevertheless, there are attempts to empirically measure trading activity in FX markets and to test for its association with spot exchange rate volatility, e.g. through FX futures trading volume and open interest. In an early contribution, Grammatikos and Saunders (1986) investigate the relation between different volatility measures of daily FX futures returns and their trading volume. In correlation and Granger-causality analyses they find a strong contemporaneous and in some cases also sequential correlation across different foreign currencies, supporting predictions from MDH and SAIH. Bessembinder and Seguin (1993) use an iterated estimation method for conditional volatility proposed by Schwert (1989) to examine how trading volume and open interest of futures for different assets such as stocks, bonds and currencies are associated with return volatility. As in previous studies for other asset classes, they find that the volatility of FX futures returns is contemporaneously positively related to volume and that unexpected volume shocks tend to have a larger impact than expected changes. These findings are consistent with MDH. Additionally, their results suggest that the relation between trading activity variables and volatility is asymmetric in that (unexpected) positive volume changes are associated with a stronger positive impact on volatility than negative ones. Similarly, Jorion (1996) analyzes the relation between options implied FX futures volatility and FX futures trading volume using time series regression techniques and finds that the strongest association exists between unexpected volume shocks and volatility. Further studies confirm these findings with FX futures, while employing other methods such as different GARCH-X specifications and VAR models (Wang, 2002; Bhargava and Malhotra, 2007; Mougoué and Aggarwal, 2011; Floros and Salvador, 2016).

Apart from using FX futures to proxy currency trading activity, in few cases it is also directly observable. For example, Hartmann (1999) uses the daily volume of the yen-US dollar currency pair as reported in a financial newspaper for currency transactions in Tokyo. Galati (2000) collects data from central banks of seven emerging markets and constructs a data set containing the daily local turnover of domestic currencies against the US dollar. Using a GARCH approach, she confirms findings from futures markets that especially unexpected trading volumes are positively related to spot volatility. She adds, however, that this finding only holds in non-crisis times, while in crisis periods the relation turns negative. Bjønnes et al. (2005) use trading volume data for transactions of Swedish krona against all other currencies as reported by primary dealer banks in Sweden. Since a majority of trades of Swedish krona is against the euro, they focus in the krona-euro exchange rate. Similar to Bessembinder and Seguin (1993) and Jorion (1996), they confirm that unexpected volume is a major driver of the volume-volatility relation in FX markets. More recently, Cespa et al. (2022) employ high frequency FX volume data for 31 currencies provided by CLS Group, which is the world's largest FX settlement institution (Cespa et al., 2022, p. 2387). Using panel regressions, they also find evidence for the positive correlation in their extensive new data set.

Other attempts to test predictions of MDH in FX markets are made not through measures of trading activity, but using other proxies for the information arrival process. For example, Bollerslev and Domowitz (1993) and Bauwens et al. (2006) use the frequency of new bid and ask quotes to measure how often new information arrives in the market. While the former do not find a significant relation between the quotation frequency and return volatility in the Deutsche mark-US dollar market, the latter do find evidence in the Norwegian krona-euro market. Apart from the currency pairs, one major difference between the two studies is their frequency: While Bollerslev and Domowitz (1993) analyze the data at an intra-daily frequency, Bauwens et al. (2006) aggregate their data to weekly observations. An excessive level of noise in FX returns data might prevent finding a relation at higher frequencies. Other examples for measuring information flow or trading activity are news headlines, e.g. from financial market news media such as "Reuters Money Market Headline News" (Melvin and Yin, 2000) or transaction and order flow data (Lyons, 2001).

To summarize, although the measurement of information arrival and trading activity in FX markets is more difficult than in exchange based asset markets, empirical results indicate that general predictions from information theories on the relation between trading activity and volatility (such as MDH) seem to hold as well in markets for foreign currencies, even when using substitute measures for trading volume and information flow.

2.3 Foreign currency positions data and exchange rates

Treasury foreign currency reports: Foreign currency position (FCP) data is collected through *Treasury Foreign Currency Reports* published by the US Treasury Department in the *Foreign Currency Positions* section of the quarterly *Treasury Bulletin* (U.S. Department of the Treasury, nd).[1] Reporting started in January 1994 for five currencies, namely Canadian dollar (CAD), Japanese yen (JPY), Swiss francs (CHF), British pound (GBP) and German mark (DEM). In 1999, euro (EUR) and US dollar (USD) were added and DEM was removed from the reports in 2002. The data comes in three frequencies, depending on the reporting institution and reported positions: Weekly and monthly reports are filed by "major US based FX market participants" holding more than $50 billion equivalent in FX contracts on the last business day of any calendar quarter during the previous year (Federal Reserve Bank of New York, 2023). The quarterly report addresses institutions that hold more than $5 billion, but less than or equal to $50 billion on any of the previously mentioned dates. The reports also differ in what is reported. For instance, while weekly reports only require a statement about net options positions by currency, monthly and quarterly reports must contain detailed information on which types of options (call or put) are bought or written. Reference date for weekly reports is Wednesday, while monthly and quarterly reports refer to the last business day in a month or a calendar quarter respectively.

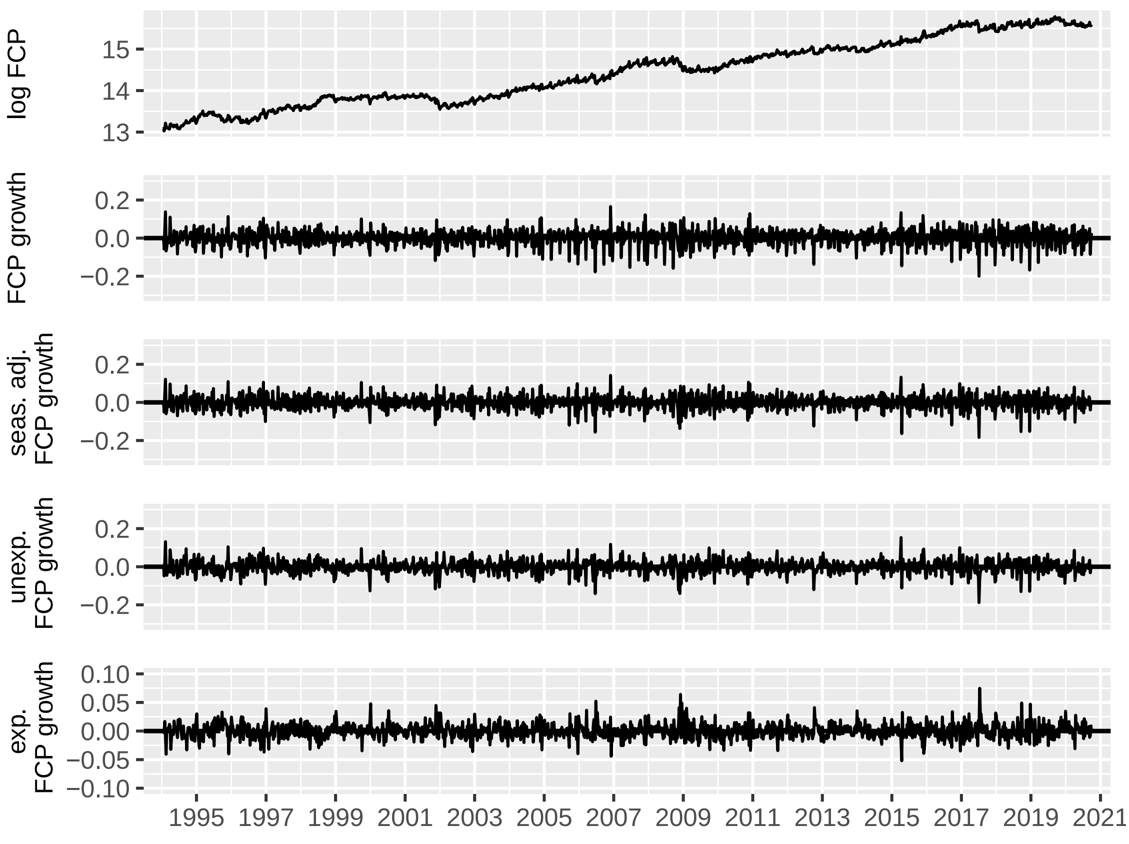
Spot, forward, futures and swap positions: I focus on the foreign currencies CAD, JPY, CHF, GBP and EUR. The DEM is not considered, because its observation period is rather short. For the first four, data is available from January 1994 to September 2020. In case of EUR, the data series starts in January 1999. For each currency, the *Foreign Currency Positions*-section in the *Quarterly Bulletin* shows two separate weekly series (long and short) for aggregated open spot, forward and futures (and swap) positions (SFF hereafter). These positions measure the aggregated nominal amount of a specified foreign currency that large market participants have contracted to receive and deliver through spot, forward, futures or swap agreements, irrespective of maturity. Weekly changes in SFF positions can be interpreted as a proxy for large market participants' foreign currency trading activity, similar to trading volume as used in previous studies. An increase in the weekly open long FCPs reflects that the notional amount of purchased foreign currency was larger than the notional amount of contracts that expired during that time and vice versa. Similarly, an increase in open short positions reflects that the notional amount of foreign currency sold was larger than that of short contracts that expired. Both situations coincide with higher relative trading activity in the respective foreign currency in the form of purchases or sales by large market participants. Hence, I use the sum instead of the two individual series as my main variables of interest in order to capture any type of trading activity, regardless of the direction of trades. In line

[1] I downloaded digital PDF copies of the *Treasury Bulletin* covering issues from September 1994 to December 2020 from the economic digital library FRASER (Federal Reserve Bank of St. Louis, 2024). For converting the PDF tables that include the data into CSV files, I used the program *tabula v.1.2.1* (Aristarán, Manuel and Tigas, Mike and Merrill, Jeremy B., 2024).

with MDH, a higher rate of information arrival in the market is expressed in higher FCP growth rates while less new information coincides with lower growth rates. From theory and previous empirical studies it follows that FCP growth rates should correlate positively and potentially non-linearly with FX volatility.

Large traders: One caveat to the suitability of the FCP data for this interpretation is that it only considers "large" market participants, defined by the amount of open FCPs on and off the balance sheet, while smaller participants are not taken into account. Due to the specific decentralized structure of the FX market, however, large market participants play a particularly important role as market makers and dealers. Therefore, changes in FCPs are still a good proxy for the overall trading in FX markets. In contrast to exchange-based stock markets, for example, over-the-counter FX markets need dealer banks and other large institutions to provide liquidity and broker deals for small participants. Hence, even though the FCP data cover only the largest US-based players, due to the fact that most of the time small participants trade foreign currencies through dealer banks, to some extent, the data reflect trades initiated by small players. There is a growing literature documenting and explaining the importance and influence of these types of players in the FX markets (cf. Gabaix and Maggiori, 2015; Schrimpf and Sushko, 2019b; Reitz and Umlandt, 2021).

Figure 2.1: Seasonal adjustment and decomposition of FCPs in GBP.



Non-stationarity and seasonality: The log of the sum of long and short SFF series for GBP is shown in the top panel of Figure 2.1. The plot indicates that the data might not only be non-stationary, but also exhibit a seasonal pattern. The focus of my analysis is in changes of FCPs in order to measure trading activity, so I calculate weekly log differences to get a series of (log-)growth rates for FCPs. Augmented Dickey-Fuller tests show that the growth rate series are stationary for all currencies. The second panel of Figure 2.1 plots the growth rates for GBP over time. Here the seasonality becomes even more apparent. In fact, analysis of autocorrelation and partial autocorrelation functions of FCP growth rates (Appendix Figures A.1 and A.2) indicates, that the data exhibit a quarterly recurring pattern. In particular, while growth rates are more or less constantly positive over the course of a calendar quarter, they contract sharply on the third Wednesday in the third month of a quarter. This pattern is found across currencies and over the whole observation period, with varying intensity. A possible explanation for the pattern is that these dates coincide with conventionally agreed-on maturity dates of a large fraction of exchange and over-the-counter traded financial derivatives, such as options, futures and swaps. These so called *International Monetary Market* (*IMM*) dates constitute by convention the maturity dates of particular standardized futures and options contracts traded at the Chicago Mercantile Exchange (CME) (CME Group, nd). Since FX derivatives such as forwards and swaps can be used to hedge FX risks of assets denominated in foreign currency, turbulence in asset markets directly affected by IMM dates can be expected to spill over to FX derivatives markets and cause this kind of sharp contraction in SFF positions.

Seasonal adjustment: In order to prevent seasonal effects from influencing estimation results in any undesirable way, I de-seasonalize FCP growth rates prior to using the series in my regression analysis. I find that a $\text{SARMA}(0,0)\,(4,0)_{13}$ of the log-growth FCP series seems to capture seasonal effects for all currencies well. I use a periodicity of 13 because during my observation period, there are on average 13 Wednesdays in a calendar quarter. The estimated residuals are used as seasonally adjusted FCP growth series in all further analyses. For GBP, they are plotted in the third panel of Figure 2.1.

Expected and unexpected changes in trading activity: Among others, Bessembinder and Seguin (1993) show that the decomposition of the proxy variables of the trading activity into expected and unexpected (or innovation) components can shed more light on the relationship between trading activity and volatility. In their work, they decompose series of futures trading volume and open interest into two components by estimating linear ARMA models and using the fitted values as expected component and residuals as unexpected component. They find that their results are mainly driven by the unexpected component of the trading volume.[2] Apart from applying only seasonally adjusted FCP growth rates as trading activity proxies in my regression analysis, I also estimate alternative specifications with expected and unexpected components as separate regressors. In line with Bessembinder and Seguin (1993), I estimate $\text{ARMA}(0,10)$ models for the seasonally adjusted FCP growth rates for all currencies and save the fitted values and residuals as expected and unexpected components, respectively. The fourth panel in Figure 2.1 shows the unexpected component for GBP while the fifth shows the expected component.

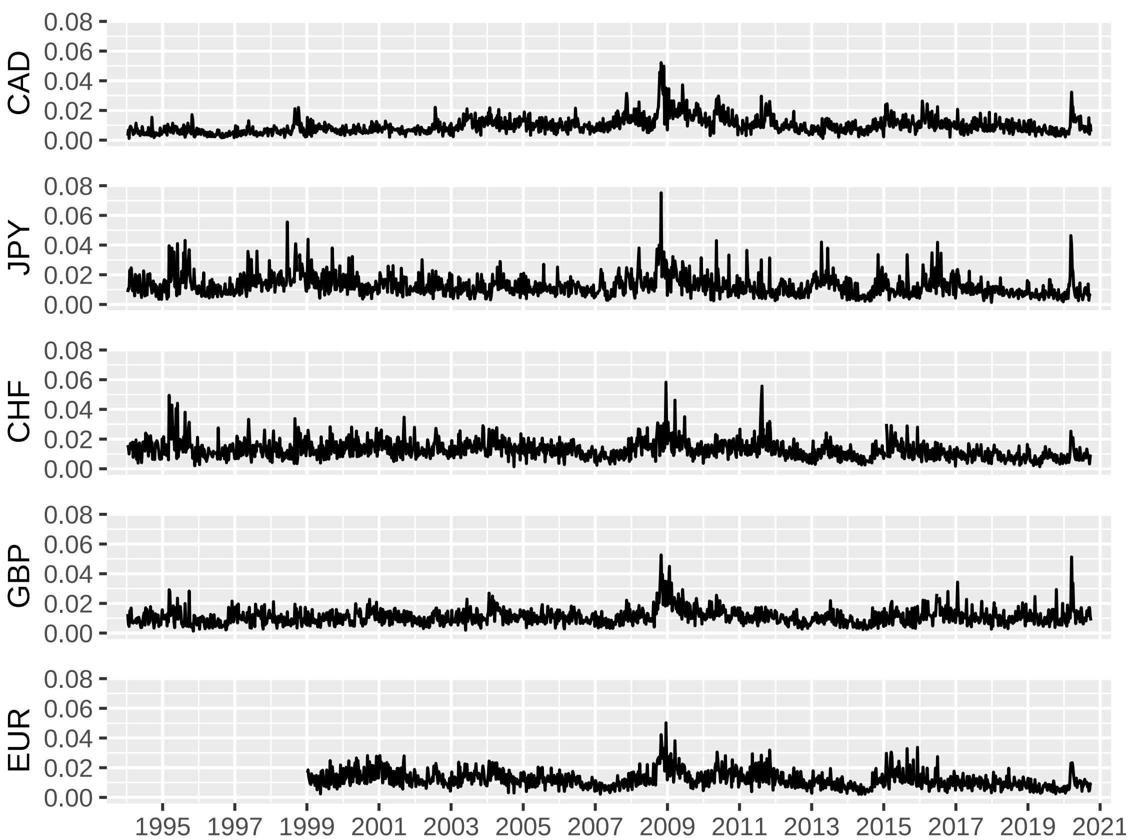
It is interesting to analyze the two components separately because they represent different forces driving changes in the FCPs. Unexpected changes may indicate the arrival of new information in the market, which causes large market participants to revise their expectations about the future and thus rebalance their positions in order to hedge risks or exploit new speculative opportunities. In line with MDH, the arrival of new information is the reason why there is a relation between trading activity and volatility. So, the unexpected component, which captures changes in trading activity due to new information, should show a strong correlation with the volatility of

[2] This type of decomposition can be interpreted as a specific form of the Hamilton filter (Hamilton, 2018), which decomposes a univariate time series into a trend and a cyclical component in the spirit of Hodrick and Prescott (1997).

the currency return. However, expected changes likely capture other drivers of FCP growth, that is, trends, undetected cyclical patterns, or lagged reactions to large changes in FCPs in previous periods caused by delayed settlements thereof. In another interpretation of MDH, Jorion (1996), Bjønnes et al. (2005) and Bauwens et al. (2006) argue that expected changes in proxy variables for trading activity may also reflect changes in the number of active traders in the market and therefore constitute an approximate measure of market liquidity.

Further studies using FCP data: Despite the fact, that the data from *Treasury Foreign Currency Reports* covers by now almost 30 years of weekly observations of large FX market participants' on- and off-balance sheet FCPs, it has not been used extensively in the previous literature. In fact, to the best of my knowledge, there are only four studies considering the data in their empirical analyses. Among the first to use it were Wei and Kim (1997), who proxy private information of large market participants with FCPs. Corsetti et al. (2002) employ the data to examine the relation between large players' net FCPs and the level of exchange rates. Goddard et al. (2015) show that investor attention is correlated with large FX market participants' trading activities, proxied by FCPs. The only study relating FCPs to exchange rate volatility is by Cai et al. (2001). They use yen positions in a setting to explain intra-daily FX volatility patterns. In contrast to my analysis however, they focus explicitly on the yen-US dollar exchange rate during the high-volatility year 1998 and use the data to proxy order flow caused by large market participants. Since none of the mentioned studies employs the FCP data to examine the exchange rate volatility and trading activity relation across different foreign currencies and for an extensive observation period, my analysis contributes to the literature by providing a new perspective on the relation in FX markets through the lens of large players' FCPs.

Figure 2.2: Weekly realized volatility.



Exchange rates and realized volatility: Daily spot exchange rates of CAD, JPY, CHF, GBP and EUR against the USD (in units of USD per unit of foreign currency) are downloaded via Refinitiv Eikon Datastream. In order to match daily exchange rate returns with weekly FCP growth, I calculate daily log growth rates and estimate weekly realized volatility for each exchange rate using

$$RV_t^{(W)} = \sqrt{\sum_{i=1}^{5} r_{t,i}^2} \tag{2.1}$$

where $r_{t,i}^2$ is the squared log-return on day i in week t. As is common in academic literature and practice, a five-working-day week is assumed to derive the weekly realized volatility. Since FCPs are usually reported on Wednesdays, my weekly measure of realized volatility starts on Thursdays and ends on Wednesdays. The log of weekly realized volatility as defined here constitutes the dependent variable in all of my main analyses. The realized volatility series for all currencies is plotted in Figure 2.2. An outlier is removed from the plot for CHF in order to have a better graphical representation of volatility dynamics over time.

It is interesting to note that volatility seems to be quite persistent in general, which in fact is a well known feature not only of foreign exchange returns, but financial asset returns in general (cf. Schwert, 1989). Moreover, as expected, in times of distress and financial crises, such as in the late 1990s and in the Global Financial Crisis 2007-2009, there is generally higher uncertainty and volatility across all currencies. Similarly, the Covid-19 pandemic-induced shock to international trade and financial markets is clearly visible in all five plots in the beginning of 2020.

Summary statistics and additional variables: Table 2.1 gives summary statistics for weekly log-returns of each currency, their weekly realized volatility, seasonally and not seasonally adjusted FCP growth rates, expected and unexpected FCP growth. The last four columns present the first four autocorrelations for each time series. Weekly realized volatility is similar in size across currencies, with CHF showing the highest mean volatility and CAD the lowest. Also, CHF exhibits the highest maximum volatility which is almost two time the second highest maximum (GBP). The reason behind the high mean and maximum volatility of the CHF is that the Swiss National Bank (SNB) started to enforce a minimum exchange rate of the CHF against the EUR on 6 September 2011 and resumed a free-floating exchange rate regime on 15 January 2015. Naturally, the initial enforcement as well as the abolition of the one-sided peg were accompanied by large jumps in the nominal value of CHF and thus with jumps in its volatility. Hence, the returns of the CHF have a relatively higher mean and maximum volatility than the other currencies. In the main dynamic regression below, the change in the Swiss exchange rate regime is not explicitly accounted for, to check the validity of the model in different regimes.[3]

Other interesting findings from summary statistics regard the dynamics of the time series. Although log returns do not exhibit substantial autocorrelation, realized volatility appears to be highly persistent. These figures confirm the previous visual findings and are in line with the literature that shows the existence of volatility clusters in asset price returns and long-memory properties of volatility of exchange rates (cf. Andersen and Bollerslev, 1997; Andersen et al., 2001). Moreover, note the strong negative first autocorrelation of the seasonally adjusted FCP growth rate. This

[3]Bauwens et al. (2006) analyze the relation between trading activity and volatility for the Norwegian krona in different exchange rate regimes and find that the positive correlation holds across different regimes. Nevertheless, in robustness checks, I estimate the benchmark regression for CHF including structural break interaction terms. The results are given in the Appendix Table A.1. They show that the results also hold during the one-sided peg period between 2011 and 2015.

Table 2.1: Summary statistics.

	mean (×100)	median (×100)	st.dev. (×100)	min	max	ac1	ac2	ac3	ac4
weekly log returns									
CAD	-0.0015	-0.0073	1.1165	-0.0608	0.0515	-0.0152	-0.0677	0.0197	-0.0243
JPY	0.0052	-0.0443	1.4235	-0.0698	0.1177	0.0033	0.0437	-0.0353	0.0119
CHF	0.0337	-0.0133	1.4840	-0.0845	0.1692	-0.0066	-0.0364	0.0089	-0.0316
GBP	-0.0110	0.0175	1.2460	-0.0989	0.0500	-0.0095	-0.0656	0.0381	-0.0171
EUR	-0.0003	0.0120	1.3251	-0.0492	0.1011	0.0204	-0.0333	0.0525	-0.0370
weekly RV									
CAD	0.9735	0.8559	0.5677	0.0011	0.0522	0.6059	0.6009	0.5908	0.5568
JPY	1.2868	1.1550	0.7414	0.0014	0.0773	0.3788	0.3413	0.3255	0.3155
CHF	1.3062	1.1871	0.7972	0.0014	0.1742	0.2627	0.2194	0.1964	0.2021
GBP	1.1072	1.0144	0.5856	0.0013	0.0922	0.3896	0.3644	0.3398	0.2966
EUR	1.2218	1.1130	0.5835	0.0021	0.0503	0.4217	0.3962	0.4203	0.4171
weekly FCP growth									
CAD	0.1812	0.8071	4.9182	-0.3213	0.2652	-0.1330	-0.0696	-0.0946	-0.0451
JPY	0.1224	0.3361	4.2106	-0.2670	0.2249	-0.2825	-0.0199	-0.0727	0.0012
CHF	0.0503	0.6848	6.2764	-0.3270	0.3542	-0.1836	-0.0417	-0.1231	0.0123
GBP	0.1810	0.5630	4.2070	-0.2000	0.1654	-0.2642	0.0426	-0.1788	0.0374
EUR	0.1256	0.5013	4.1492	-0.2800	0.1845	-0.3112	0.1200	-0.2110	0.0570
weekly FCP growth (seasonally adjusted)									
CAD	0.0267	0.0504	3.8172	-0.2452	0.2802	-0.2517	-0.0244	-0.0001	-0.0156
JPY	0.0557	0.1550	3.9721	-0.2701	0.2261	-0.3315	-0.0015	-0.0285	-0.0241
CHF	0.0035	0.0677	5.2154	-0.2880	0.3114	-0.2910	0.0189	-0.0617	0.0187
GBP	0.0565	0.1333	3.7114	-0.1837	0.1418	-0.3249	0.0505	-0.1184	0.0227
EUR	0.0378	0.1472	3.6100	-0.2632	0.1540	-0.3608	0.1240	-0.1105	0.0335
weekly expected FCP growth									
CAD	0.0267	0.0293	1.0563	-0.0755	0.0601	0.0906	0.0679	0.0619	0.0179
JPY	0.0542	0.0497	1.5293	-0.0544	0.1069	-0.0312	0.0019	0.0535	-0.1741
CHF	0.0027	-0.0220	1.6604	-0.1051	0.0867	-0.0341	0.1638	-0.0849	-0.0950
GBP	0.0558	0.0447	1.3380	-0.0519	0.0748	-0.0666	0.3186	-0.0636	-0.0163
EUR	0.0400	0.0091	1.3565	-0.0489	0.0971	-0.3350	0.2991	-0.1027	-0.0559
weekly unexpected FCP growth									
CAD	0.0000	-0.0092	3.6683	-0.2253	0.2739	-0.0011	0.0015	0.0006	0.0044
JPY	0.0015	0.0288	3.6612	-0.2553	0.1508	0.0003	-0.0008	-0.0015	0.0004
CHF	0.0008	0.0408	4.9454	-0.2626	0.2795	0.0001	-0.0004	0.0010	0.0010
GBP	0.0006	0.0876	3.4565	-0.1881	0.1532	-0.0018	0.0013	-0.0015	0.0019
EUR	-0.0022	0.1329	3.3353	-0.2499	0.1233	-0.0040	0.0052	-0.0029	0.0015

finding is expected due to the way that open FCPs are included in the *Treasury Foreign Currency Reports*: Since both long- and short-positions enter open SFF positions positively, negative changes in FCPs are almost exclusively caused by settlements of previously signed long or short contracts. Any newly signed contract (either long or short) with a maturity that exceeds the next reporting date will initially appear as an increase in FCPs. Once the contract settles, its nominal amount of currency purchased or sold will disappear from the FCPs again, leading to its decrease.[4] Therefore, initial increases in FCPs are usually followed by delayed decreases, causing a measurable negative autocorrelation in FCP growth rates. The fact that only the first-order autocorrelation is strongly negative can be explained using data from the recent BIS Triennial Survey 2022. The survey results show that about 70% of the FX turnover occurs in spot, outright forwards and

[4] The only exception from this mechanism are FX futures contracts. They are reported on a net basis, so a decrease can also be caused by netting open positions, instead of settlements.

FX swaps with a maturity of no more than seven days (Bank for International Settlements, 2022, p. 10). Hence, a large proportion of open FCPs settle within one week after contracts are signed. As a consequence, these contracts do not appear anymore in subsequent SFF positions as reported by the *Treasury Foreign Currency Reports*, causing a strong negative first-order autocorrelation in weekly FCP growth rates.

2.4 Heterogeneous autoregressive model of realized volatility

Corsi (2009) proposes a parsimonious model of volatility of financial asset returns consisting of volatility components measured over different time horizons. Based on the empirical finding that asset return volatility measured at low frequency tends to influence high-frequency volatility more than conversely (i.a. Müller et al., 1997), Corsi (2009) motivates his approach by the idea that traders in financial markets are heterogeneous in terms of trading frequencies. While short-term traders such as dealers, market makers and speculators usually trade at very high (intra-daily or daily) frequencies, long-term traders such as insurance companies, pension funds and other institutional investors trade at much lower (weekly, monthly, or even lower) frequencies (Corsi, 2009, p. 178). He argues that for short-term traders, volatility of financial returns measured at low (i.e. weekly, monthly, quarterly) frequencies is relevant as it indicates expected future trends and risks and induces them to trade and consequently cause volatility at high frequencies. However, high-frequency volatility is less relevant for long-term trading strategies. This reasoning leads to "volatility cascades" from low frequencies to high frequencies (Corsi, 2009, p. 179). In his modeling approach, Corsi (2009) assumes that the unobserved partial volatility at a given frequency is a function of the past volatility realized on the same time scale and the expected longer-term volatility for the next period. On a daily time scale, the following representation of the realized volatility is derived.

$$RV_t^{(D)} = c + \beta^{(D)} RV_{t-1}^{(D)} + \beta^{(W)} RV_{t-1}^{(W)} + \beta^{(M)} RV_{t-1}^{(M)} + \varepsilon_t \tag{2.2}$$

where $RV_t^{(D)}$ is the daily realized volatility as estimated by

$$RV_t^{(D)} = \sqrt{\sum_{i=1}^{M} r_{t,i}^2} \tag{2.3}$$

and $r_{t,i}^2$ is the squared return on day t in the intra-daily (e.g. hourly with $M = 24$) interval i. The lower frequency volatility components $RV_t^{(W)}$ and $RV_t^{(M)}$ are realized weekly and monthly volatility as estimated by[5]

$$RV_t^{(W)} = \frac{1}{5} \sum_{j=0}^{4} RV_{t-j}^{(D)} \tag{2.4}$$

$$RV_t^{(M)} = \frac{1}{22} \sum_{j=0}^{21} RV_{t-j}^{(D)}. \tag{2.5}$$

[5] Corsi (2009, p. 177) states that due to "Jensen's inequality, the aggregated volatility, as defined here, cannot be exactly interpreted as the realized volatility over the specific time interval." However, in empirical applications the differences are negligible.

The regression equation (2.2) can be estimated via OLS. Using simulations, Corsi (2009) demonstrates that his heterogeneous autoregressive model of realized volatility (HAR-RV) is able to parsimoniously describe important stylized empirical facts of return volatility, such as long memory of squared and absolute returns and fat tails of return distributions (Andersen et al., 2000, 2001). Moreover, its volatility forecasting performance at short horizons appears to be at least on par with various empirical models up to ARFIMA models. The versatility of HAR-RV is demonstrated not least by the fact that a large number of extensions have been developed, e.g. to include jump components (Andersen et al., 2007), leverage effects (Patton and Sheppard, 2015) and structural breaks (Ma et al., 2015).

The HAR-RV approach is well suited to analyze the relationship between FCPs and the volatility of the return of the FX. Not only does its simple structure allow for an extension by further exogenous factors, such as growth rates of FCPs, but its empirical representation of "volatility cascades" fits the reasoning of a relation implied by MDH. As Corsi (2009, p. 179) states, low-frequency volatility likely affects high-frequency volatility through short-term traders, who infer expected future trends and risks from it. Hence, to measure the relation between trading activity due to the arrival of new fundamental information in the market, one needs to control for volatility caused by short-term traders who adapt to changes in long-term volatility. Accordingly, using OLS, for each currency, I estimate the following regression equation:

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j\in K} \beta^{(j)} \log\left(RV_{t-1}^{(j)}\right) + \sum_{j=1}^{L} \gamma_j z_\tau + \sum_{j=1}^{P} \delta_j x_\tau + \varepsilon_t \tag{2.6}$$

where $K = \{W, M, Q, Y\}$ represents the frequency (i.e. weekly, monthly, etc.) of the realized volatility measure.

Compared to equation (2.2), some features of HAR-RV are modified: First, the realized volatility measures enter the regression in natural logs. I use the natural logs of the realized volatilities in order to mitigate effects of single large outliers in volatility and avoid issues resulting from skewness of the distribution variance data (cf. Taylor, 2008, Ch. 3; Bauwens et al., 2006). Second, due to the frequency of the FCP data, the highest volatility frequency considered is weekly instead of daily. Third, control variables z_τ and different measures of trading activity x_τ (described below) are added as exogenous regressors. The weekly realized volatility is estimated as in equation (2.1), while the lower frequencies considered in the regression are monthly (M), quarterly (Q) and yearly (Y), which are estimated by

$$\log\left(RV_t^{(M)}\right) = \frac{1}{4}\sum_{j=0}^{3} \log\left(RV_{t-j}^{(W)}\right) \tag{2.7}$$

$$\log\left(RV_t^{(Q)}\right) = \frac{1}{12}\sum_{j=0}^{11} \log\left(RV_{t-j}^{(W)}\right) \tag{2.8}$$

$$\log\left(RV_t^{(Y)}\right) = \frac{1}{52}\sum_{j=0}^{51} \log\left(RV_{t-j}^{(W)}\right). \tag{2.9}$$

As control variables, I include the one-period lagged trend defined as the previous week's average FX return, as well as the CBOE Volatility Index (VIX) in all regressions. A trend proxy as defined here is included because Bauer and Herz (2004) show theoretically and empirically in a noise trading model that trend is a powerful predictor of the volatility of the exchange rate return. The VIX volatility index measures the expected volatility of the US stock market as derived from stock market index options (Cboe Exchange, nd), but has also been used as a more general indicator of global risk aversion (e.g. Bekaert et al., 2013). I include VIX to capture uncertainty and volatility from other markets than the FX market that might spill over to FX markets and affect traders behavior at the same time. The regression is estimated using different specifications and transformations of FCPs, which are described below.

2.5 Results

2.5.1 Linear contemporaneous and predictive regressions

Linear contemporaneous regression: In specification (2.10), the growth of the FCP enters linearly using the seasonally adjusted weekly (log-)growth rate Δfcp_t as a regressor. Seasonal adjustment using SARMA is described in section 2.3.

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j\in K} \beta^{(j)} \log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_t + \delta\Delta fcp_t + \varepsilon_t. \tag{2.10}$$

This regression serves as a benchmark to evaluate the contemporaneous relationship between short-term realized volatility and FCP growth rates, while controlling for volatility cascades and other influences such as trend and stock market volatility. If growth rates in FCPs are indeed a proxy for the rate of information arrival as proposed by MDH, then the coefficient δ is expected to be significantly positive across currencies. The parameters are estimated with OLS and standard errors using the Newey-West estimator (Newey and West, 1987, 1994). The estimation results for the regression equation (2.10) are presented in Table 2.2, columns (1) through (5).

The results confirm the predictions by MDH and previous findings in the empirical literature on stock and futures markets' volume-volatility relation. The estimates of the coefficients for Δfcp_t are positive and significant for the five regressions. The greatest impact is measured for JPY with a point estimate of approximately 2.58, while in the other four regressions, the coefficients range from 1.18 to approximately 1.83. These estimation results can be interpreted as an indication for MDH to also apply in FX markets, even when changes in large traders' FX positions are used as an information flow proxy instead of more traditional variables such as total volume, open interest, order flow or imbalances. Interestingly, with the exception of $trend_t$, the point estimates for all other variables in the regression are comparable in size between currencies: The coefficients for vix_t are highly significant and positive, with only little variation between currencies. This result indicates the potential existence of volatility spillovers between US stock markets and FX markets (cf. Diebold and Yilmaz, 2012).

Moreover, the impact of monthly realized volatility on weekly volatility is also highly significant and positive for all but one currency. The same holds for quarterly realized volatility. Yearly and lagged weekly realized volatility seem to be less important for explaining weekly volatility, as their coefficients are barely significant. These findings support the relevance of incorporating lower frequency volatility measures to account for potential volatility cascades. The only variable departing from estimation patterns found across currencies is $trend_t$. The coefficients are negative but insignificant for CAD, GBP and EUR, while they are strongly positive and significant for JPY and CHF. Although the specifications estimated in this study do not allow for a more thorough investigation of this finding, one conjecture to explain it might relate to JPY and CHF (besides the USD) taking the role as safe haven currencies in international financial markets. For example, Ranaldo and Söderlind (2010) find that especially CHF and JPY appreciate against the USD, when the FX volatility increases.

Table 2.2: Estimation results for the linear contemporaneous and predictive regressions (2.10) and (2.11).

	$\log RV_t^{(W)}$									
	CAD	JPY	CHF	GBP	EUR	CAD	JPY	CHF	GBP	EUR
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	-0.675^{***}	-1.218^{***}	-1.177^{***}	-1.524^{***}	-1.250^{***}	-0.606^{***}	-1.053^{***}	-1.066^{***}	-1.486^{***}	-1.090^{***}
	(0.166)	(0.281)	(0.292)	(0.257)	(0.281)	(0.168)	(0.292)	(0.309)	(0.260)	(0.296)
$\log RV_{t-1}^{(W)}$	-0.007	0.081^{**}	0.036	0.037	0.024	-0.018	0.036	0.018	0.029	0.008
	(0.039)	(0.034)	(0.033)	(0.031)	(0.035)	(0.040)	(0.033)	(0.035)	(0.032)	(0.037)
$\log RV_{t-1}^{(M)}$	0.455^{***}	0.365^{***}	0.134	0.244^{***}	0.187^{**}	0.475^{***}	0.406^{***}	0.154^{*}	0.240^{***}	0.222^{**}
	(0.080)	(0.071)	(0.086)	(0.064)	(0.085)	(0.081)	(0.072)	(0.087)	(0.064)	(0.086)
$\log RV_{t-1}^{(Q)}$	0.140	0.235^{**}	0.518^{***}	0.354^{***}	0.508^{***}	0.142	0.239^{**}	0.522^{***}	0.370^{***}	0.519^{***}
	(0.092)	(0.097)	(0.103)	(0.077)	(0.111)	(0.093)	(0.100)	(0.102)	(0.077)	(0.110)
$\log RV_{t-1}^{(Y)}$	0.309^{***}	0.081	0.083	0.074	0.044	0.303^{***}	0.102	0.093	0.070	0.037
	(0.066)	(0.082)	(0.083)	(0.066)	(0.078)	(0.065)	(0.082)	(0.081)	(0.066)	(0.075)
$trend_{t-1}$	-8.336^{*}	16.213^{***}	9.626^{**}	-3.634	-6.117	-10.708^{**}	14.247^{***}	9.342^{**}	-4.145	-6.566
	(4.721)	(4.418)	(4.014)	(4.109)	(4.107)	(5.065)	(4.459)	(3.961)	(4.200)	(4.175)
vix_t	0.009^{***}	0.007^{***}	0.008^{***}	0.009^{***}	0.009^{***}					
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)					
Δfcp_t	1.380^{***}	2.576^{***}	1.176^{***}	1.831^{***}	1.355^{***}					
	(0.296)	(0.357)	(0.257)	(0.331)	(0.314)					
vix_{t-1}						0.007^{***}	0.004^{**}	0.005^{***}	0.007^{***}	0.006^{***}
						(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Δfcp_{t-1}						0.337	0.647^{**}	0.304	-0.092	0.399
						(0.297)	(0.306)	(0.222)	(0.294)	(0.317)
N	1,393	1,392	1,393	1,393	1,132	1,393	1,392	1,393	1,393	1,132
R^2	0.473	0.307	0.275	0.252	0.369	0.457	0.266	0.253	0.224	0.350
Adjusted R^2	0.471	0.304	0.271	0.248	0.365	0.454	0.262	0.250	0.220	0.346

Notes: ***Significant at 1% level; **Significant at 5% level; *Significant at 10% level. Newey-West standard errors in parentheses.

Since all regressions are estimated with the log of weekly realized volatility as dependent variable, the coefficient estimates can be interpreted in terms of a percentage change of volatility for a unit change in FCP growth. For example, an increase in FCPs in CAD by one unit (that is, 100%) is associated with an increase in volatility by 138% and for JPY it increases by 258%. These estimates indicate that the relationship is not only statistically significant, but also economically significant. However, a unit increase, i.e. a doubling of FCPs is an unrealistic assumption to assess the economic relation between trading activity and volatility. A more intuitive and standardized approach is to consider, for example, a standard deviation shock in FCPs. The standard deviations, together with the values of the estimated regression parameters and the predicted effects on realized volatility are reported in Table 2.3. In the case of CAD in regression (2.10), for example, a one-standard deviation increase in FCPs of approximately 3.82% is associated with an increase in weekly realized volatility of approximately $1.38 \cdot 3.82\% \approx 5.27\%$. The effects for the other currencies are of similar magnitudes, i.e. between 4.8% and 10.24%. These results substantiate the economic relevance of the relation. Although 5% or 10% increases in exchange rate volatility appear minor, they can entail large shifts in the portfolios of international financial market actors in an attempt to hedge the additional risk. This becomes even more apparent in light of recent developments with regard to the enormous growth in off-balance sheet US dollar debt positions outside the US through FX swaps, forwards and currency swaps (Borio et al., 2017, 2022).

Table 2.3: Predicted effect of a one standard deviation shock in trading activity variable to weekly realized volatility in %.

	St.Dev.	Par.Est.	Effect		St.Dev.	Par.Est.	Effect
Model (2.10): Δfcp_t				**Model (2.13):** Δfcp_t^+			
CAD	3.8183	1.3798	5.2683	CAD	3.8183	2.0180	7.7055
JPY	3.9723	2.5757	10.2313	JPY	3.9723	4.8385	19.2199
CHF	5.2168	1.1761	6.1355	CHF	5.2168	2.1629	11.2834
GBP	3.7126	1.8306	6.7963	GBP	3.7126	3.9590	14.6985
EUR	3.6111	1.3546	4.8914	EUR	3.6111	2.1841	7.8870
Model (2.11): Δfcp_{t-1}				**Model (2.13):** Δfcp_t^-			
CAD	3.8181	0.3374	1.2882	CAD	3.8183	0.8090	3.0890
JPY	3.9729	0.6470	2.5705	JPY	3.9723	0.5743	2.2814
CHF	5.2162	0.3040	1.5860	CHF	5.2168	0.3006	1.5683
GBP	3.7126	-0.0924	-0.3431	GBP	3.7126	-0.1087	-0.4037
EUR	3.6105	0.3986	1.4392	EUR	3.6111	0.6827	2.4653
Model (2.12): $expc_t$				**Model (2.18):** $unex_t^+$			
CAD	1.0566	-2.0290	-2.1439	CAD	3.6694	2.5005	9.1752
JPY	1.5299	-1.6712	-2.5568	JPY	3.6614	5.6644	20.7395
CHF	1.6610	-0.9132	-1.5169	CHF	4.9467	2.4398	12.0688
GBP	1.3385	0.2298	0.3075	GBP	3.4576	3.9665	13.7146
EUR	1.3571	-0.7954	-1.0795	EUR	3.3363	2.5292	8.4381
Model (2.12): $unex_t$				**Model (2.18):** $unex_t^-$			
CAD	3.6694	1.6573	6.0812	CAD	3.6694	0.8702	3.1931
JPY	3.6614	3.2764	11.9963	JPY	3.6614	1.2434	4.5525
CHF	4.9467	1.4057	6.9536	CHF	4.9467	0.4719	2.3344
GBP	3.4576	2.0666	7.1456	GBP	3.4576	0.3481	1.2035
EUR	3.3363	1.7058	5.6912	EUR	3.3363	1.0699	3.5696

Linear predictive regression: MDH predicts a contemporaneous relation between trading activity and return volatility. As mentioned above, various empirical studies indicate such a contemporaneous correlation for different assets and using different methods. SAIH, on the other hand, predicts also a lead-lag relation between the two variables. Results from previous empirical analyses however, are less clear about such a relation. Only few studies find a lead-lag relation empirically (e.g. Grammatikos and Saunders, 1986; Mougoué and Aggarwal, 2011). To test for such a lead-lag relation between exchange rate volatility and FCP growth rates, I estimate the following regression equation (2.11), where Δfcp_t is defined as before:

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j \in K} \beta^{(j)} \log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_{t-1} + \delta \Delta fcp_{t-1} + \varepsilon_t. \tag{2.11}$$

Note that not only Δfcp_t enters the equation lagged by one period, but also VIX. Since all RHS variables are lagged, the regression is purely predictive. It serves to evaluate the predictive power of FCP growth for short-term realized volatility. Since results from previous empirical studies are not clear about the lead-lag relation between trading activity and volume, it is to be expected that coefficient δ is either not significant or weakly positive. Estimation results are presented in Table 2.2, columns (6) through (10). Again Newey-West standard errors (Newey and West, 1987, 1994) are given in parentheses.

The strong positive relation measured in the contemporaneous specification (2.10) vanishes almost completely. Only for JPY the coefficient for Δfcp_{t-1} remains significantly positive, with an increase of volatility by 2.57% ($\approx 0.647 \cdot 3.9721\%$) per standard deviation of FCP growth. For the other currencies the estimates are mostly positive, but not significant. Table 2.3 shows that the measured effects of a one standard deviation change in previous period's FCP growth range from -0.34% to 2.57%, i.e. much smaller than in the contemporaneous regression. This indicates, as expected, that the relation between short-term realized volatility and FCP growth rates is indeed first and foremost contemporaneous, as predicted by MDH. Although the SAIH explicitly evolves around the lead-lag relationship between information arrival in asset markets, trading activity and their impact on volatility, results from specification (2.11) do not support the hypothesis. The reason is that even if information arrived sequentially in the market, due to the rapid development of new information and communication technologies and fully automatic trading systems during the past 30 years (King et al., 2012; Schrimpf and Sushko, 2019a), its frequency would likely still be relatively high (i.e. intra-daily or daily) in comparison to the weekly regressions employed here. Weekly indicators of trading activity, such as FCP growth, are too slow to measure meaningful changes in activity and volatility due to lagged dissemination of information. Unlike the estimates for coefficient δ, the other coefficients are comparable in size and significance to their counterparts in the estimation of regression equation (2.10). Even the coefficient for vix_{t-1}, which, in contrast to before now measures the relation between lagged VIX and exchange rate volatility is significantly positive. I conclude from these results that in general, FCP growth does not help predicting exchange rate volatility over a weekly horizon, which is in line with most of the empirical literature.

Decomposed FCP growth: I.a. Bessembinder and Seguin (1993) show that their results are mainly driven by the unexpected component of trading activity. They demonstrate this by decomposing trading volume into an expected and an unexpected component using linear univariate time series techniques. As described in section 2.3, I do the same by estimating an ARMA$(0, 10)$ model for FCP growth and using the fitted values as expected component and residuals as unexpected or innovation component of FCP growth. To examine which component drives the results from

Table 2.4: Estimation results for the linear model with decomposed FCP growth (2.12) and the non-linear model of FCP growth (2.13).

	$\log RV_t^{(W)}$									
	CAD	JPY	CHF	GBP	EUR	CAD	JPY	CHF	GBP	EUR
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$Constant$	-0.660^{***}	-1.156^{***}	-1.157^{***}	-1.501^{***}	-1.226^{***}	-0.699^{***}	-1.286^{***}	-1.202^{***}	-1.674^{***}	-1.262^{***}
	(0.165)	(0.277)	(0.287)	(0.259)	(0.278)	(0.166)	(0.279)	(0.287)	(0.254)	(0.279)
$\log RV_{t-1}^{(W)}$	-0.023	0.045	0.021	0.026	0.011	-0.006	0.082^{**}	0.037	0.032	0.024
	(0.039)	(0.033)	(0.034)	(0.031)	(0.035)	(0.039)	(0.034)	(0.033)	(0.031)	(0.035)
$\log RV_{t-1}^{(M)}$	0.459^{***}	0.382^{***}	0.144^{*}	0.248^{***}	0.197^{**}	0.453^{***}	0.353^{***}	0.124	0.249^{***}	0.184^{**}
	(0.081)	(0.070)	(0.087)	(0.064)	(0.085)	(0.080)	(0.071)	(0.085)	(0.064)	(0.084)
$\log RV_{t-1}^{(Q)}$	0.165^{*}	0.278^{***}	0.545^{***}	0.369^{***}	0.518^{***}	0.140	0.228^{**}	0.540^{***}	0.342^{***}	0.510^{***}
	(0.092)	(0.098)	(0.102)	(0.077)	(0.110)	(0.092)	(0.101)	(0.101)	(0.076)	(0.110)
$\log RV_{t-1}^{(Y)}$	0.300^{***}	0.071	0.066	0.070	0.043	0.309^{***}	0.097	0.074	0.064	0.046
	(0.066)	(0.083)	(0.083)	(0.066)	(0.077)	(0.066)	(0.081)	(0.081)	(0.064)	(0.077)
$trend_{t-1}$	-7.886^{*}	15.970^{***}	10.110^{**}	-3.510	-6.168	-8.492^{*}	14.979^{***}	9.005^{**}	-4.275	-6.425
	(4.684)	(4.339)	(3.960)	(4.107)	(4.037)	(4.722)	(4.417)	(3.928)	(4.085)	(4.069)
vix_t	0.009^{***}	0.007^{***}	0.008^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.007^{***}	0.008^{***}	0.009^{***}	0.009^{***}
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$unex_t$	1.657^{***}	3.276^{***}	1.406^{***}	2.067^{***}	1.706^{***}					
	(0.315)	(0.395)	(0.266)	(0.362)	(0.346)					
$expc_t$	-2.029^{*}	-1.671^{**}	-0.913	0.230	-0.795					
	(1.104)	(0.810)	(0.715)	(0.790)	(0.852)					
Δfcp_t^{+}						2.018^{***}	4.839^{***}	2.163^{***}	3.959^{***}	2.184^{***}
						(0.547)	(0.673)	(0.535)	(0.572)	(0.597)
Δfcp_t^{-}						0.809^{**}	0.574	0.301	-0.109	0.683
						(0.396)	(0.521)	(0.293)	(0.481)	(0.446)
N	1,393	1,392	1,393	1,393	1,132	1,393	1,392	1,393	1,393	1,132
R^2	0.478	0.323	0.280	0.254	0.373	0.474	0.319	0.279	0.263	0.371
Adjusted R^2	0.475	0.319	0.275	0.250	0.369	0.471	0.315	0.275	0.258	0.366

Notes: ***Significant at 1% level; **Significant at 5% level; *Significant at 10% level. Newey-West standard errors in parentheses.

specification (2.10), I include both as exogenous regressors in the following regression equation (2.12):

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j\in K}\beta^{(j)}\log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_t + \delta_1 unex_t + \delta_2 expc_t + \varepsilon_t, \tag{2.12}$$

where $unex_t$ denotes unexpected innovations in FCP growth and $expc_t$ is the expected component. If changes in FCPs are proportional to trading volume and information arrival rate, then from previous empirical and theoretical literature it can be expected that the estimate of coefficient δ_1 is stronger positive and more often significant than the estimate of δ_2. Estimation results for regression equation (2.12) are presented in Table 2.4, columns (1) through (5).

In comparison to Table 2.2, some interesting results emerge. It appears that among the two variables for FCP growth, the unexpected component $unex_t$ is more relevant for explaining variation over time in weekly realized volatility across all currencies. The coefficients are all highly significant and even more positive than in the benchmark regression, ranging from 1.4 to 3.2. In terms of predicted effects for a standard deviation change in the unexpected component in Table 2.3, the impact increases in comparison to the benchmark model (2.10) and ranges from about 5.7% to 12%. At the same time, the estimates for the expected component $expc_t$ are negative for almost all currencies, but barely significant with the exception of JPY. The negative coefficients for $expc_t$ can be explained using the argument by Jorion (1996), Bjønnes et al. (2005) and Bauwens et al. (2006) that expected changes in trading volume represent changes in the number of active traders and consequently, liquidity in the market, which should have a weakly negative impact on volatility, if any at all. Overall, these results confirm previous findings, such as in Bessembinder and Seguin (1993) and Jorion (1996), for the case of FX markets: Unexpected innovations in the trading activity proxy are the main driver of co-variation between volatility and trading activity in general. The other coefficient estimates are qualitatively and quantitatively again very similar to the results in Table 2.2. Assuming that unexpected FCP growth is driven by new information arriving in the market, the unexpected component as used in regression specification (2.12) is a more appropriate proxy for the news arrival rate than the original FCP growth rate series in regression specification (2.10). Hence, the regression results in Table 2.4 can be interpreted as additional evidence for the MDH in FX markets implying that a common process of information arrival is the root of the co-movement between trading activity and volatility.

2.5.2 Asymmetry and news impact curve

Asymmetric impact of FCP growth: Bessembinder and Seguin (1993), Wang (2002), Bjønnes et al. (2005) and others have investigated, whether the relation between their trading activity proxies and return volatility is non-linear. They find strong evidence for asymmetry, in particular that positive shocks to trading activity have a stronger impact on volatility than negative ones. To analyze this possibility in a setting with FCPs as trading activity proxy, I estimate the following regression equation (2.13):

$$\begin{aligned}\log\left(RV_t^{(W)}\right) =& \alpha + \sum_{j\in K}\beta^{(j)}\log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_t + \\ & \delta_1 \Delta fcp_t^{+} + \delta_2 \Delta fcp_t^{-} + \varepsilon_t,\end{aligned} \tag{2.13}$$

where I define the interaction terms as

$$\Delta fcp_t^+ \equiv I_{\{\Delta fcp_t \geq \overline{\Delta fcp}\}} \Delta fcp_t \tag{2.14}$$
$$\Delta fcp_t^- \equiv (1 - I_{\{\Delta fcp_t \geq \overline{\Delta fcp}\}}) \Delta fcp_t. \tag{2.15}$$

The expression I represents the indicator function, taking the value 1 if FCP growth Δfcp_t is above or equal to its unconditional average over the whole observation period $\overline{\Delta fcp}$ and 0 if it is below (cf. Bessembinder and Seguin, 1993). In this setup, I allow positive and negative (i.e. above and below average) changes in FCPs to affect volatility asymmetrically. Estimation results for regression equation (2.13) are presented in Table 2.4, columns (6) through (10).

The results confirm that positive and negative changes in FCPs have a different impact on realized volatility. While the coefficient estimates for positive FCP growth rates are positive and significant across all currencies, for negative FCP growth they are substantially smaller and only significant in the case of CAD. Quantitatively, the coefficient estimates of δ_1 are even larger than the estimates of the coefficient for Δfcp_t in specification (2.10). This is also reflected in the predicted effects of standard deviation growth in FCPs: With a minimum impact of about 7.7% for CAD and a maximum effect of about 19% for JPY for positive shocks, these figures show that especially in times of high frequency news arrival, exchange rate volatility reacts more than proportionally. The coefficient of the other variables are again barely changed in comparison to previous regressions. Generally, these results match expectations from the previous literature using other trading activity proxies than changes in FCPs.

The asymmetry in the relation between news and asset price volatility is a well known topic in the literature. Various GARCH specifications have been developed to explicitly describe non-linear reactions of volatility to news events, such as the prominent E-GARCH (Nelson, 1991) and GJR-GARCH (Glosten et al., 1993) models. However, there is an important difference between the interpretation of “news” in these GARCH specifications and the regressions estimated in this study. The term “news” in a GARCH context refers to the unexpected innovations in the mean process of the returns, i.e. news can be “good” or “bad” if the sign of the innovation is either positive or negative. Accordingly, conditional volatility can react asymmetrically to them, depending on the sign. In the context of MDH and trading activity, the term “news” refers to the information arrival rate, i.e. the number of news events in a given period of time that causes market participants to adjust their positions in relevant assets. Hence, a positive growth rate is not necessarily “good”, neither is a negative growth rate “bad” news. Its interpretation is monotonic in that a higher growth rate means more news, while lower rates mean less news. This perspective explains why the estimated coefficients in Table 2.4 indicate a more monotonic relation between FCP growth rates and realized volatility than the relation between innovations and conditional volatility in GARCH models, where positive and negative shocks usually increase volatility.

Asymmetric impact of unexpected FCP growth: To analyze whether positive and negative shocks in unexpected FCP growth rates have different effects, I employ the transformations (2.14) and (2.15) also for the unexpected component of FCP growth. In particular, define

$$unex_t^+ \equiv I_{\{unex_t \geq \overline{unex}\}} unex_t \tag{2.16}$$
$$unex_t^- \equiv \left(1 - I_{\{unex_t \geq \overline{unex}\}}\right) unex_t, \tag{2.17}$$

Table 2.5: Estimation results for the model with non-linear model of unexpected FCP growth (2.18) and the news-impact-curve (2.20).

	$\log RV_t^{(W)}$									
	CAD	JPY	CHF	GBP	EUR	CAD	JPY	CHF	GBP	EUR
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	−0.686***	−1.185***	−1.174***	−1.596***	−1.237***	−0.664***	−1.127***	−1.138***	−1.549***	−1.219***
	(0.163)	(0.277)	(0.281)	(0.257)	(0.276)	(0.163)	(0.276)	(0.281)	(0.256)	(0.276)
$\log RV_{t-1}^{(W)}$	−0.021	0.048	0.020	0.021	0.011	−0.021	0.048	0.020	0.021	0.011
	(0.039)	(0.033)	(0.034)	(0.031)	(0.035)	(0.039)	(0.033)	(0.034)	(0.031)	(0.035)
$\log RV_{t-1}^{(M)}$	0.458***	0.367***	0.137	0.249***	0.195**	0.458***	0.367***	0.137	0.249***	0.195**
	(0.080)	(0.070)	(0.086)	(0.064)	(0.084)	(0.080)	(0.070)	(0.086)	(0.064)	(0.084)
$\log RV_{t-1}^{(Q)}$	0.162*	0.281***	0.562***	0.370***	0.521***	0.162*	0.281***	0.562***	0.370***	0.521***
	(0.092)	(0.100)	(0.100)	(0.075)	(0.109)	(0.092)	(0.100)	(0.100)	(0.075)	(0.109)
$\log RV_{t-1}^{(Y)}$	0.301***	0.087	0.061	0.062	0.043	0.301***	0.087	0.061	0.062	0.043
	(0.066)	(0.083)	(0.082)	(0.065)	(0.077)	(0.066)	(0.083)	(0.082)	(0.065)	(0.077)
$trend_{t-1}$	−7.964*	15.491***	9.839**	−4.113	−6.446	−7.964*	15.491***	9.839**	−4.113	−6.446
	(4.686)	(4.277)	(3.919)	(4.110)	(4.030)	(4.686)	(4.277)	(3.919)	(4.110)	(4.030)
vix_t	0.009***	0.008***	0.008***	0.009***	0.009***	0.009***	0.008***	0.008***	0.009***	0.009***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$expc_t$	−1.867*	−1.616*	−0.857	0.268	−0.830	−1.867*	−1.616*	−0.857	0.268	−0.830
	(1.097)	(0.829)	(0.727)	(0.794)	(0.855)	(1.097)	(0.829)	(0.727)	(0.794)	(0.855)
$unex_t^+$	2.500***	5.664***	2.440***	3.966***	2.529***					
	(0.616)	(0.666)	(0.539)	(0.662)	(0.701)					
$unex_t^-$	0.870**	1.243**	0.472	0.348	1.070**					
	(0.425)	(0.607)	(0.344)	(0.537)	(0.518)					
z_t						0.062***	0.126***	0.072***	0.075***	0.060***
						(0.012)	(0.013)	(0.014)	(0.012)	(0.012)
$(\lvert z_t\rvert - \mu_{\lvert z_t\rvert})$						0.030*	0.081***	0.049***	0.063***	0.024
						(0.015)	(0.019)	(0.018)	(0.017)	(0.017)
N	1,393	1,392	1,393	1,393	1,132	1,393	1,392	1,393	1,393	1,132
R^2	0.479	0.334	0.284	0.261	0.375	0.479	0.334	0.284	0.261	0.375
Adjusted R^2	0.476	0.330	0.279	0.257	0.370	0.476	0.330	0.279	0.257	0.370

Notes: ***Significant at 1% level; **Significant at 5% level; *Significant at 10% level. Newey-West standard errors in parentheses.

where $\overline{unex}$ is the unconditional mean of the unexpected component over the whole observation period. Similar to specification (2.13), I estimate a regression equation where the interaction terms are included as exogenous regressors:

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j\in K} \beta^{(j)} \log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_t + \delta_1 unex_t^{+} + \delta_2 unex_t^{-} + \delta_3 expc_t + \varepsilon_t \tag{2.18}$$

From previous results and the literature it can be expected that again the unexpected component and in that regard the positive shocks are most relevant, i.e. that coefficient δ_1 is significantly positive. Estimation results for regression equation (2.18) are presented in Table 2.5, columns (1) through (5).

The results confirm the prediction. Although the coefficient values for the other control variables are again barely changed in comparison to previous estimations, the coefficient estimate for unexpected positive shocks in FCP growth is significant and positive across all currencies. However, the coefficients for negative shocks are also positive, however insignificant for CHF and GBP. Regarding the effect of a one-standard deviation change in the unexpected component, positive shocks would have the largest impact among all estimated regressions so far. Table 2.3 shows that for JPY, an unexpected FCP growth of 3.66% is associated with an increase in volatility of approximately 20%. For the other currencies, the impact lies between 8.4% and 13.7%. These figures highlight that the effects are not only statistically significant but also economically relevant. In contrast, estimates for the expected component are negative for all but one currency, but only in two cases significant at the 10% level. In general, these results show that the relation between trading activity and volatility is mainly driven by unexpected shocks and that, in absolute terms, the impact of unexpected positive shocks is larger than that of negative ones. With regard to the information arrival rate, the findings underline that if new fundamental information were to arrive in a cluster over a short period of time, short-term realized volatility of exchange rates would increase more than proportionally.

News impact curve: The estimation results of the two asymmetric specifications motivate a more detailed analysis of the relationship between volatility and FCP growth. In particular, if unexpected changes in FCP growth rates are interpreted as a proxy for news or the rate of arrival of new information, then the relationship between volatility and news can be graphically illustrated in a type of news-impact curve (NIC), similar to the concept proposed by Engle and Ng (1993) for asymmetric GARCH models. To construct the NIC, I proceed as follows:

1. Standardize the unexpected component of FCP growth by

$$z_t = \frac{unex_t - \mu_{unex}}{\sigma_{unex}}, \tag{2.19}$$

   where μ_{unex} and σ_{unex} are the arithmetic mean and standard deviation of $unex_t$ over the whole observation period.

2. Using OLS, estimate the regression equation

$$\log\left(RV_t^{(W)}\right) = \alpha + \sum_{j\in K} \beta^{(j)} \log\left(RV_{t-1}^{(j)}\right) + \gamma_1 trend_{t-1} + \gamma_2 vix_t + \delta_1 z_t + \delta_2 \left(|z_t| - \mu_{|z_t|}\right) + \delta_3 expc_t + \varepsilon_t, \tag{2.20}$$

where $K = \{W, M, Q, Y\}$ and $\mu_{|z_t|}$ is the arithmetic mean of the absolute value of the standardized unexpected component of FCP growth.

3. For $unex_t$ in a predefined range $[unex_{min}, unex_{max}]$, construct the NIC as a function RV of $unex_t$ by

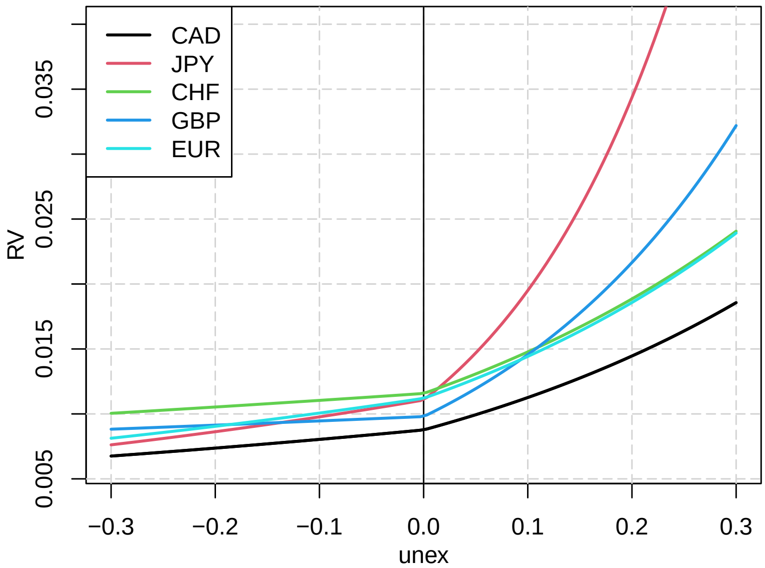
$$RV\left(unex_t\right) = \begin{cases} A \cdot \exp\left(\left(\widehat{\delta}_1 + \widehat{\delta}_2\right) \frac{unex_t - \mu_{unex}}{\sigma_{unex}}\right) & \text{for} \quad unex_t \geq \mu_{unex} \\ A \cdot \exp\left(\left(\widehat{\delta}_1 - \widehat{\delta}_2\right) \frac{unex_t - \mu_{unex}}{\sigma_{unex}}\right) & \text{for} \quad unex_t < \mu_{unex}, \end{cases} \tag{2.21}$$

with

$$A = \exp\left(\widehat{\alpha} + \sum_{j \in K} \widehat{\beta}^{(j)} \overline{\log\left(RV^{(j)}\right)} + \widehat{\gamma}_1 \overline{trend} + \widehat{\gamma}_2 \overline{vix} - \widehat{\delta}_2 \mu_{|z_t|} + \widehat{\delta}_3 \overline{expc}\right). \tag{2.22}$$

A hat over a parameter represents their estimated value and a bar over a variable represents their arithmetic mean.

Figure 2.3: News impact curves.



The regression results from equation (2.20) are presented in Table 2.5, columns (6) through (10) and the NICs for all currencies are plotted in Figure 2.3. The regression results provide a picture consistent with the previous estimates. The coefficient estimates for δ_2 are all positive and significant at the 5% level for JPY, CHF and GBP and at the 10% level for CAD. This indicates that at least for these currencies, the asymmetries in the relation between FCP growth shocks and volatility are significant. Figure 2.3 presents the asymmetry graphically. The x-axis represents the values of an unexpected shock to FCP growth $unex_t$, while the y-axis measures the behavior of weekly realized volatility RV. As noted before, across all currencies the relation appears to be monotonous: Positive shocks coincide with increases in volatility, while negative shocks coincide with decreases. Due to the previously explained differences in the interpretation of news, this picture is in contrast to the usual NICs constructed from GARCH models, where positive and negative news shocks lead to increased volatility. However, the plot gives a good indication of how news arrival rate and realized volatility are related. In particular, it is interesting to note the variation in the non-linearity across currencies. For example, in the case of CAD and EUR, the curves are generally flatter than for the other currencies. A strong positive news shock is associated with a weaker reaction to realized volatility. At the same time, the slope of the curves for negative shocks is not much smaller than for positive ones. In contrast to that, the curves for JPY and GBP are much steeper. Even relatively weak unexpected increases in FCP growth rates are associated with stronger increases in volatility. Although the cross-currency differences in the curves are quite substantial, it is difficult to obtain singular explanations for them. Possible reasons may be related to the particular role that a foreign currency has in international financial markets, e.g. as a safe haven currency, or certain events that caused strong disruptions in volatility and foreign currency trading activity, such as financial crises (cf. Galati, 2000). In order not to overstretch the analyses in this study, I leave the answer to this question for future research.

2.6 Conclusion

In this chapter I investigate the relation between large FX market actors' trading activity and nominal spot exchange rate volatility. For that purpose, I use data on weekly growth rates in aggregated FCPs to proxy trading activity and the rate at which new information arrives in the market over a period of about 27 years. Estimation results of different HAR-RV specifications with additional exogenous regressors show that there is a significant positive contemporaneous relation between trading activity and volatility across five major currencies against the USD. My findings cannot support a lead-lag relation between the two variables, i.e., trading activity does not help predict volatility over a one week horizon. Non-linear specifications indicate that for some currencies the relation is asymmetric in that the above-average growth in FCPs is stronger positively related to volatility than below-average growth rates. Using linear time series techniques to decompose FCP growth into expected growth and unexpected shocks, I find that unexpected shocks are the main driver of the positive relation, while the impact of expected growth rates is less significant and generally negative. If unexpected growth rates are interpreted as trading activity shocks resulting from the arrival of new information about economic fundamentals in the market, these results are consistent with information based theories of the trading activity-volatility relation such as MDH. Thereby, my findings extend the results from the literature regarding the trading volume-volatility relation in equity and futures markets to the FX market.

The innovation of this chapter is twofold: First, since the FX market is comparably large, decentralized and opaque, data on trading activity, volume or trading positions is usually difficult to collect. For this reason most of the empirical literature addressing MDH and other theories focused on equity and futures markets, for which such data is more readily available. To the best of my knowledge this study is the first to use the FCP data collected in the *Treasury Foreign Currency Reports* by the Federal Reserve Bank of New York to proxy trading activity in relation with spot exchange rate volatility over the whole 27 year observation period. Second, depending on specific research questions and available data, existing studies on the relation between exchange rate volatility and FX trading activity have employed various methods to estimate volatility and trading activity such as linear regressions, different GARCH models and VAR analysis. This study is the first to employ the HAR-RV by Corsi (2009) extended with exogenous regressors in this context. The model suits the theoretical foundation of MDH well, because it is able to control for persistence in asset price volatility through volatility cascades, while at the same allowing for an expansion by exogenous variables such as trading activity proxy variables.

The results in this chapter are of interest especially for regulators, policy makers and FX market participants in general. Exchange rate volatility and the risks involved in cross-currency investing are an integral ingredient in the decision making of firms which for example invest or are funded from abroad or receive payments for exports in a foreign currency. The firms involved have to account for and properly hedge the risk that an exchange rate moves in an unfavorable way due to its high volatility by signing respective derivative contracts. For that purpose it is vital to measure volatility precisely and to understand its dynamics and drivers. Hence, by highlighting the relevance of large players' trading activity and the importance of the information arrival rate for short-term exchange rate volatility I contribute to this assessment process. Policy makers and regulators, on the other hand, are often interested in the risks for an economy resulting from the instability of a financial system as a whole. As described at the beginning of this chapter, since the Global Financial Crisis, US dollar funding of non-US banks and financial institutions has grown rapidly. Since accounting regulations do not treat this kind of foreign currency denominated debt the same way other debt positions are treated, this type of debt is in fact "missing" from balance sheets (Borio et al., 2017, p. 39). Hence, for policy makers and regulators it is difficult to assess the size, sustainability, currency and term structure of these obligations and to detect potential threats to the stability of financial systems. In order to predict, dampen or even prevent financial crises, it is therefore not only necessary to measure the risk of an adverse event and the exposure of the economy, but also to understand probable triggers, such as increased exchange rate volatility. On this account, the empirical findings in this chapter can contribute to regulators' and policy makers' understanding of exchange rate volatility and the role the dissemination of new information (e.g. through new information technologies) can play in the (de-)stabilization of international financial markets.

A.1 Appendix: Figures and tables

Figure A.1: ACF of FCP (log-)growth.

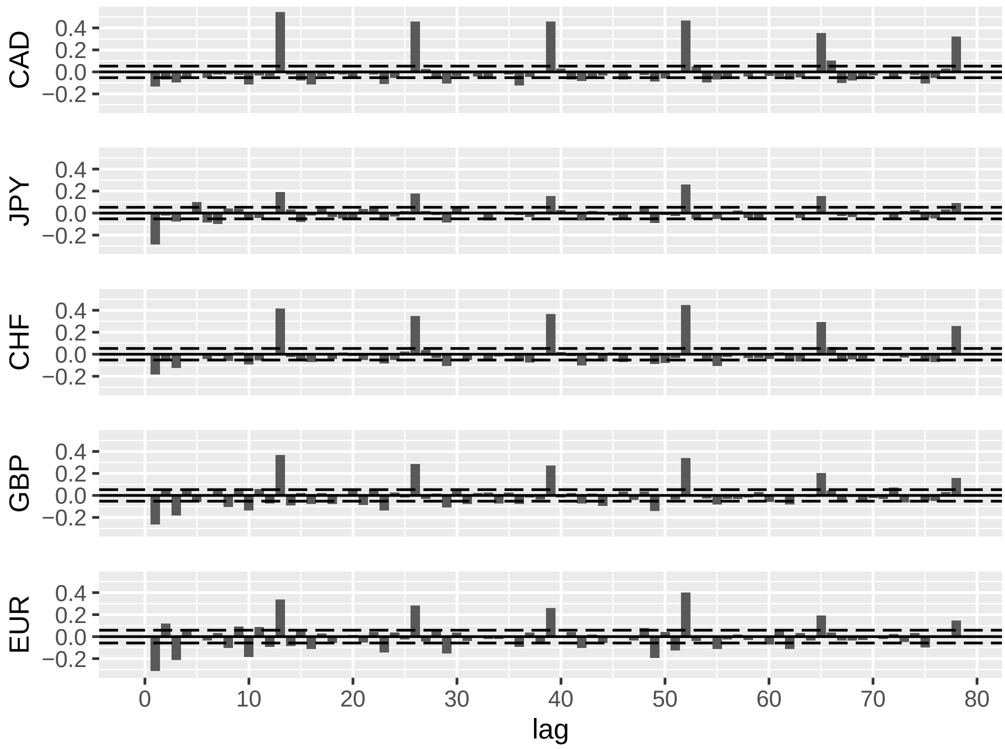


Figure A.2: PACF of FCP (log-)growth.

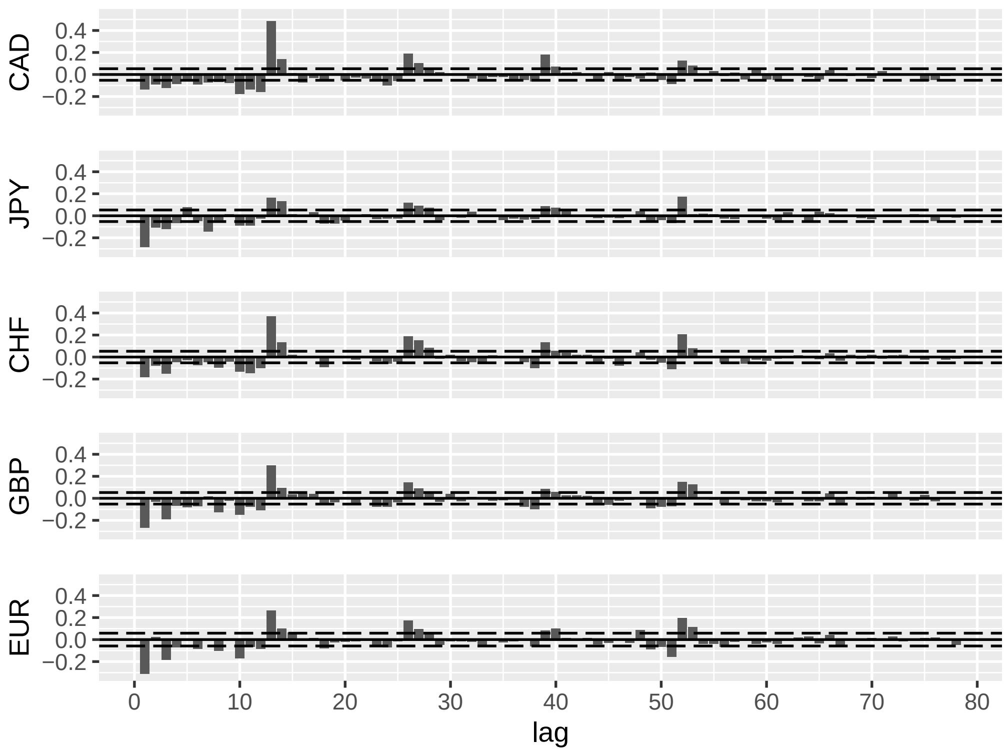


Table A.1: Estimation results for CHF including dummy-interaction terms to account for minimum exchange rate regime between 2011 and 2015.

	$\log RV_t^{(W)}$	
	CHF	CHF
	(1)	(2)
$Constant$	-1.204^{***}	-1.184^{***}
	(0.295)	(0.289)
$\log RV_{t-1}^{(W)}$	0.037	0.021
	(0.034)	(0.034)
$\log RV_{t-1}^{(M)}$	0.133	0.143
	(0.086)	(0.087)
$\log RV_{t-1}^{(Q)}$	0.510^{***}	0.537^{***}
	(0.102)	(0.102)
$\log RV_{t-1}^{(Y)}$	0.084	0.067
	(0.084)	(0.084)
$trend_{t-1}$	9.379^{**}	9.878^{**}
	(4.017)	(3.959)
vix_t	0.008^{***}	0.008^{***}
	(0.002)	(0.002)
D_t	-0.045	-0.046
	(0.038)	(0.037)
Δfcp_t	1.235^{***}	
	(0.274)	
$D_t \times \Delta fcp_t$	-0.538	
	(0.804)	
$unex_t$		1.477^{***}
		(0.285)
$expc_t$		-0.856
		(0.778)
$D_t \times unex_t$		-0.630
		(0.804)
$D_t \times expc_t$		-0.824
		(2.061)
N	1,393	1,393
R^2	0.276	0.281
Adjusted R^2	0.271	0.275

Notes: ***Significant at 1% level; **Significant at 5% level; *Significant at 10% level. Newey-West standard errors in parentheses. The dummy-variable D_t takes on the value 1 if an observation falls in the period starting 6 September 2011 and ending 15 January 2015, and 0 otherwise.

3

Large Creditor Signaling and Learning

Abstract

I investigate the role of a large player in a sequential global game of regime change. In a credit rollover setting, the large creditor is given the choice to roll over an investment early or postpone her decision. Rolling over early sends a positive signal to small creditors, who are thereby induced to roll over as well. At the same time, postponing allows the large creditor to observe and learn from the small ones in order to make a better informed decision. She faces a trade-off between being at the front or at the rear of the crowd of creditors. By being a focal point for small creditor coordination, her actions have a substantial impact on the probability of coordination failure and thus the failure of the investment project. By highlighting the importance of signaling, my findings open up a new perspective on the idea of catalytic finance and the impact of a lender-of-last-resort in self-fulfilling debt crises.

3.1 Introduction

Coordination problems are at the core of many economic and social phenomena. A well-known feature of coordination games is the existence of multiple equilibria and the potential of coordination failure if the players are not able to jointly coordinate on choosing the action that leads to the Pareto-optimal equilibrium. In this chapter, I investigate how a large player's choice between signaling her expectations to a group of smaller players and learning from their actions affects the players' behavior and the probability of coordination failure. I show that there is a trade-off between signaling and learning and that optimal decisions depend on the relative precision of private information.

I address this scenario within the framework of a global coordination game as proposed and popularized by Carlsson and Van Damme (1993) and Morris and Shin (1998). In contrast to traditional coordination games with multiple equilibria, it allows to pin down a unique equilibrium in so-called threshold strategies so that comparative statics analysis delivers valid insights. As an applied example, I use a simplified credit rollover game to illustrate the equilibrium mechanisms and implications.[6] In this game, one large and a continuum of small creditors have invested their capital in an investment project and, at an interim stage of the project, they are asked to roll over their investment or withdraw. The large creditor first chooses whether she wants to roll over or

[6] In the following, I use the terms "loan" and "credit" interchangeably to refer to liquidity provided by a lender or creditor for a limited period of time.

postpone her decision until after the small creditors have made their choice.[7] The small creditors observe her decision and afterwards simultaneously choose to roll over or withdraw. The project succeeds if a sufficiently large proportion of creditors rolls over relative to the fundamental state of the investment project.

In many examples of coordination problems, coordination failure is associated with high economic and social costs. For example, financial crises such as banking crises and bank runs, sovereign default and currency crises are often the result of the failure of market participants to coordinate in choosing the stabilizing action. To avoid crises and minimize their costs, policymakers across a variety of countries have agreed on prudential policy measures such as liquidity reserve requirements for banks (e.g. through the Basel Accords) or the establishment of intergovernmental and international organizations to provide assistance for illiquid governments (e.g. IMF or the European Stability Mechanism). However, in order to implement such policies to full effect and further minimize the possibility of future runs on banks or public debt, it is essential to understand the causes of coordination failure in financial markets and what tools there are to guide the players to the socially preferable equilibrium action. In this regard, large players, such as hedge funds and banks, but also governments, central banks and international institutions and organizations such as the IMF can play a crucial role. Either by being able to provide a large amount of liquidity or by their actions being observed by smaller market participants, the decisions of large players can have substantial effects on the outcome of financial crises. Policy-makers can exploit this impact to calm markets down, e.g., by setting the appropriate incentives or through cleverly designed communication strategies. In fact, the aforementioned examples show that policymakers themselves often act as large players in volatile markets in order to avoid the spread of panic and provide stability. Government bail-outs of troubled financial institutions or central banks stepping up as lenders-of-last-resort can be interpreted as large players acting in financial markets, who guide small players' expectations and reduce uncertainty.

Starting point for my investigation is the global game model proposed by Corsetti et al. (2004), who ask whether "one Soros makes a difference" and analyze the impact of a large currency speculator on speculative attacks. They show that the sheer existence of a large player can destabilize currency markets by making small speculators more aggressive and thereby increase the probability of a currency crash. In their work, the large player has to decide whether she wants to attack before the small speculators or not at all. To justify this fixed sequence of actions, the authors argue that small speculators have a dominant strategy to move late because they believe that, because they are small, they cannot influence others' actions when attacking early but learn from others if they move late. The large speculator, knowing that in equilibrium small speculators want to move late, has an incentive to move early because she can thereby signal her position to small speculators and potentially induce them to join her attack, making an overall successful attack more likely. I challenge this argument. Corsetti et al. (2004) impose a fixed sequence of decisions (first large player, then small players) by implicitly assuming that moving late and observing others are mutually exclusive. While adapting their model for a credit rollover setting, I relax this assumption by applying the strategic waiting mechanism introduced by Dasgupta (2007) to the large player and giving her the choice of whether she wants to move *before* small players or *after* observing their actions. In my proposed model, the large player therefore faces a trade-off between signaling to and learning from small players. Moreover, I refute the argument that a small player would not want to move early because they believe that they had no influence on other players'

[7] For better understanding, I refer to the large player as "she" and "her" and an individual small player as "he" and "him" throughout the rest of this chapter.

actions. I show that if the large player's size decreases to (nearly) zero, her incentive to move early even increases.

Moreover, using the model by Corsetti et al. (2004) as a benchmark game without the option to wait, I find the following additional results:

a) The large creditor rolls over early more often if she has the option to wait than if she has not.

b) If the large creditor's private signal is fully revealing, the option to wait is irrelevant for the creditors' equilibrium strategies. If it is completely uninformative, the option to wait affects equilibrium strategies only up to the LC's size, but not beyond.

c) Early rollover by the large creditor increases the number of small creditors who roll over and decreases the probability of coordination failure, compared to postponing the decision or withdrawing.

d) The mass of small creditors that follow the large creditor increases if the precision of the large creditor's signal increases relative to theirs. This holds regardless of the large creditor's size, i.e., also for a "small" large creditor.

e) The option to wait decreases the conditional probability of coordination failure, compared to the case where the large creditor cannot postpone the decision.

These findings show that, by rolling over early, large players have significant influence and can calm markets down, either by providing fundamental information through signaling or by presenting a focal point for coordination due to strategic complementarity. The implications for signaling are particularly interesting with respect to the effects of catalytic finance in the context of a lender-of-last-resort (cf. Morris and Shin, 2006; Corsetti et al., 2006). So far, the literature has focused on direct private bail-in effects as a consequence of a lender-of-last-resort (e.g. the IMF) bailing out troubled debtors. However, they ignore that a bail-out conveys fundamental information and provides orientation to smaller creditors. My analysis contributes to filling this gap in the literature. Moreover, by investigating how the relative precision of information affects the creditors' incentives and, as a consequence, the probability of coordination failure, I contribute to the understanding of how transparency and information dissemination policies can help to stabilize volatile markets and guide market participants' expectations to a sustainable equilibrium.

The rest of this chapter is organized as follows: Section 3.2 provides background on the relevant literature. Sections 3.3 and 3.4 derive the two games with and without the option of waiting, as well as their equilibrium conditions. Section 3.5 briefly describes how the models are solved and which parameter specifications are used for numerical simulations, while Section 3.6 presents and discusses comparative statics results. The final section 3.7 concludes and discusses the possible implications of my findings.

3.2 Literature

The models proposed in this chapter relate to contributions that examine the impact of large players on financial crises from the perspective of coordination and so-called global games (e.g. Morris and Shin, 2003; Corsetti et al., 2004; Rochet and Vives, 2004; Morris and Shin, 2006). In this view, financial crises are coordination problems that are caused by payoff externalities, which may lead, for example, to panic-based runs and self-fulfilling debt crises. A large market participant, such as an institutional investor, a central bank, or an international financial institution (e.g. IMF) thereby often acts as a focal point or "center of gravity" for small-player coordination.

The global game approach was introduced by Carlsson and Van Damme (1993) and popularized by Morris and Shin (1998) as a way to deal with the indeterminacy of the equilibrium of classical models of self-fulfilling bank runs, debt crises and speculative currency attacks (cf. Diamond and Dybvig, 1983; Calvo, 1988; Obstfeld, 1995, 1996). Their method discards the assumption of common knowledge of the fundamental state of the economy and replaces it with private information. The authors show that a unique equilibrium emerges, even if private information becomes arbitrarily precise. An important branch of the literature on global games investigates the equilibrium characteristics in models featuring private, as well as public information, dynamics and signaling. Morris and Shin (2004) show that with exogenous public information, a unique equilibrium prevails only if private information is sufficiently precise. Angeletos and Werning (2006), Angeletos et al. (2006) and Angeletos et al. (2007) find that similar results hold if the public signal is generated endogenously. In their models, public signals consist of prices in a financial market, a policy carried out by a policymaker, or the history of previously played rounds of a dynamic global game. In all cases, the authors show that multiple equilibria may reemerge.

Other global games that incorporate a large player consider the impact of a lender-of-last-resort (LOLR) on credit markets. Rochet and Vives (2004) propose a bank-run model and investigate the potential impact of a central bank acting as a LOLR. They show that a solvent bank can be illiquid due to a coordination failure in the interbank market and that the provision of liquidity by a LOLR can prevent the occurrence of such a coordination failure. Corsetti et al. (2006) and Morris and Shin (2006) look at sovereign debt crises and investigate how an international financial institution such as the IMF could help prevent inefficient debt runs by providing liquidity to a sovereign debtor. Both studies focus on the catalytic effects of an IMF intervention, i.e. the private bail-in that follows a partial bail-out by an international LOLR (cf. Krahnke, 2023; Zwart, 2007). The underlying argument is that by reducing the debtor country's pressure to sell illiquid assets at a cost, an IMF intervention improves the overall prospects of a successful aversion of sovereign default (bail-out). This, in turn, has the effect that fewer creditors are willing to run, i.e. more creditors are willing to roll over their short-term loans, which further decreases the default probability (bail-in). As a consequence, even a partial bail-out of an illiquid debtor country by the IMF could induce private investors to voluntarily provide liquidity and thereby prevent a default, as long as the fundamentals are not too bad. The basic assumption behind this interpretation of catalytic finance is that the actions of the international LOLR and private creditors are strategic complements, i.e. a private investor's incentive to roll over his loan increases if the IMF intervenes and other creditors roll over as well. My model is closely related to this type of literature in that it considers how a large player's actions affect small creditors' behavior and causes a herd-like bail-in. However, in addition to strategic complementarity, I propose signaling as a second channel through which the large player induces small creditors to roll over. In this view, the large creditor's actions not only have a direct effect on the probability of a debt crisis, but also convey information with regard to economic fundamentals and present a focal point for coordination. If small creditors interpret the large creditor's decision to roll over as an optimistic signal about the fundamental state, then, on average, they update their expectations to be more optimistic as well. As a consequence, more small creditors are induced to follow the large creditor's lead and roll over. This signaling channel has so far not been included in global games considering the impact of a LOLR.

In a sequential global game with endogenous learning, Dasgupta (2007) investigates the impact of giving small investors the option to put their capital in an investment project early or to wait and decide about investing later. While the payoff for investing early is generally higher than for investing late, waiting gives the advantage of learning from others about the fundamentals of the project. The author shows that giving players the option to wait reduces the incidence of coordination failure in comparison to a simultaneous move game. The contribution by Dasgupta (2007) represents the basis for the learning-and-signaling mechanism I employ in my model. In this sense, the analysis in this chapter is also related to the literature on herd behavior and strategic delay in financial markets.[8]

Moreover, the models proposed in this chapter can be interpreted as a formalization of the so-called endorsement or certification effect (cf. Booth and Smith, 1986; Titman and Trueman, 1986). In this context, an "influential" player affects small players in their decision by endorsing a particular action or investment opportunity. An example of such an endorsement would be Warren Buffett, who publicly announces to buy a particular commodity or stock. Small and potentially less informed investors observe this and conjecture that he might have better information on the future profitability of the commodity or stock and therefore also decide to buy it (Hirshleifer and Teoh, 2003, p. 47f.). In my analysis, when rolling over early, the large creditor endorses the investment project and convinces a fraction of small creditors to roll over as well. Interestingly, such an endorsement effect does not necessarily require the endorser to actually buy the asset he endorses, as long as he can credibly substantiate why he believes that it is profitable (reputation). In a similar context, credit rating agencies can be understood as "influential" players who certify, e.g., a particular company or a country's government by issuing a credit rating. Using a global game framework, Carlson and Hale (2006) and Holden et al. (2018) discuss the impact of credit rating agencies on the coordination of creditors in global games of debt rollover. My model relates to this literature in that by assuming that the large creditor becomes vanishingly small, her action does not directly affect the outcome of the coordination game, but only through signaling, which would correspond to the "certification" impact of a rating agency.

3.3 Benchmark game Γ_{NW} without option to wait

3.3.1 Setup

In order to analyze how the option to wait affects the players' behavior and the probability of coordination failure, I compare the results to a benchmark model in which I assume that the large player has to decide in the first stage of the game whether she wants to roll over or withdraw completely. For the sake of brevity I denote this game by Γ_{NW} and the game featuring the option to wait by Γ.[9] The game Γ_{NW} corresponds to the "sequential move game" presented in Corsetti et al. (2004, p. 103ff.), which I reformulate in the form of a credit rollover game instead of a speculative attack game for reasons of comparability.[10]

There are two types of players: A continuum of homogeneous risk-neutral small players, called "small creditors" (SCs) indexed by $i \in (0, 1-\lambda)$ and a single risk-neutral player called "large creditor" (LC) of size $\lambda \in (0,1)$. Each creditor is invested in a risky investment project and at some interim stage of the project, they review their investment and get the choice to either roll over their loan or withdraw from it. If the project is successful, it pays a positive return at the end of the game. If it fails, it pays 0. Rolling over is risky as the project might be liquidated

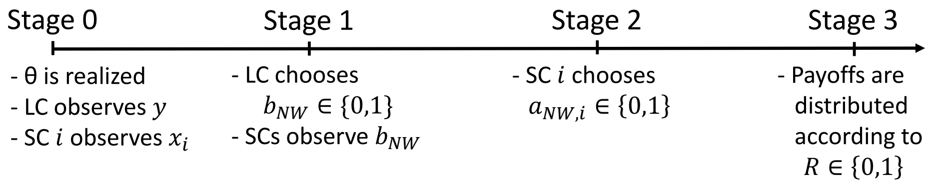
[8]See Hirshleifer and Teoh (2003) for a comprehensive literature overview.

[9]The subscript NW stands for "No Waiting". Throughout the analysis, it is used to distinguish actions and strategies in Γ_{NW} from Γ, for which I omit the subscripts.

[10]The games Γ and Γ_{NW} presented in this chapter are essentially identical, with the exception being the sequence of actions. Therefore, this subsection represents the foundation for understanding both games.

prematurely, either due to bad fundamentals or due to liquidity issues, caused by too many creditors withdrawing their funds. The creditors' choice of rolling over or withdrawing is modeled as a game of three stages, which are summarized in the timeline in Figure (3.1).

Figure 3.1: Sequence of actions in benchmark game without option to wait.



Before stage 1, the fundamental state θ of the project is drawn from an improper uniform prior distribution over the real line. It can be interpreted as an index of the general performance and outlook of the project. Higher values of θ represent a better outlook for the project, while lower values mean fewer chances of successful project completion. Creditors cannot observe θ. Instead, an SC i receives a private signal $x_i = \theta + \varepsilon_i$, where $\varepsilon_i \overset{iid}{\sim} N\left(0, \beta^{-1}\right)$, while the LC receives a private signal $y = \theta + \varepsilon_y$, where $\varepsilon_y \sim N\left(0, \gamma^{-1}\right)$ and independent of all ε_i. The parameters β and γ are signal precisions. As usual in the global games literature, I assume that noise distributions (i.e. ε_y and ε_i) are common knowledge, that is, they are known to every player and every player knows that everybody knows.[11] In stage 1, the LC gets to choose between actions "roll over" (denote $b_{NW} = 1$) and "withdraw" (denote $b_{NW} = 0$). In general, creditors' payoffs depend on their decision to roll over or to withdraw and on the outcome of the project, i.e. whether it succeeds (denote $R = 1$) or fails (denote $R = 0$). Rollover is always associated with fixed costs or expenditures $t > 0$, e.g., the initial investment sum or monitoring costs.[12] Hence, if the LC rolls over and the project is finished successfully, she receives a net payoff of $c_1 - t > 0$, once the project is complete. If the LC rolls over and the project fails anyway, the project pays 0 and she receives a net payoff of $-t$. Withdrawing does not incur any costs and thus the payoff is 0.

SCs observe the LC's action in stage 1. Therefore, it is also considered common knowledge. In stage 2, each SC i chooses between actions "roll over"(denote $a_{NW,i} = 1$) and "withdraw" (denote $a_{NW,i} = 0$). Like the LC, if a small player i rolls over and the project is successful, he receives a net payoff of $c_1 - t$ at the end of the game. If he rolls over and the project fails, he receives a net payoff of $-t$. If he withdraws, he receives a safe payoff of 0. In stage 3, the outcome of the project is determined, depending on the fundamental state and the percentage of creditors who withdraw their investment. If the fundamentals are strong relative to the fraction of creditors who withdraw, then the project is completed successfully. If they are weak, it fails. Depending on the outcome of the project and individual decisions, the payoffs are distributed to creditors accordingly at the end of stage 3. Table 3.1 summarizes the conditional payoffs for small and large creditors.

[11] This assumption is necessary for the tractability of equilibrium derivations. If the precision parameters γ and β were private information, the players would not only have to second-guess the average signals of the other players, but also the uncertainty associated with them. Even though the derivation of equilibrium conditions would probably still be possible, it would become disproportionately more complicated while the value of new insights remains questionable.

[12] This is a common assumption in the global games literature: The risky action (here rolling over) is usually associated with some fixed costs.

Table 3.1: Payoffs in game Γ_{NW}.

Player	SC i		LC	
Action	$a_{NW,i} = 0$	$a_{NW,i} = 1$	$b_{NW} = 0$	$b_{NW} = 1$
$R = 0$	0	$-t$	0	$-t$
$R = 1$	0	$c_1 - t$	0	$c_1 - t$

3.3.2 Equilibrium

In the global games literature where players can choose between two actions, it is common to derive equilibria in so-called threshold strategies (Morris and Shin, 2003).[13] These strategies consist of thresholds for private signals, so that it is optimal for the player to choose one action if his signal is above the threshold and the other action if it is below. In particular, according to Morris and Shin (2003, p. 60), in a binary action global game such as those presented in this chapter, a threshold strategy m around threshold k is a reaction function that maps each possible private signal x into the binary action space $\{0, 1\}$, such that[14]

$$m(x) = \begin{cases} 0, & \text{if} \quad x > k, \\ 1, & \text{if} \quad x \leq k. \end{cases} \tag{3.1}$$

Morris and Shin (2003) show that in a simple setting, threshold strategies are the only strategies surviving iterated deletion of dominated strategies. However, their argument does not generally apply to more complicated settings such as the dynamic model proposed here. In order to keep the equilibrium derivation straightforward and comparative statics results tractable, I nevertheless restrict my analysis to Perfect Bayesian Equilibria (PBE) in threshold strategies, even though other types of equilibria may exist in these games. For Γ_{NW}, the PBE in threshold strategies is defined as follows:

Definition 3.1 (Equilibrium in Γ_{NW})**.** The PBE in threshold strategies in Γ_{NW} consists of the following elements:

1. Threshold strategies:
   - A threshold strategy around y^*_{NW}, such that rolling over ($b_{NW} = 1$) maximizes the LC's expected payoff if $y > y^*_{NW}$ and withdrawing ($b_{NW} = 0$) otherwise.
   - A threshold strategy around x^*_{NW} and x^{**}_{NW}, such that rolling over ($a_{NW,i} = 1$) maximizes SC i's expected payoff if a) $b_{NW} = 1$ and $x_i > x^*_{NW}$, or b) $b_{NW} = 0$ and $x_i > x^{**}_{NW}$ and withdrawing ($a_{NW,i} = 0$) otherwise.
2. Creditors' aggregate action: The mass of creditors who roll over is given by $\ell_{NW} = b_{NW} \cdot \lambda + A$, where $A = \int_0^{1-\lambda} a_{NW,i} di$ is the mass of SCs who roll over. The mass of creditors who withdraw is $1 - \ell_{NW}$.
3. Critical mass conditions:
   - A fundamental threshold θ^*_{NW}, such that conditional on $b_{NW} = 1$, the project succeeds ($R = 1$) iff $\theta > \theta^*_{NW}$.

[13] Also referred to as switching or cutoff strategies.

[14] The threshold k is usually derived by equating the expected payoff for one action with the expected payoff from the alternative action and solving the resulting indifference condition for the private signal. If the solution is unique, then the expected payoff for one action is greater than the expected payoff from the alternative action for all signals below the threshold and vice versa for all signals above the threshold. I use the same technique to find the creditors' equilibrium threshold strategies.

- A fundamental threshold θ^{**}_{NW}, such that conditional on $b_{NW} = 0$, the project succeeds ($R = 1$) iff $\theta > \theta^{**}_{NW}$.

4. Beliefs:

- The LC forms beliefs $f(R \mid y)$ using Bayes' Law.
- SC i forms beliefs $f(R \mid x_i, y > y^*_{NW})$ and $f(R \mid x_i, y \leq y^*_{NW})$ using Bayes' Law.

Large Creditor: In stage 1, the LC knows that if she rolls over early, the project succeeds if $\theta > \theta^*_{NW}$ and fails if $\theta \leq \theta^*_{NW}$. Given her private signal y, she believes that the project succeeds with probability $Pr(\theta > \theta^*_{NW} \mid y)$ and therefore her expected payoff for rolling over is

$$(c_1 - t)Pr\left(\theta > \theta^*_{NW} \mid y\right) - tPr\left(\theta \leq \theta^*_{NW} \mid y\right) = c_1 Pr\left(\theta > \theta^*_{NW} \mid y\right) - t. \tag{3.2}$$

At the same time, her payoff for withdrawing is 0. To maximize her expected payoff, she wants to roll over if $c_1 Pr(\theta > \theta^*_{NW} \mid y) - t > 0$ and withdraw otherwise.[15] She is indifferent if

$$c_1 Pr(\theta > \theta^*_{NW} \mid y^*_{NW}) = t \quad \Leftrightarrow \quad c_1 \Phi\left(\sqrt{\gamma}\left(y^*_{NW} - \theta^*_{NW}\right)\right) = t \tag{3.3}$$

where y^*_{NW} is the value of y that solves the indifference equation and Φ is the cumulative distribution function of the standard normal distribution. The second equation follows by using the LC's posterior $\theta \mid y \sim N(y, \gamma^{-1})$. Since the LHS of the equation is increasing monotonously in y and converges to $c_1 (> t)$ for $y \to \infty$ and to 0 for $y \to -\infty$, for a given threshold θ^*, there is a unique value y^*_{NW} that solves the indifference equation, such that $c_1 \cdot Pr(\theta > \theta^*_{NW} \mid y) > t$ if $y > y^*_{NW}$ and $c_1 \cdot Pr(\theta > \theta^*_{NW} \mid y) \leq t$ if $y \leq y^*_{NW}$. Hence, LC is said to follow a threshold strategy around y^*_{NW}, such that it is optimal to roll over if $y > y^*_{NW}$ and withdraw otherwise.

Small Creditors: An SC i's decision to either roll over or withdraw depends on two pieces of information: his private signal x_i and the LC's action b_{NW}. Suppose SC i observes the LC roll over. Then he knows that in equilibrium $y > y^*_{NW}$ and that the project is successful if $\theta > \theta^*_{NW}$. Hence, i's expected payoff from rolling over is

$$\begin{aligned}&(c_1 - t) \cdot Pr\left(\theta > \theta^*_{NW} \mid y > y^*_{NW}, x_i\right) - tPr\left(\theta \leq \theta^*_{NW} \mid y > y^*_{NW}, x_i\right)\\ &= c_1 Pr\left(\theta > \theta^*_{NW} \mid y > y^*_{NW}, x_i\right) - t,\end{aligned} \tag{3.4}$$

while for withdrawing it is 0. In equilibrium, i wants to roll over if $c_1 \cdot Pr(\theta > \theta^*_{NW} \mid y > y^*_{NW}, x_i) - t > 0$ and withdraw otherwise. He is indifferent if

$$c_1 \cdot Pr\left(\theta > \theta^*_{NW} \mid y > y^*_{NW}, x^*_{NW}\right) = c_1 \frac{Pr\left(\theta > \theta^*_{NW}, y > y^*_{NW} \mid x^*_{NW}\right)}{Pr\left(y > y^*_{NW} \mid x^*_{NW}\right)} = t, \tag{3.5}$$

where on the LHS Bayes' Rule is applied. The value x^*_{NW} solves the indifference equation and constitutes the threshold for the SC's threshold strategy, such that the expected payoff for rolling over is greater than the expected payoff for withdrawing for every $x > x^*_{NW}$ and vice versa for

[15] Throughout the rest of this chapter I assume that players withdraw/wait if the expected payoffs for the two actions are equal and roll over only if the expected payoff is strictly greater. This has no effect on the general results of the models but makes the representation of probabilities somewhat less cumbersome.

$x \leq x^*_{NW}$. Moreover, using the assumption that private signals are normally distributed with mean θ and precision β, the indifference equation can be rewritten as

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{NW} - \theta^*_{NW}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^*_{NW} - y^*_{NW}\right); \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^*_{NW} - y^*_{NW}\right)\right)} = t, \tag{3.6}$$

where Φ_2 is the CDF of the bi-variate normal distribution with means normalized to 0, variances to 1 and its third argument is the correlation.[16]

Analogously, if the LC does not roll over early, i's expected payoff for rolling over is given by $c_1 Pr\left(\theta > \theta^{**}_{NW} \mid y \leq y^*_{NW}, x_i\right) - t$. SC i wants to roll over if his expected payoff exceeds the expected payoff for withdrawing and accordingly, he is indifferent if

$$c_1 \cdot Pr\left(\theta > \theta^{**}_{NW} \mid y \leq y^*_{NW}, x_i\right) = c_1 \frac{Pr\left(\theta > \theta^{**}_{NW}, y \leq y^*_{NW} \mid x^{**}_{NW}\right)}{Pr\left(y \leq y^*_{NW} \mid x^{**}_{NW}\right)} = t. \tag{3.7}$$

For this indifference equation, x^{**}_{NW} constitutes the threshold for the threshold strategy and the expected payoff for rolling over is greater than the expected payoff for withdrawing for every $x > x^{**}_{NW}$ and vice versa for $x \leq x^{**}_{NW}$. Again, using the distributional assumption about private signals, we can reformulate the indifference equation as

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{NW} - \theta^{**}_{NW}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^*_{NW} - x^{**}_{NW}\right); -\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^*_{NW} - x^{**}_{NW}\right)\right)} = t. \tag{3.8}$$

An SC i is said to follow a threshold strategy around x^*_{NW} and x^{**}_{NW}, such that it is optimal to roll over if the LC rolls over and $x_i > x^*_{NW}$, or if the LC withdraws and $x_i > x^{**}_{NW}$ and withdraw otherwise. The index i is omitted for signal thresholds because SCs are homogeneous in their preferences and therefore the same equilibrium threshold strategy applies for all SCs.

However, it should be noted that I was not able to show analytically, that the indifference equations (3.6) and (3.8) always have a unique solution x^*_{NW} and x^{**}_{NW}. Since the concept of a threshold equilibrium is built on the assumption that there are unique thresholds that define threshold strategies, I conducted extensive numerical simulations to ensure that x^*_{NW} and x^{**}_{NW} are indeed unique, at least for the scenarios considered in this chapter.[17] For the sake of further argumentation and to prove additional results later, I therefore make the following assumption:

Assumption 3.1 (Uniqueness of x^*_{NW} and x^{**}_{NW})**.** Each of the indifference equations (3.6) and (3.8) has a unique solution x^*_{NW} and x^{**}_{NW}, which constitutes the threshold for the equilibrium strategy of the SCs.

Fundamental Thresholds: The outcome of the project depends on two factors: the fundamental state θ and the fraction of creditors who withdraw from the project, denoted by $1 - \ell_{NW} = 1 - b_{NW}\lambda - A$. The project succeeds ($R = 1$) if the fundamentals are sufficiently strong, relative to the fraction of creditors who withdraw, i.e. if $\theta > 1 - \ell_{NW}$. It fails ($R = 0$) if fundamentals are too weak, i.e. if $\theta \leq 1 - \ell_{NW}$. Since $\theta \in \mathbb{R}$ and $\ell_{NW} \in [0, 1]$, there are three cases in which the project fails or succeeds:

- $\theta \leq 0$: The fundamental state is so weak that the project fails even if all creditors roll over.

[16] Mind that in order to derive this expression, I made use of the fact that $y \mid x_i \sim N\left(x_i, (\beta + \gamma) / (\beta\gamma)\right)$. See Appendix B.4 for a derivation.

[17] See Appendix B.4 for information how the simulations were conducted.

- $\theta > 1$: The fundamental state is so strong that the project succeeds even if no creditor rolls over.
- $\theta \in (0, 1]$: The project succeeds only if a sufficient proportion of creditors rolls over.

The outcome in the first two cases only depend on the fundamental state, while the creditors' actions are irrelevant. However, in the third case, the outcome depends on the players, hence it constitutes the focus of my analysis. It captures the cases where the creditors have to coordinate on rolling over in order to render the project successful. If they fail to coordinate, the project fails. Therefore, I denote the cases where $\theta \in (0, 1]$ and $\ell_{NW} \leq 1 - \theta$ as coordination failure.

It is necessary to distinguish between failure of the project due to fundamentals and due to coordination failure because in the latter, the project is merely illiquid but not necessarily insolvent and could therefore continue operation if sufficient funds were available. Hence, project failure due to coordination failure is avoidable if coordination is sufficiently facilitated or promoted, e.g. through transparency requirements or policy dissemination measures taken by a policy-maker who wants to minimize the probability of project failure.

Given the creditors' threshold strategies derived above, by the law of large numbers the fraction of SCs who roll over can be written as the probability that an individual (marginal) creditor rolls over:

$$A(\theta, b_{NW}) = \int_0^{1-\lambda} a^*_{NW}(x_i, b_{NW}) di \tag{3.9}$$

$$= \begin{cases} (1-\lambda)Pr(x_i > x^*_{NW} \mid \theta) & \text{if } b_{NW} = 1 \\ (1-\lambda)Pr(x_i > x^{**}_{NW} \mid \theta) & \text{if } b_{NW} = 0, \end{cases} \quad = \begin{cases} (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta - x^*_{NW})\right) & \text{if } b_{NW} = 1 \\ (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta - x^{**}_{NW})\right) & \text{if } b_{NW} = 0, \end{cases}$$

where $a^*_{NW}(x_i, b_{NW})$ denotes the SC's equilibrium threshold strategy around x^*_{NW} and x^{**}_{NW}. With b_{NW} fixed, $A(\theta, b_{NW})$ and therefore $\ell_{NW} = \ell_{NW}(\theta, b_{NW})$ are monotonically increasing functions of θ, which implies that the fraction of creditors who withdraw $1 - \ell_{NW}(\theta, b_{NW})$ is strictly monotonically decreasing in θ. Hence, for any b_{NW} there is always a unique intersection of $\ell_{NW}(\theta, b_{NW})$ with the 45-degree line which defines the solution for the equation $\theta = 1 - \ell_{NW}(\theta, b_{NW})$. Conditional on the LC rolling over, the fundamental threshold θ^*_{NW} is defined as the solution to the critical mass condition

$$\theta^*_{NW} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^*_{NW} - x^*_{NW})\right), \tag{3.10}$$

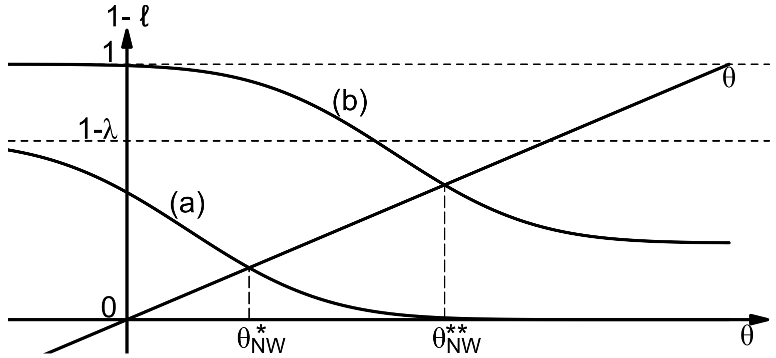
whereas conditional on the LC withdrawing, the fundamental threshold θ^{**}_{NW} is defined as the solution to the critical mass condition

$$\theta^{**}_{NW} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^{**}_{NW} - x^{**}_{NW})\right). \tag{3.11}$$

The interpretation of these thresholds is straight-forward: Conditional on the LC's action, if the fundamental value is realized above the respective threshold, fundamentals are strong enough to finish the project successfully, whereas if they are realized below (or on) the threshold, the project fails. This relation is illustrated in Figure (3.2).

Curve (a) represents the fraction of creditors who withdraw, conditional on the LC rolling over as a function of θ. Curve (b) depicts the fraction if the LC does not roll over. The fundamental thresholds are the intersections between the two curves and the 45 degree line. Note that curve (a) lies entirely below curve (b), so that $\theta^*_{NW} \leq \theta^{**}_{NW}$. The reason is that given the LC rolls over, the fraction of creditors who withdraw is accordingly smaller because on the one hand, the LC does

Figure 3.2: Proportion of creditors who withdraw $1-\ell$ at θ in game Γ_{NW}.



not withdraw and on the other hand, SCs interpret this as an optimistic signal and want to roll over more often as well.

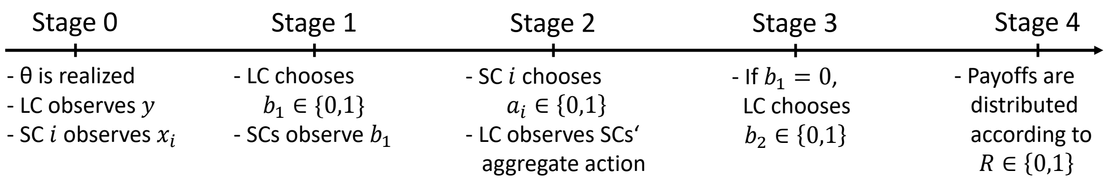
The solution (y^*_{NW}, x^*_{NW}, x^{**}_{NW}, θ^*_{NW}, θ^{**}_{NW}) to the system of non-linear equations (3.3), (3.6), (3.8), (3.10) and (3.11) describes the PBE in threshold strategies in Γ_{NW} and in subsequent sections these thresholds are used as the main indicators for how the creditors' equilibrium behavior changes if we vary the parameters of the game.

3.4 Game Γ with option to wait

3.4.1 Setup

In general, the setup of the game Γ with the option to wait is very similar to the game Γ_{NW} without waiting. In order to implement the new assumption, I use the framework proposed by Dasgupta (2007), who analyzes how small investors in a global game behave, if they are given the option to postpone their decision to reinvest in a project. Accordingly, I apply his setup to a large player.

Figure 3.3: Sequence of actions in the game with the option to wait.



As before, there is an LC of size λ and a crowd of SCs of aggregate size $1-\lambda$. They are invested in an investment project and at an interim stage of the project, they get to review their investment and decide about rolling over or withdrawing. The outcome of the project depends on the fundamental of the project and the percentage of creditors who withdraw. However, unlike in Γ_{NW}, the decision is modeled as a game of four instead of three stages, which are summarized in Figure 3.3. The major difference from the benchmark game Γ_{NW}, is that in stage 1 the LC gets to decide between the two actions "roll over early" (denote $b_1 = 1$) or "wait" (denote $b_1 = 0$).[18] If she rolls over early and the project is finished successfully, she receives a payoff of $c_1 - t > 0$, once

[18]The LC does not have the option to withdraw in stage 1 because it would always be preferable to wait. As long as the net payoffs for success or failure are the same in both stages, waiting gives at least the same payoff as withdrawing, but with the chance of higher payoff if the project succeeds. Hence, from the LC's perspective, the first stage can be considered as a gamble between a risky and a safe action, where the safe option allows to play again at a later stage. The risk-neutral LC always prefers to wait rather than withdraw in stage 1.

the project is complete and $-t$ if it fails. Once the LC rolled over early, she is committed to the project and cannot change her decision later. As in Γ_{NW}, each individual SC i observes the LC's action in stage 1 and decides in stage 2 whether to roll over (denote $a_i = 1$) or withdraw (denote $a_i = 0$). If the LC decides to wait in stage 1, in stage 3 she observes the SCs' aggregate action $A = \int_0^{1-\lambda} a_i di$. She then chooses between actions "roll over late" (denote $b_2 = 1$) and "withdraw" (denote $b_2 = 0$). If she decides to roll over late and the project is successful, she receives a net payoff of $c_2 - t > 0$, where $c_2 \leq c_1$ and $-t$ if it fails. If she withdraws, she receives 0. In stage 4, the outcome of the project is determined, depending on the fundamental state and the fraction of creditors who withdraw their loan. As before, if the fraction of creditors who withdraw is smaller than the fundamental state, then the project is successful and otherwise it fails. In table 3.2 the conditional payoffs for small and large creditors for each case are summarized.

Table 3.2: Payoffs in game Γ with the option to wait.

Player	SC i		LC		
Action	$a_i = 0$	$a_i = 1$	$b_1 = 0$ $b_2 = 0$	$b_1 = 0$ $b_2 = 1$	$b_1 = 1$
$R = 0$	0	$-t$	0	$-t$	$-t$
$R = 1$	0	$c_1 - t$	0	$c_2 - t$	$c_1 - t$

3.4.2 Equilibrium

Analogous to Γ_{NW}, the PBE in threshold strategies for the game Γ with the option to wait is defined as follows:

Definition 3.2 (Equilibrium in Γ)**.** The PBE in threshold strategies in Γ consists of the following elements:

1. Threshold strategies:
   - A threshold strategy around y^*, such that rolling over early ($b_1 = 1$) maximizes the LC's expected payoff if $y > y^*$ and waiting ($b_1 = 0$) otherwise.
   - A threshold strategy around s^*, such that rolling over late ($b_2 = 1$) maximizes the LC's expected payoff if $s > s^*$ and withdrawing ($b_2 = 0$) otherwise. $s = s(y, A)$ is a sufficient statistic with respect to θ, which summarizes all information that is relevant for the LC's decision in stage 3.
   - A threshold strategy around x^* and x^{**}, such that rolling over ($a_i = 1$) maximizes SC i's expected payoff if a) $b_1 = 1$ and $x_i > x^*$, or b) $b_1 = 0$ and $x_i > x^{**}$ and withdrawing ($a_i = 0$) otherwise.
2. Creditors' aggregate action: The mass of creditors who roll over is given by $\ell = (b_1 + b_2) \cdot \lambda + A$, where $A = \int_0^{1-\lambda} a_i di$ is the mass of SCs who roll over. The mass of creditors who withdraw is $1 - \ell$.
3. Critical mass conditions:
   - A fundamental threshold θ^*, such that conditional on $b_1 = 1$, the project succeeds ($R = 1$) iff $\theta > \theta^*$.
   - A fundamental threshold $\overline{\theta}^{**}$, such that conditional on $b_2 = 1$, the project succeeds ($R = 1$) iff $\theta > \overline{\theta}^{**}$.

- A fundamental threshold $\underline{\theta}^{**}$, such that conditional on $b_2 = 0$, the project succeeds ($R = 1$) iff $\theta > \underline{\theta}^{**}$.

4. Beliefs:

- The LC forms beliefs $f(R \mid y)$ and $f(R \mid s)$ using Bayes' Law.
- SC i forms beliefs $f(R \mid x_i, y > y^*)$ and $f(R \mid x_i, y \leq y^*)$ using Bayes' Law.

Large Creditor: With regards to the LC's decisions, the model is easiest to solve backwards, i.e. by starting with her decision in stage 3, given she decided to wait in stage 1. In stage 3, the LC observes the aggregate action of SCs $A = \int_0^{1-\lambda} a_i di$. Her information on the realized value of θ consists of y and A. As shown by Dasgupta (2007), these two signals can be summarized in a sufficient statistic $s = s(y, A)$ such that we can express the LC's optimization problem in terms of the one-dimensional statistic s instead of the signal vector (y, A). Analogous to Γ_{NW}, we can then express her threshold strategy in terms of a threshold s^* for the statistic s, so that it is optimal for her to roll over whenever $s > s^*$ and withdraw if $s \leq s^*$. However, unlike Dasgupta (2007), I assume that the LC observes the mass of SCs that roll over perfectly instead of a noisy signal of it. This assumption implies that by observing A, the LC can perfectly infer the realization of θ. To see why, recall that due to the law of large numbers, the mass of SCs who roll over can be written in terms of the probability that an individual SC rolls over. For a given threshold strategy around x^{**}, this means $A = (1-\lambda)\Phi(\sqrt{\beta}(\theta - x^{**}))$. Since this expression is completely deterministic at stage 3, the realized value of

$$\theta = x^{**} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{A}{1-\lambda}\right). \tag{3.12}$$

Compared to A, the noisy signal y does not contain any additional information compared to θ. Hence, in this specific setup with perfect learning $s = \theta$. Since the LC knows that conditional on rolling over late, the project is successful if $\theta > \overline{\theta}^{**}$, she wants to roll over whenever $\theta > \overline{\theta}^{**}$ and withdraw otherwise. It follows that she chooses a threshold strategy around $s^* = \overline{\theta}^{**}$. Due to the assumption that the LC learns the realization of θ by observing the aggregate action of the SC, the action of the LC in stage 3 and the outcome of the project are perfectly congruent: The LC rolls over late if and only if the project will be successful and she withdraws if and only if the project will fail.[19]

In stage 1, the LC wants to roll over early if her expected payoff for rolling over is greater than her expected payoff from waiting and vice versa. The expected payoff from rolling over early is $c_1 \cdot Pr\left(\theta > \theta^* \mid y\right)$. In order to determine an expression for the expected payoff for waiting, the LC has to predict her own behavior in stage 3, conditional on her information in stage 1. For that, suppose that she waits in stage 1. She knows that she will roll over late and the project will be successful if $\theta > \overline{\theta}^{**}$ and that she withdraws and the project fails if $\theta \leq \overline{\theta}^{**}$. Hence, the expected payoff from rolling over late is

$$(c_2 - t) \cdot Pr\left(\theta > \overline{\theta}^{**} \mid y\right) - t \cdot Pr\left(\theta \leq \overline{\theta}^{**} \mid y\right) = c_2 \cdot Pr\left(\theta > \overline{\theta}^{**} \mid y\right) - t. \tag{3.13}$$

[19]If the LC were to only observe a noisy signal of SCs' aggregate action, she would not know θ and there would be cases where she chooses the objectively wrong action, i.e. the action that yields the ex-post lower payoff. The assumption that the LC can perfectly infer θ from observing A greatly simplifies the equilibrium derivation as well as the comparative statics analysis, while still giving insight into the implications that the option to wait has on the creditors' behavior. Nevertheless, in Chapter 4 of this thesis I investigate whether the results change if I assume that the LC only observes a noisy signal of the mass of SCs who roll over, i.e. the case where she can potentially make the wrong choice in stage 3. I find that the assumption has no qualitative impact.

To maximize the expected payoff, the LC wants to roll over early if her expected payoff from rolling over is greater than her expected payoff from waiting, that is, if $c_1 \cdot Pr(\theta > \theta^* \mid y) > c_2 \cdot Pr(\theta > \overline{\theta}^{**} \mid y) + t \cdot Pr(\theta \leq \overline{\theta}^{**} \mid y)$ and vice versa. She is indifferent if

$$c_1 \cdot Pr\left(\theta > \theta^* \mid y^*\right) = c_2 \cdot Pr\left(\theta > \overline{\theta}^{**} \mid y^*\right) + t \cdot Pr\left(\theta \leq \overline{\theta}^{**} \mid y^*\right), \tag{3.14}$$

$$\Leftrightarrow \quad c_1 \cdot Pr\left(\theta > \theta^* \mid y^*\right) = t + (c_2 - t)\, Pr\left(\theta > \overline{\theta}^{**} \mid y^*\right), \tag{3.15}$$

where y^* solves the indifference equation.[20] Therefore, in stage 1 it is optimal for her to roll over early if $y > y^*$ and wait if $y \leq y^*$. Since $\theta \mid y \sim N(y, \gamma^{-1})$, the indifference equation (3.14) can be written as

$$c_1 \cdot \Phi\left(\sqrt{\gamma}\,(y^* - \theta^*)\right) = c_2 \cdot \Phi\left(\sqrt{\gamma}\left(y^* - \overline{\theta}^{**}\right)\right) + t \cdot \Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**} - y^*\right)\right), \tag{3.16}$$

$$\Leftrightarrow \quad c_1 \cdot \Phi\left(\sqrt{\gamma}\,(y^* - \theta^*)\right) = t + (c_2 - t)\, \Phi\left(\sqrt{\gamma}\left(y^* - \overline{\theta}^{**}\right)\right). \tag{3.17}$$

Notice the difference between the indifference condition (3.16) in the game with the option to wait and the condition (3.3) in the benchmark game. The expressions for the expected payoffs for rolling over on the LHSs of both equations are identical, however, the RHSs differ because the expected payoffs for the respective alternative actions differ.

Small Creditors: Given that the LC rolls over early, the individual SC i knows that in equilibrium $y > y^*$ and that the project succeeds if $\theta > \theta^*$. Hence, i's expected payoff from rolling over is $c_1 \cdot Pr(\theta > \theta^* \mid y > y^*, x_i) - t$, while for withdrawing it is 0. Optimally i wants to roll over if $c_1 \cdot Pr(\theta > \theta^* \mid y > y^*, x_i) - t > 0$ and withdraw otherwise. He is indifferent if

$$c_1 \cdot Pr\left(\theta > \theta^* \mid y > y^*, x^*\right) = c_1 \frac{Pr\left(\theta > \theta^*, y > y^* \mid x^*\right)}{Pr\left(y > y^* \mid x^*\right)} = t, \tag{3.18}$$

where on the LHS I apply Bayes' Rule and x^* solves the equation, such that the expected payoff for rolling over is greater than the expected payoff for withdrawing for every $x > x^*$ and vice versa for $x \leq x^*$. Moreover, using $\theta \mid x_i \sim N(x_i, \beta^{-1})$, the indifference equation can be rewritten as

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^* - \theta^*),\, \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*);\, \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*)\right)} = t. \tag{3.19}$$

Generally, for the case that the LC waits in stage 1, the derivation of an SC's strategy differs from the benchmark model because the SC essentially has to predict the LC's action in stage 3. However, since I assume that the LC observes θ perfectly in stage 3, for an SC it suffices to predict the outcome of the project, because the LC will always choose "correctly". Hence, the derivation of the equilibrium threshold coincides again with game Γ_{NW}. Accordingly, given the LC waits, SC i's expected payoff for rolling over is given by

$$\begin{aligned} &c_1 \cdot \left(Pr\left(\theta > \overline{\theta}^{**} \mid y \leq y^*, x_i\right) + Pr\left(\underline{\theta}^{**} < \theta \leq \overline{\theta}^{**} \mid y \leq y^*, x_i\right)\right) - t \\ &= c_1 \cdot Pr\left(\theta > \overline{\theta}^{**} \mid y \leq y^*, x_i\right) - t, \end{aligned} \tag{3.20}$$

[20]In Appendix B.3 I show that y^* is the unique solution to the equation and that the expected payoff for rolling over early is greater than the expected payoff for waiting for every $y > y^*$ and vice versa for $y \leq y^*$.

where the last equality follows because in equilibrium $\overline{\theta}^{**} \leq \underline{\theta}^{**}$.[21] SC i wants to roll over if his expected payoff exceeds the expected payoff for withdrawing and accordingly, he is indifferent if

$$c_1 \cdot Pr\left(\theta > \overline{\theta}^{**} \mid y \leq y^*, x_i\right) = c_1 \frac{Pr\left(\theta > \overline{\theta}^{**}, y \leq y^* \mid x^{**}\right)}{Pr\left(y \leq y^* \mid x^{**}\right)} = t, \tag{3.21}$$

The value x^{**} constitutes the threshold such that the expected payoff for rolling over is greater than the expected payoff for withdrawing for every $x > x^{**}$ and vice versa for $x \leq x^{**}$. Using the distributional assumption about private signals, the indifference equation is parameterized as

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**} - \theta^{**}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^* - x^{**}\right); -\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^* - x^{**}\right)\right)} = t. \tag{3.22}$$

SCs are said to follow a threshold strategy around x^* and x^{**} where they roll over if the private signal realizes above the respective threshold and withdraw if it realizes on or below the threshold. Notice that the SCs' equilibrium threshold strategy in Γ is essentially identical to the one in Γ_{NW}. This is because the only decision by the LC that is relevant for an SC's choice is her action in the stage 1. Since the LC always chooses the action that coincides with the outcome of the project in stage 3, SCs do not have to additionally predict what the LC might do in stage 3 if she waits in stage 1.

As in case of Γ_{NW}, it was not possible for me to prove analytically that x^* and x^{**} are unique, which is why I resort to the same numerical simulation procedure as for the indifference conditions in Γ_{NW}.[22] The results indicate that the two thresholds are indeed unique, which is why I make the following assumption for the rest of this chapter:

Assumption 3.2 (Uniqueness of x^* and x^{**})**.** Each of the indifference equations (3.19) and (3.22) has a unique solution x^* and x^{**}, which constitutes the threshold for the equilibrium strategy of the SCs.

Fundamental Thresholds: The project succeeds ($R = 1$) if the fundamentals are strong relative to the percentage of creditors who withdraw ($\theta > 1 - \ell$) and fails ($R = 0$) otherwise. Again, in contrast to project failure due to weak fundamentals, I denote the case that $\theta \in (0, 1]$ and $\ell \leq 1 - \theta$ as coordination failure. Analogous to game Γ_{NW}, for each action of the LC in stage 1, the fraction of creditors who roll over $\ell = \ell(\theta, b_1, b_2)$ is a monotonously increasing function of θ. It follows that for each pair (b_1, b_2) there is a unique value of θ, such that $\theta = 1 - \ell(\theta, b_1, b_2)$.[23] As before, using the law of large numbers, these values are defined by the critical mass conditions

$$\theta^* = 1 - \lambda - (1 - \lambda)\Phi\left(\sqrt{\beta}\left(\theta^* - x^*\right)\right), \tag{3.23}$$

$$\overline{\theta}^{**} = 1 - \lambda - (1 - \lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**} - x^{**}\right)\right), \tag{3.24}$$

$$\underline{\theta}^{**} = 1 - (1 - \lambda)\Phi\left(\sqrt{\beta}\left(\underline{\theta}^{**} - x^{**}\right)\right). \tag{3.25}$$

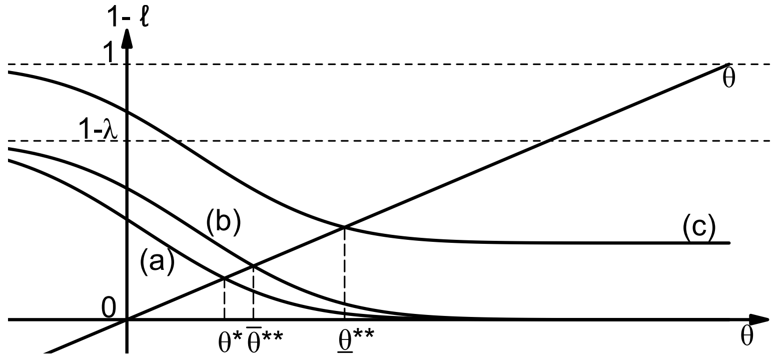
Given the LC rolls over early, rolls over late or withdraws, the project succeeds if $\theta > \theta^*$, or if $\theta > \overline{\theta}^{**}$, or if $\theta > \underline{\theta}^{**}$ and fails otherwise. The derivation of these fundamental thresholds is illustrated in Figure 3.4.

[21] For proof see Appendix B.3.

[22] See Appendix B.4 for information how the simulations were conducted.

[23] To see why there is a unique solution, cf. game Γ_{NW}.

Figure 3.4: Fraction of creditors who withdraw $1-\ell$ at θ in game Γ.



The curves (a), (b) and (c) represent the fraction of creditors who withdraw $1-\ell$ as a function of θ, conditional on the LC rolling over early (a), rolling over late (b) and withdrawing (c). Their intersections with the 45-degree line define the fundamental thresholds. Notice that $\theta^* < \overline{\theta}^{**} < \underline{\theta}^{**}$. This ordering is intuitive because if the LC rolls over early, she not only commits herself to rolling over with her size $\lambda > 0$, but also sends a positive signal to SCs who are then more willing to roll over as well. If the LC rolls over late, she also commits her size λ to rolling over, however only after sending a pessimistic signal by waiting in stage 1. SCs are less willing to also roll over so that the fundamental range for project success ($\theta > \overline{\theta}^{**}$) is smaller than if the LC rolled over early.

Notice that the threshold $\underline{\theta}^{**}$ has no relevance for the equilibrium behavior of creditors or the project outcome. Since I assume that the LC knows the realization of θ in stage 3, she knows that the project succeeds if $\theta > \overline{\theta}^{**}$ and she rolls over. Hence, she has an incentive to roll over whenever $\theta > \overline{\theta}^{**}$. At the same time if $\theta \leq \overline{\theta}^{**}$, the project can never succeed, even if she rolls over. Hence, she withdraws whenever $\theta \leq \overline{\theta}^{**}$. It follows that there is never a case where $\theta \in (\overline{\theta}^{**}, \underline{\theta}^{**})$ and the LC withdraws so that neither for SCs nor for the project outcome the threshold $\underline{\theta}^{**}$ is binding. This is different if the LC observes the mass of SCs who roll over A with noise. In that case there is a chance that she observes such a favorable signal of A that she believes that $\theta > \overline{\theta}^{**}$, even though it is actually smaller than $\overline{\theta}^{**}$.[24] Hence, in my equilibrium analysis below I solely focus on the fundamental thresholds θ^* and $\overline{\theta}^{**}$ while leaving $\underline{\theta}^{**}$ aside, as it does not affect the players' equilibrium behavior.

Analogous to Γ_{NW}, the solution (y^*, x^*, x^{**}, θ^*, $\overline{\theta}^{**}$, $\underline{\theta}^{**}$) to the system of non-linear equations (3.16), (3.19), (3.22), (3.23), (3.24) and (3.25) describes the PBE in threshold strategies in Γ. In the equilibrium analysis below, these thresholds are compared to the thresholds from Γ_{NW} and used as the main indicators for the creditors' equilibrium behavior.[25]

[24] This scenario is discussed in Chapter 4.

[25] One note of caution: The subscript NW in all equilibrium variables of game Γ_{NW} are used not only to distinguish them from the corresponding variables in game Γ. They should also make clear that the values of the variables in equilibrium need not be identical. Due to the difference in the structure of the two games, the equilibrium behavior of the players is generally not the same (even if the parameters are the same) and thus the values of thresholds such as θ^* and θ^*_{NW} are not identical. In fact, the idea of the comparative equilibrium analysis conducted in later sections of this chapter builds on the idea of investigating the difference between equilibrium variables from the two games for identical sets of parameters.

3.5 Solution and parameter specification

The solution $(y^*_{NW}, x^*_{NW}, x^{**}_{NW}, \theta^*_{NW}, \theta^{**}_{NW})$ to the system of non-linear equations

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{NW} - \theta^*_{NW}\right)\right) = t, \tag{3.26}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{NW} - \theta^*_{NW}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^*_{NW} - y^*_{NW}\right); \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^*_{NW} - y^*_{NW}\right)\right)} = t, \tag{3.27}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{NW} - \theta^{**}_{NW}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^*_{NW} - x^{**}_{NW}\right); -\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^*_{NW} - x^{**}_{NW}\right)\right)} = t, \tag{3.28}$$

$$\theta^*_{NW} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta^*_{NW} - x^*_{NW}\right)\right), \tag{3.29}$$

$$\theta^{**}_{NW} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta^{**}_{NW} - x^{**}_{NW}\right)\right) \tag{3.30}$$

describes the PBE in threshold strategies in game Γ_{NW} without the option to wait for the LC.[26] The solution $(y^*, x^*, x^{**}, \theta^*, \overline{\theta}^{**}, \underline{\theta}^{**})$ to the system

$$c_1 \cdot \Phi\left(\sqrt{\gamma}\left(y^* - \theta^*\right)\right) = c_2 \cdot \Phi\left(\sqrt{\gamma}\left(y^* - \overline{\theta}^{**}\right)\right) + t \cdot \Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**} - y^*\right)\right), \tag{3.31}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^* - \theta^*\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^* - y^*\right); \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(x^* - y^*\right)\right)} = t, \tag{3.32}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**} - \overline{\theta}^{**}\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^* - x^{**}\right); -\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\left(y^* - x^{**}\right)\right)} = t, \tag{3.33}$$

$$\theta^* = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta^* - x^*\right)\right), \tag{3.34}$$

$$\overline{\theta}^{**} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**} - x^{**}\right)\right), \tag{3.35}$$

$$\underline{\theta}^{**} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\underline{\theta}^{**} - x^{**}\right)\right) \tag{3.36}$$

describes the PBE in threshold strategies in game Γ, where the LC has the option to wait in stage 1.[27] Due to the number of dimensions to consider and the non-linear structure of the equations, neither of the two systems can be solved analytically. Therefore, I analyze equilibria from the following two perspectives: First, I investigate how players behave if I let γ go to infinity or to zero, while I keep β fixed. Second, away from the limit I resort to numerical methods to solve the models and derive comparative statics results.[28] More specifically, while keeping all other parameters fixed, I solve the two models for small increments of γ in the range $[0.1, 10]$ and plot the resulting equilibrium variables on the y-axis against (the log of) γ/β on the x-axis. I repeat this procedure for different specifications of the other parameters, as explained below.

As is common in the global games literature, I focus on the investigation of the effect of changes in information precision, or more specifically, the relative information precision γ/β. Since learning and signaling constitute the central mechanism by which creditors make their decisions, the "quality" of information in terms of its precision play the key role in understanding the different

[26] Subscript NW indicates game without option to wait, i.e. *no waiting*.

[27] A comprehensive summary of all equilibrium thresholds and their interpretation is given in Table B.1 in Appendix B.1.

[28] I use the (Quasi-)Newton's algorithm (cf. Judd, 1998, Ch. 5) as implemented in the `nleqslv`-command in the R-package of the same name (Hasselman, 2023). See Appendix B.4 for more information on the exact setting of control arguments of the R-command.

incentives in the two games. Throughout the analysis, the parameters β, c_1 and t are kept constant, while λ and c_2 are varied in order to check the sensitivity of results. More specifically, the SCs' private information precision β is fixed at 1 in order to interpret γ in terms of fractions or multiples of β. Similarly, c_1 is fixed at 1 in order to present payoffs c_2 and t as fractions or multiples of c_1. The costs for investment t are set to 0.3 to keep a balance between two arguments: On the one hand, the payoff for withdrawing should be non-negligible, so that the safe action of withdrawing is a valid alternative to the risky action of rolling over, i.e. t should not be too large. On the other hand, some interpretations of results tend to be more tractable if t is set sufficiently small relative to c_1, in particular if $t < 0.5c_1$. Therefore, all analyzed scenarios consider the following ordering of payoffs: $c_1 > c_2 > 0.5c_1 > t > 0$. For the baseline scenarios, c_2 is fixed at 0.9999. The rationale for setting c_2 close to c_1 is to reduce the impact that a substantially smaller payoff for rolling over late might have on the LC's decision in stage 1. The main focus of my analysis is to explore the LC's signaling and learning motives with respect to her information precision, which would be disturbed if the payoff for waiting was too small, relative to c_1. However, attempts at solving the models numerically while c_2 is equal to c_1 turn out to be unstable in many cases which is why I resort to an appropriate approximation and set c_2 marginally smaller than c_1.[29] Nevertheless, in an additional exercise I solve the models for $c_2 \in \{0.6, 0.8\}$ in order to see how previous results are affected by the assumption. For the LC's size λ I use the values 0 and 0.5. While $\lambda = 0.5$ can be seen as representative for the classical scenario where the LC is actually large, the case of $\lambda = 0$ has interesting implications with regards to the definition of a "large creditor". In particular, the LC is no longer "large" in the sense that she has a substantial impact on the project outcome simply by rolling over a large fraction of funds, but only by being observed by the other market participants. Considering this scenario allows to boil down the LC's impact on the project outcome to her ability to credibly signal her expectations to SCs.[30]

For the analysis of the investors' behavior for different parameter sets, it is not intuitive to look at plain equilibrium thresholds. A threshold y^* gives an idea of the values of the private signal y for which the LC would rather roll over early than wait. However, this threshold has little meaning if the distribution of the private signal itself changes by changing the parameters of the game, as would be the case if we change the signal precision γ. Hence, rather than comparing the equilibrium thresholds, I compare the ex-ante probabilities that a creditor takes one action or another.[31]

The LC's decision in stage 3 is trivial, which is why my analysis focuses on her behavior in stage 1. It is described by the probability that the LC rolls over early, given a realization of the fundamental value of the project $\hat{\theta}$. For simplicity, I denote this probability by P^y_{NW} and P^y respectively. It is calculated by

$$P^y_{NW} \equiv Pr\left(y > y^*_{NW} \mid \hat{\theta}\right) = \Phi\left(\sqrt{\gamma}\left(\hat{\theta} - y^*_{NW}\right)\right), \tag{3.37}$$

$$P^y \equiv Pr\left(y > y^* \mid \hat{\theta}\right) = \Phi\left(\sqrt{\gamma}\left(\hat{\theta} - y^*\right)\right). \tag{3.38}$$

Note that in order to compute the above measure of the LC's behavior, it is necessary to fix the realization of the fundamental value $\hat{\theta}$, under which the private signal is realized. In my

[29] Besides technical limitations, there may also be intuitive reasons why $c_2 < c_1$. E.g. to incorporate the risks that the LC waits too long and misses her opportunity to roll over. Also, there are of course opportunity costs and interests to be paid that reduce the present value of a future investment.

[30] The parameters, as well as their values used for the numerical simulations and their interpretation are summarized in Table B.2 in the appendix.

[31] For completeness, the graphical results for the plain equilibrium thresholds are given in Appendix B.1.

numerical simulation exercises, I consider realizations of -0.5, 0 and 0.5 as representative of a bad, intermediate and good fundamental state.

In order to understand the LC's incentives to roll over early, it is also necessary to understand how the SCs react to her decision. Analogously to the LC, the probabilities that an individual SC i rolls over, given the LC's decision in stage 1, are given by

$$Pr\left(x_i > x^*_{NW} \mid \hat{\theta}\right) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*_{NW}\right)\right) \text{ and } Pr\left(x_i > x^* \mid \hat{\theta}\right) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*\right)\right), \quad (3.39)$$

$$Pr\left(x_i > x^{**}_{NW} \mid \hat{\theta}\right) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}_{NW}\right)\right) \text{ and } Pr\left(x_i > x^{**} \mid \hat{\theta}\right) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}\right)\right). \quad (3.40)$$

Notice that since there is a continuum of SCs, the law of large numbers applies so that this probability can be interpreted as the fraction of SCs who decide to roll over. This perspective allows us to quantify the percentage of SCs that roll over if and only if the LC rolls over early. It is given by the difference between the fraction of those SCs who roll over if the LC rolls over early and those who received a private signal sufficiently well to make them roll over even if the LC waits. For simplicity, I call this measure the "herd-index", which is defined as

$$H_{NW} \equiv \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*_{NW}\right)\right) - \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}_{NW}\right)\right), \quad (3.41)$$

$$H \equiv \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*\right)\right) - \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}\right)\right). \quad (3.42)$$

The herd-index represents the fraction of those SCs, who always take the same action as the LC in stage 1, which is a classical example of herd behavior (cf. Banerjee, 1992; Bikhchandani et al., 1992; Hirshleifer and Teoh, 2003). It is an important measure for the incentive for the LC to roll over early.

The interpretation of the results of comparative statics with respect to the probability of coordination failure is less intuitive. Recall that coordination failure is the incidence of a failure of the project, even though fundamentals could have led to success if sufficiently many creditors rolled over their loans. In terms of equilibrium thresholds, this is the case if θ is greater than 0 but smaller than or equal to the respective fundamental threshold. Hence, the probability of coordination failure corresponds to the probability that the fundamental value realizes in this interval. However, since I assume that the fundamental value is drawn from an improper uniform prior over the real line, it is not possible to give an exact expression for this probability. Nevertheless, in order to draw conclusions about the probability of coordination failure, I rely on the heuristic argument that an increase in any of the fundamental thresholds coincides with an increase in the range of fundamentals that lead to coordination failure and thus to an increase in the probability of coordination failure. Even if this simplification appears very restrictive at first glance, the results of my model are consistent in their sign and relative size with the results of a model with a properly defined fundamental distribution, as long as this distribution does not change with any of the parameters considered in the comparative statics exercises. Hence, throughout my analysis, I use the various fundamental thresholds as measures for the conditional probability of coordination failure.

3.6 Comparative statics results

3.6.1 The effect of information precision in the limit

To get an idea of how the LC's information precision γ relative to β affects the players' equilibrium behavior, let us assume that $\gamma \to \infty$, while $\beta < \infty$ is constant. The result of this thought experiment is summarized in the following proposition[32]:

Proposition 3.1. *Suppose $\gamma \to \infty$ and $\beta < \infty$ is constant. The equilibrium thresholds in game Γ_{NW} converge to*

$$y^*_{NW} \to \theta^*_{NW} \to 0, \quad x^*_{NW} \to -\infty, \quad x^{**}_{NW} \to \infty, \quad \theta^{**}_{NW} \to 1,$$

while for game Γ they converge to

$$y^* \to \theta^* \to 0, \quad x^* \to -\infty, \quad x^{**} \to \infty, \quad \overline{\theta}^{**} \to 1-\lambda, \quad \underline{\theta}^{**} \to 1.$$

Proposition 3.1 states the same result for both games: As the private signal y becomes infinitely precise, the SCs will completely ignore their private signals x_i and follow the LC in her decision in stage 1, i.e., herd-index $H \to 1$. The reason is that the LC's action becomes a perfect predictor of the outcome of the project. It follows that the only decision relevant to the LC is her choice in stage 1. If she rolls over, all SCs follow her and if she waits or withdraws, SCs withdraw as well. Accordingly, this translates into fundamental thresholds. Given that the LC and all SCs roll over, the project can only fail if it would have failed anyway, i.e. if $\theta \leq 0$. However, since in this case the LC would not roll over in the first place, this scenario never occurs. At the same time, if the LC and all SCs withdraw, the project can only succeed if $\theta > 1$. Again, since in this case the LC would have rolled over, this scenario also never occurs. What remains is that the LC and SCs always roll over and the project always succeeds if $\theta > 0$ and fails if $\theta \leq 0$. One interesting consequence is that in this experiment, coordination failure is not possible because the LC is always perfectly informed, i.e. she always predicts the outcome correctly and all SCs follow her in her action. Hence, there is no uncertainty, neither with regards to the fundamental state nor with regards to the other players' information.

On the other hand, if we let $\gamma \to 0$, results can be summarized in the following proposition[33]:

Proposition 3.2. *Suppose $\gamma \to 0$ and $\beta > 0$ is constant. The equilibrium thresholds in game Γ_{NW} converge to*

$$y^*_{NW} \to \begin{cases} -\infty & \text{if} \quad 0.5c_1 > t, \\ (1-\lambda)\frac{t}{c_1} & \text{if} \quad 0.5c_1 = t, \\ \infty & \text{if} \quad 0.5c_1 < t, \end{cases} \qquad x^*_{NW} \to (1-\lambda)\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}},$$

$$x^{**}_{NW} \to \lambda + (1-\lambda)\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}}, \qquad \theta^*_{NW} \to (1-\lambda)\frac{t}{c_1}, \qquad \theta^{**}_{NW} \to \lambda + (1-\lambda)\frac{t}{c_1},$$

[32]For proof see Appendix B.3.
[33]For proof see Appendix B.3.

while for Γ they converge to

$$y^* \to \begin{cases} -\infty & \text{if} \quad c_1 > c_2 + t, \\ (1-\lambda)\frac{t}{c_1} & \text{if} \quad c_1 = c_2 + t, \\ \infty & \text{if} \quad c_1 < c_2 + t, \end{cases} \qquad x^* \to x^{**} \to (1-\lambda)\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}},$$

$$\theta^* \to \overline{\theta}^{**} \to (1-\lambda)\frac{t}{c_1}, \qquad \underline{\theta}^{**} \to \lambda + (1-\lambda)\frac{t}{c_1}.$$

With regards to game Γ_{NW}, Proposition 3.2 says that the LC's private signal y conveys no information about θ, so that for her betting on the project's success or failure amounts to flipping a coin. Hence, she always rolls over if the ex-ante expected payoff from rolling over is greater than the ex-ante expected payoff from withdrawing ($0.5c_1 > t$) and she never rolls over in case ($0.5c_1 < t$). In the special case that $0.5c_1 = t$, the LC's threshold y^*_{NW} converges to θ^*_{NW} so that she rolls over whenever $y > \theta^*_{NW} \to (1-\lambda)t/c_1$ and withdraws otherwise. For an individual SC this means that the LC's action also does not convey any information about the fundamental state θ, so that they take her action only into account as her size λ is added to the fraction of creditors who roll over or not. The difference between x^*_{NW} and x^{**}_{NW} accordingly is λ. The same holds for fundamental thresholds, which only adjust for the LC's size λ. An interesting consequence is that if the LC becomes infinitely small, i.e. if $\lambda \to 0$, the game collapses to the canonical global coordination game with private information and without large player as presented in Morris and Shin (2003).

For game Γ the picture looks similar. As before, if the private signal y becomes uninformative with regards to the fundamental state, from the LC's perspective the probability of success amounts to a coin toss, regardless of her own action in stage 1. Hence, her decision to roll over early or not depends entirely on payoffs c_1, c_2 and t. She always wants to roll over if her payoff from rolling over early is greater than the payoff from waiting $c_1 > c_2 + t$ and wait if $c_1 < c_2 + t$. If $c_1 = c_2 + t$, she again rolls over early if $y > \theta^* \to (1-\lambda)t/c_1$ and withdraws otherwise. Much like in Γ_{NW}, since the LC's decision does not contain information about the fundamental state, SCs do not update their beliefs and thus their decision only relies on private signals. Hence, thresholds x^* and x^{**} converge. This also translates into fundamental thresholds, which again only adjust for the LC's size λ.

Another important consequence from Propositions 3.1 and 3.2 considers the SCs' herd behavior as measured by the herd-index[34]:

Corollary 3.1. *a) In the limit, as $\gamma \to 0$ and $\beta > 0$ is constant, the herd-index converges to*

$$\lim_{\gamma\to 0} H_{NW} = \Phi(M) - \Phi\left(M - \sqrt{\beta}\lambda\right), \quad \text{where} \quad M = \sqrt{\beta}\left(\theta - (1-\lambda)\frac{t}{c_1} - \Phi^{-1}\left(\frac{t}{c_1}\right)\right)$$

$$\text{and} \quad \lim_{\gamma\to 0} H = 0,$$

for a realized fundamental state θ.

[34] For proof see Appendix B.3.

b) As $\gamma \to \infty$ and $\beta < \infty$ is constant, the herd-index converges to

$$\lim_{\gamma \to \infty} H_{NW} = \lim_{\gamma \to \infty} H = 1.$$

c) It holds that $H_{NW} \geq 0$ and $H \geq 0$ for all $\theta \in \mathbb{R}$.

Corollary 3.1 is worth noting for two reasons. First, it demonstrates intuitively how herd behavior changes in the limit if we let $\gamma \to 0$ and $\gamma \to \infty$, which will become useful when explaining some of the numerical simulation results below. In particular, note that as the LC becomes infinitely better informed than SCs ($\gamma \to \infty$), in both games they will ignore their private signals completely and follow the LC in her action. In this case, the LC manages to perfectly coordinate all SCs to take the same action as she does and thus there is no coordination failure. Second, in general, $\lim_{\gamma \to 0} H_{NW} \neq \lim_{\gamma \to 0} H$. Only if $\lambda \to 0$ the two values converge. This finding highlights one major difference between the two games, which is the irreversibility of the LC's choice in stage 1 in Γ_{NW}. If she decides to withdraw, she cannot roll over later and the SCs discount her from the fraction of creditors who roll over for good. If $\lambda > 0$, it follows that there are cases where the fundamental state falls in the interval $(0, \lambda]$ so that a success would have been possible if the LC did not withdraw. However, without the LC, success is not possible even if all SCs roll over. This decreases the overall probability of success so that SCs are less willing to roll over. As a consequence, even if the LC is completely uninformed about θ, due to the strategic complementarity of rolling over, the SCs are still willing to follow her and the herd-index does not converge to 0 in Γ_{NW}. In Γ this is different because the LC has the choice to roll over later if she learns that the project will succeed. Hence, for the SCs her action in stage 1 is not only uninformative about the fundamental state, but also about the overall probability of success. As a consequence, an individual SC will not alter his choice in response to the LC choosing one or another action in stage 1 and his strategy is the same for both cases. Consequently, there is no herding.

Finally, the notion that $H_{NW} \geq 0$ and $H \geq 0$ for all realized fundamental values θ is important as well. Intuitively, it says that herding is never negative, which is a reasonable result. However, it also implies that SCs consider an early rollover by the LC an unambiguously optimistic signal about the fundamental state, whereas withdrawing or waiting is always a pessimistic signal. To see this, consider

$$H_{NW} \geq 0 \quad \Leftrightarrow \quad Pr\left(x > x^*_{NW} \mid \theta\right) \geq Pr\left(x > x^{**}_{NW} \mid \theta\right) \quad \text{and} \tag{3.43}$$

$$H \geq 0 \quad \Leftrightarrow \quad Pr\left(x > x^* \mid \theta\right) \geq Pr\left(x > x^{**} \mid \theta\right). \tag{3.44}$$

That is, the fraction of SCs who roll over if the LC rolls over early is always at least as large as the fraction of SC who roll over even if the LC does not roll over in stage 1.

3.6.2 The probability of early rollover and herd behavior

The LC's incentives. In order to understand how the probability of an early rollover by the LC changes in reaction to changes in the games' parameters, it is necessary to understand her incentives to roll over early. In the game without option to wait Γ_{NW}, rolling over in stage 1 is the only way to participate in a potential success of the project. Therefore, her only consideration is how likely a success is, given that she rolls over early. If, conditional on her private signal y, this probability is sufficiently large, she rolls over and otherwise she withdraws. Since this is her only decision, there is no choice between signaling to SCs and rolling over, the two actions are identical.

Hence, signaling is not a conscious decision, but a by-product of her decision to participate in the project or not. This is different in the game with the option to wait. Here the two choices can be distinguished: The LC has the choice to roll over early and thereby signal to SCs that she bets on the success of the project or to wait and observe the realized fundamental in order to make a better informed decision at a later stage. Hence, unlike in game Γ_{NW}, the LC can participate in the project without rolling over early, so the decisions to roll over and signaling are separated.

Generally, her incentive to roll over early depends on payoffs c_1, c_2 and t as well as how SCs react to her action. If the payoff c_2 for success after waiting is much smaller than the payoff c_1 for success after an early rollover, the LC is obviously more inclined to roll over early than to wait, simply because the expected payoff is smaller. The same holds for t. However, since the focus of my analysis lies on signaling and learning effects, the more interesting factor is how SCs react to observing the LC's action. SCs want to follow her action for two reasons: First, they know that the LC only rolls over if she receives a sufficiently good private signal y. If she rolls over, they become more optimistic about θ, so that they are more willing to roll over as well (see Corollary 3.1). I denote this the *signaling channel* of the LC's decision. Second, the LC's and SCs' actions are strategic complements. That means, the more creditors roll over, the more likely is a success of the project and the higher is the expected payoff for rolling over. It follows that the incentive to roll over increases with the number of creditors who do the same. If the LC rolls over early, SCs know not only that she rolled over, but also that other SCs observed her action and are more willing to do so as well (due to signaling and strategic complementarity), so that everyone of them is more willing to roll over. This self-reinforcing mechanism constitutes the *strategic complementarity channel* of the LC's decision.

The LC's alternative action to rolling over early is to wait and observe the SCs' actions and decide later whether she wants to participate in the project. Since I assume that the LC observes the realized fundamental state in stage 3, waiting is more advantageous for the LC from an information perspective. At the same time, waiting in stage 1 sends a pessimistic signal about the fundamental state so that fewer SCs are willing to roll over and the probability of success decreases. The choice to roll over early or wait depends on whether the net benefit of rolling over early (sending good signal to SCs and observing only noisy private signal) outweighs the net benefit of waiting (sending bad signal to SCs and observing θ perfectly). The important factor for both alternatives is the effect the LC's action has on the SCs' decision, i.e., how many SCs want to follow her. This insight is summarized in the following corollary:

Corollary 3.2. *In game Γ and for $c_2 \to c_1$, the LC wants to roll over early if and only if $(1-\lambda)H$ is sufficiently large.*

Proof. Recall that, conditional on an early rollover, the condition for project success is $\theta > 1 - \ell(\theta, 1, b_2)$. The equilibrium fraction of creditors who withdraw can be written as

$$\begin{aligned} 1 - \ell(\theta, 1, b_2) &= 1 - \lambda - (1-\lambda) Pr(x > x^* \mid \theta) \\ &= \underbrace{1 - \lambda - (1-\lambda) Pr(x > x^{**} \mid \theta)}_{=1-\ell(\theta,0,1)} - (1-\lambda) \underbrace{[Pr(x > x^* \mid \theta) - Pr(x > x^{**} \mid \theta)]}_{=H}. \end{aligned}$$

Using this expression in the LC's expected payoffs gives

$$c_1 \cdot Pr(\theta > 1 - \ell(\theta, 0, 1) - (1-\lambda)H \mid y)$$

if she rolls over early and

$$c_2 \cdot Pr(\theta > 1 - \ell(\theta, 0, 1) \mid y) + t \cdot Pr(\theta \leq 1 - \ell(\theta, 0, 1) \mid y)$$

if she waits. Suppose that $c_2 \to c_1$ and recall from Corollary 3.1 that $H \geq 0$. In this case, the payoff for rolling over early is greater than the payoff for waiting only if $(1-\lambda)H$ is large enough to increase the probability of success sufficiently. Conversely, if $H = 0$ (or $\lambda \to 1$), the payoff for rolling over early is always smaller than the payoff for waiting and thus the LC always wants to wait. □

Corollary 3.2 says that if payoffs for early and late rollover are (nearly) equal ($c_2 \to c_1$), the main determinants of the LC's decision in stage 1 are her size λ and the fraction of SCs who follow her, H. A large fraction of SCs who roll over if and only if the LC rolls over early means that the LC's decision has a sizeable impact on the probability of success. An early rollover motivates a large portion of SCs to also roll over, which increases the probability of success accordingly. Waiting, on the other hand, deters SCs from rolling over so that the probability of success decreases. Hence, an exogenous increase in H increases the LC's expected payoff for rolling over early, while it decreases the expected payoff for waiting. Any change in parameters that affects the number of SCs who follow the LC also affects her trade-off between rolling over early and waiting.

Numerical Simulation Results. Figure 3.5 shows the simulation results for the behavior of the creditors. Panels (a) and (b) plot the probability of an early rollover by the LC against (the log of) γ/β for λ equal to 0.5 and 0, while panels (c) and (d) plot the herd-index. In all four panels, the payoff c_2 is fixed at 0.9999. Within each panel, the blue lines represent the equilibrium variables in game Γ_{NW}, while the black lines represent the respective variables in game Γ. Additionally, to compute the probability of early rollover and the herd-index, realized fundamental values $\hat{\theta}$ were specified and their values are represented by the different line types, as presented in the legend in panel (a).[35]

The plots suggest the following observations about the probability that the LC rolls over early and the SCs' herd behavior:

Simulation Result 3.1. *Probability of early rollover and herd-index:*

a) $P^y_{NW} > P^y$ holds for all γ/β.

b) The relation between P^y_{NW} and γ/β is monotone. The relation between P^y and γ/β may be non-monotone.

c) If c_2 increases, the probability of an early rollover P^y decreases.

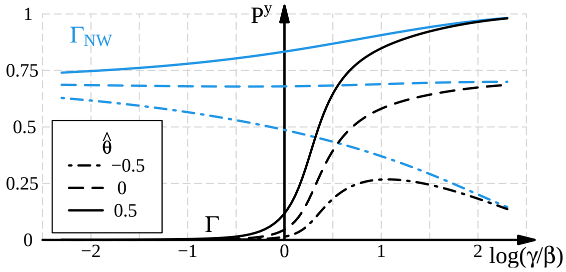
d) If the LC's size λ increases, in Γ the probability of an early rollover P^y decreases. In Γ_{NW} the probability P^y_{NW} increases.

e) If γ/β increases, the fraction of SCs who roll over if and only if the LC rolls over early H and H_{NW} increases monotonically.

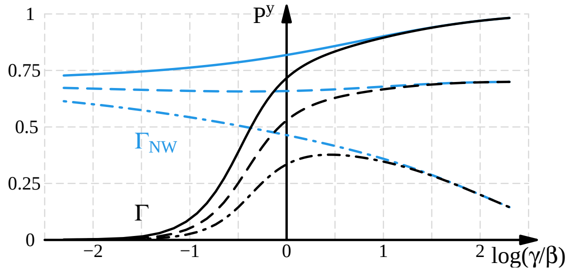
Result 3.1a) says that the probability of an early rollover is always larger if the LC does not have the option to wait than if she does. By giving the LC the option to delay her decision, the action not to roll over in stage 1 becomes relatively more attractive for her because she has

[35] For comparison, the same plots with c_2 equal to 0.8 and 0.6 are given in Appendix B.1 in Figures B.1 and B.2.

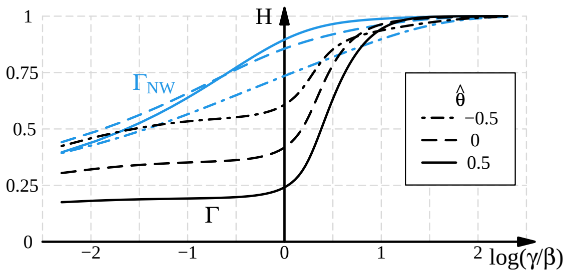
Figure 3.5: Effect of change in $\log(\gamma/\beta)$ on probability of early rollover and herd-index for $c_2 = 0.9999$.



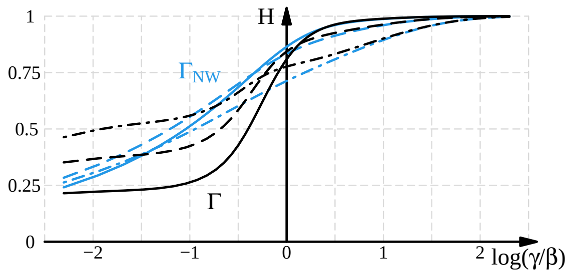
(a) $\lambda = 0.5$, $c_2 = 0.9999$.



(b) $\lambda = 0$, $c_2 = 0.9999$.



(c) $\lambda = 0.5$, $c_2 = 0.9999$.



(d) $\lambda = 0$, $c_2 = 0.9999$.

Note: Panels (a) and (b): Black = P^y and blue = P^y_{NW}. Panels (c) and (d): Black = H and blue = H_{NW}.

the opportunity to correct her initial decision at a later stage of the game, after observing an additional signal. As discussed in Corollary 3.1, in game Γ_{NW}, rolling over in stage 1 is her only chance of participating in a success. Hence, if she receives a mediocre signal, she wants to roll over as long as she sees a success being sufficiently likely. However, if she receives the same mediocre signal in the game with the option to wait, she might want to delay her decision in order to receive another signal and make a better informed guess about the outcome of the project. In the specific set-up in this chapter, this relation is most pronounced because I assume that the second signal the LC receives is infinitely more precise than the first one. Due to this information advantage, her incentive to roll over early is naturally smaller if she can wait than if she cannot and the probability of early rollover is lower. The only incentive to roll over early is to mobilize SCs to do the same and increase the probability of success until the expected payoff for rolling over early is greater than for waiting, as stated in Corollary 3.2.

Supplementary to this, Result 3.1b) emphasizes that the relative information precision γ/β is an important factor for the decision making of the LC. It shows that there are two channels through which a change in γ/β affects the probability of an early rollover: First, it affects the precision of the LC's signal with regard to the fundamental state. I call this the *fundamental information channel*. By increasing γ, the LC rolls over falsely less often if the fundamentals are weak and withdraws less often if they are strong because she knows with higher precision whether θ is actually good or bad. This explains why the probability of an early rollover increases in game Γ_{NW} if $\theta > 0$ and decreases if $\theta < 0$ (see panels (a) and (b) in Figures 3.5, B.1 and B.2). The second channel concerns the SCs' decision to follow the LC or not. I call this the *herd behavior channel* and its effects are observed in panels (c) and (d) of Figure 3.5 (as well as Figures B.1 and B.2 in Appendix B.1.). If γ increases, SCs know that the LC has relatively better information and chooses the right decision more often. Hence, SCs are more often willing to ignore their private signal and simply follow the LC: Herd indices H and H_{NW} increase (cf. Result 3.1e)). Given that the LC rolls over early, this increases the probability of success, so it becomes relatively more attractive to roll over early than to wait. In game Γ_{NW} the impact of the herd behavior channel is only quantitative, because rolling over and signaling are the same decision. In game Γ, however, the herd behavior channel affects the LC's strategy qualitatively. While for $\theta > 0$ the probability of an early rollover increases in γ in both games, in Γ the relation is non-monotone if $\theta < 0$. The reason is that the two channels have the opposite sign. If $\theta < 0$, a more precise signal implies that the LC wants to roll over less often because she is more certain that the fundamental state is actually too weak to support the project's success. Therefore, the probability of an early rollover decreases. At the same time, a larger γ/β means that more SCs roll over if the LC rolls over early, increasing the conditional probability of success. The probability of early rollover increases. Consequently, for $\theta < 0$ these two forces pull in different directions. In Figure 3.5 this can be seen in the non-monotonicity of the probability of early rollover for $\hat{\theta} = -0.5$: While for low γ/β the herd behavior channel dominates and the probability increases in γ/β, the fundamental information channel starts to dominate for higher values of γ/β and the probability decreases again, as the LC learns that θ is actually too weak.

This result has important implications for potential policy measures. Suppose that in order to decrease the probability of coordination failure, a policy-maker chooses to disseminate more precise fundamental information to the LC to make her roll over early. Depending on the actual fundamental state and the precision of the LC's initial information, this policy measure may have different effects. This is particularly problematic if the policy-maker does not know the realization of the fundamental state. If the LC's information is not too precise, an increase in information precision causes the LC and SCs with her to roll over more often. In the event of bad fundamentals, this means that more creditors will not be able to fully recover their investment as the project fails in any case. Depending on the policy-maker's objective function, this may well be considered a suboptimal outcome.

Result 3.1c) is straightforward to interpret: An exogenous increase in c_2 increases the expected payoff for waiting relative to the expected payoff for rolling over early, so the incentive to wait increases. Hence, the probability of rolling over early decreases. Result 3.1d) predicts that an increase in the LC's size decreases her incentive to roll over early if she has the option to wait, while it increases the incentive if she cannot wait. The effect in game Γ becomes clear when recalling Corollary 3.2. A larger LC means that there are fewer SCs that could potentially follow the LC and thus the difference between the effects of an early rollover and a late rollover on the probability of success decreases. If c_2 is close to c_1, the information advantage granted by waiting outweighs the signaling advantage for rolling over early so that the LC is more willing to wait. In the game without the option to wait, this looks different. Withdrawing is only more preferable if the initial private signal is sufficiently bad. The probability of an early rollover increases in λ because the strategic uncertainty between SCs decreases. If a larger LC rolls over early, a smaller fraction of SCs is needed to render the project successful and thus the probability of success increases. Therefore, for the LC it is relatively better to roll over early than to withdraw.

An interesting special case to consider here is when $\lambda \to 0$, i.e., when the LC is "small" in the sense that she cannot change the probability of project success by her size alone but only through signaling to SCs. This scenario is depicted in Figure 3.5 in panel (b). The traditional argument for catalytic finance as stated in Corsetti et al. (2006) and Morris and Shin (2006) is that if e.g. a government is solvent but illiquid, an intervention by a lender-of-last-resort such as the IMF may avert a sovereign default, even if the provided funds only cover a portion of all liquidity needs. The reason is that even a partial bail-out may reduce the probability of default sufficiently to crowd in private investors who provide more liquidity to cover the remaining needs. This argument is purely based on the strategic complementarity of the IMF's action to bail-out and the private investors action to bail-in. However, it ignores uncertainty and the signaling effect of the IMF's action. Figure 3.5 and Results 3.1 show that it does not even need a partial bail-out by the large player (the IMF in this case) in order to make small players bail-in, as long as the large player can credibly signal that she believes that the country is not going to default. The case when $\lambda \to 0$ shows this because even as the LC has no direct impact on the probability of success, she still manages to convince SCs to follow her and roll over if she does, which in turn reduces the probability of a failure of the project. However, one caveat in this interpretation is that for this result to hold, the LC's incentive must be to maximize her expected payoff from the investment. For large international institutions such as the IMF or other lenders-of-last-resort such as central banks, this is not necessarily the case because they do not operate for the purpose of maximizing profits, but to avert the outbreak and spread of financial and economic crises. Hence, for such institutions it is difficult to credibly signal that they believe that the fundamentals of the troubled country are actually not bad, which reduces the effectiveness of this signaling channel. Nevertheless, the result also highlights that if policy-makers believe that a country's fundamentals are actually good, the provision of high quality information to large players in international financial markets (γ increases) can improve the prospect of the troubled country, because they will more likely provide liquidity and thereby crowd-in smaller market participants as well.

Moreover, this finding is at odds with the previously stated arguments in Corsetti et al. (2004, p. 104) that no small player would have the incentive to move first in such a sequential setting. In fact, their conclusion that a small player would not want to move first is a consequence of their assumption that a small player is not observed by others and not that he is small. In this sense, the results for the case when $\lambda \to 0$ help to separate the size of a player in terms of her direct impact on the outcome from her action being observed by others. One implication of this finding is that even players who do not or cannot affect the outcome of a coordination game directly, can have an impact simply by virtue of signaling. Real-world examples of such players are credit rating agencies that publicly signal their stance toward the creditworthiness of a company or a government but do not actively buy or sell their stocks or bonds.[36] Other examples include experts or opinion leaders of any kind who can influence crowds or populations by signaling their opinions e.g. in settings of social uprisings or political unrest (cf. Edmond, 2013; Loeper et al., 2014).

3.6.3 The conditional probability of coordination failure

The effect of LC's decision in stage 1. Recall that throughout this chapter coordination failure is defined as the incidence that the fundamental value is sufficiently strong to support the success of the project (i.e. greater than 0), but too weak to make sufficiently many creditors roll over their loans. For an arbitrary equilibrium fundamental threshold $\tilde{\theta}$ as derived from the two games, the conditional probability of coordination failure is thus given by $Pr(0 < \theta \leq \tilde{\theta})$. Consequently, the probability of project success and failure is $Pr(\theta > \tilde{\theta})$ and $Pr(\theta \leq \tilde{\theta})$. However, notice that due to the assumption that the fundamental state is drawn from an improper uniform distribution over the real line, we cannot express these probabilities in terms of parameters of the two games. Instead, for my comparative statics analysis, I have to use the various fundamental thresholds as proxies for these probabilities. The argument is that if any of the thresholds increase, the probabilities of coordination or project failure also increase.

[36] However, in order to model this particular case one would have to adapt the credit rating agencies' incentives and preferences as they do not necessarily fully coincide with those of creditors or bond-holders (cf. Carlson and Hale, 2006; Holden et al., 2018).

With these pre-condition set, we can say the following about the conditional probabilities of coordination failure[37]:

Proposition 3.3. *The conditional probability of coordination failure is smaller if the LC rolls over early than if she withdraws/waits. More specifically, $\theta^{**}_{NW} > \theta^{*}_{NW}$ and $\overline{\theta}^{**} > \theta^{*}$.*

Proposition 3.3 in part follows from Corollary 3.1 in that SCs always consider an early rollover by the LC as an optimistic signal about θ, whereas waiting or withdrawing is a pessimistic signal. Consequently, if the LC rolls over early, the fraction of SCs who roll over is larger than if she withdraws (or waits) so that coordination failure is less likely. An important implication of this result is that for a policy-maker who wants to minimize the ex-ante probability of coordination failure, an early rollover by the LC is desirable as long as fundamentals are sufficiently strong $\theta > 0$. If fundamentals are too weak ($\theta \leq 0$), more creditors who roll over means that the project fails at a higher cost for all creditors.

Numerical Simulation Results. Figure 3.6 shows the numerical simulation results for the different fundamental thresholds. Panels (a) and (b) show the threshold if the LC rolls over early, that is, θ^* (black) and θ^*_{NW} (blue) for different values of λ. Panels (c) and (d) show the thresholds for the case in which she waits or withdraws in stage 1, that is, $\overline{\theta}^{**}$ (black) and θ^{**}_{NW} (blue). The different types of lines represent different values of payoff c_2. The following observations can be made from these plots:

Simulation Result 3.2. *Conditional probability of coordination failure:*

a) $\theta^*_{NW} > \theta^*$ *and* $\theta^{**}_{NW} > \overline{\theta}^{**}$ *for all* γ/β.

b) If γ/β *increases,* $\overline{\theta}^{**}$ *and* θ^{**}_{NW} *increase and* θ^*_{NW} *decreases monotonically. The relation between* θ^* *and* γ/β *may be non-monotone.*

c) If c_2 *increases,* θ^* *and* $\overline{\theta}^{**}$ *decrease.*

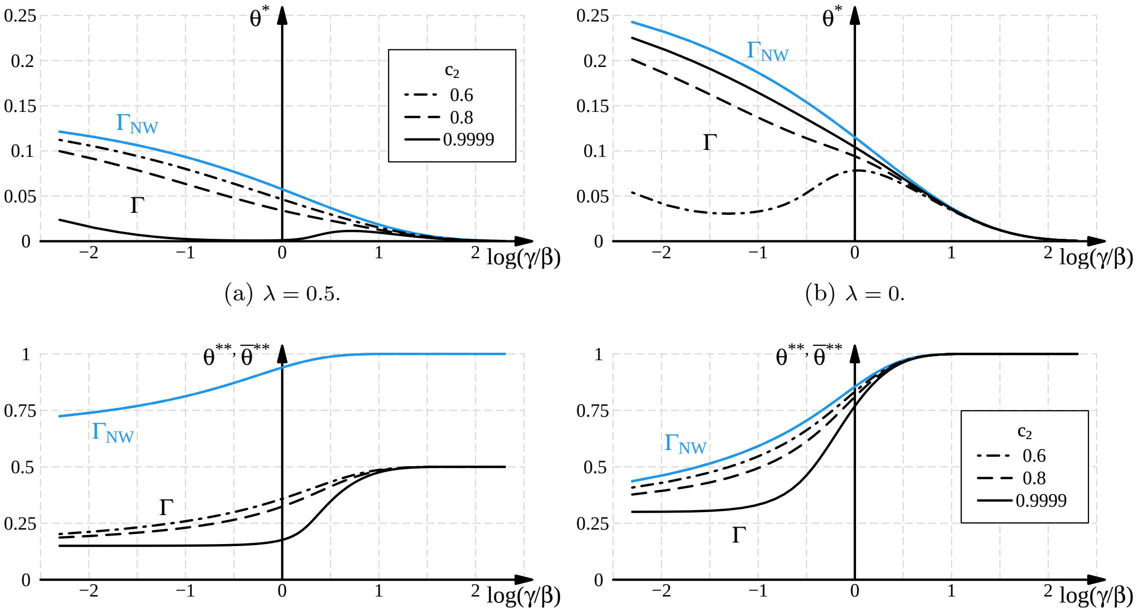
d) If λ *increases,* θ^*, θ^*_{NW} *and* $\overline{\theta}^{**}$ *decrease, while* θ^{**}_{NW} *increases.*

Result 3.2a) implies that if the LC has the option to wait, the conditional probability of coordination failure is always lower than if she does not. This finding is consistent with the results in Dasgupta (2007). The LC is generally less willing to roll over early if she has the option to wait than if she has not. Accordingly, if she rolls over early, SCs consider this an even more optimistic signal for θ than if the LC cannot wait so that more SCs roll over and coordination failure is less likely. At the same time, not rolling over early is less of a pessimistic signal in the game with the option to wait because waiting offers a clear informational advantage over rolling over early, which is not there in the game without option to wait. Hence, if the LC would roll over for a mediocre private signal in Γ_{NW}, she would want to wait in Γ. Consequently, conditional on the LC not rolling over in stage 1, SCs roll over more often in Γ than in Γ_{NW}. It follows that coordination failure is less likely.

The result highlights the importance of the credibility of the LC's commitment. As Result 3.1a) shows, if the LC has the option to wait, not rolling over in stage 1 is c.p. more preferable than if she cannot wait. In turn, this means that the opportunity costs for rolling over are also higher. If the LC nevertheless rolls over early, an SC interprets this as a sign that the LC must

[37]For proof see Appendix B.3.

Figure 3.6: Effect of change in $\log(\gamma/\beta)$ on fundamental thresholds.



(a) $\lambda = 0.5$.

(b) $\lambda = 0$.

(c) $\lambda = 0.5$.

(d) $\lambda = 0$.

Note: Panels (a) and (b): Black = θ^* and blue = θ^*_{NW}. Panels (c) and (d): Black = $\overline{\theta}^{**}$ and blue = θ^{**}_{NW}.

have received a very good private signal because otherwise she would not have put herself at risk of bearing these higher (opportunity) costs if the project fails. Her commitment to the project is more credible and more SCs roll over as well. The project is thus more likely to succeed, given fundamentals are not bad. This relation is important to keep in mind if policy-makers want to exploit the signaling channel to reduce the probability of coordination failure because it shows that putting their own "skin in the game" amplifies the effect of the signal.

Result 3.2b) states that the probability of coordination failure conditional on the LC not rolling over in stage 1 increases in γ/β in both games. To see why, consider the following: If the LC has a precise private signal and the realized fundamental value is not too bad ($\theta > 0$), then she is likely to roll over early because a large fraction of SCs will roll over if she does and the probability that θ is smaller than the fraction of creditors who roll over is small. If, however, the LC does not roll over early, the project is more likely to fail due to coordination failure because many SCs will have followed the LC and only few will have rolled over so that it is likely that θ is smaller than the fraction of creditors who withdraw. The argument shows that with better private information, the LC can coordinate SCs better because they follow her. However, if the LC chooses the "false" action, the project is more likely to fail because a large fraction of SCs will have done so as well. This insight holds regardless of whether the LC has the option to wait or not. It is for the same reasons that the probability of coordination failure conditional on the LC rolling over (i.e. θ^*_{NW}) in game Γ_{NW} decreases in γ/β: A large number of SCs follow the LC and if she rolls over, it is less likely that θ is smaller than the fraction of creditors who withdraw.

Mind, however, that this is not necessarily true in game Γ. The observation that θ^* may be non-monotone in γ/β (as can be seen e.g. in panels (a) and (b) in Figure 3.6) is a consequence from the ambiguous behavior of the LC if fundamentals are weak as described in Result 3.1b). If fundamentals are bad ($\theta < 0$) and if γ/β is small or in an intermediate range, the LC's incentive to roll over early because of the positive signaling effect increases faster in γ/β than the incentive to wait due to the increasingly certain realization that fundamentals are bad. That means, for increasing γ the LC is more often willing to roll over early even though she learns that fundamentals might be bad (herd behavior channel > fundamental information channel). This in turn dilutes the information content of the LC's action in stage 1 with respect to the fundamental state so that conditional on the LC rolling over early, SCs are *less* willing to roll over and thus coordination failure becomes *more* likely. To make this counter-intuitive behavior more clear: Given that γ/β is in low or intermediate range, SCs internalize that for some values of y the LC has an incentive to roll over early only to convince SCs to follow her and not because she believes that the fundamental value might be strong. Hence, for this range of γ/β the action to roll over early as a means of signaling is less effective than for higher values of γ/β, so that it is more likely that the project fails due to the failure of creditors to coordinate on rolling over.

For a policy-maker this insight has meaningful consequences. If he chooses to provide more precise information to an LC in order to reduce the probability of coordination failure, this might in fact *increase* the probability of coordination failure, depending on the initial precision of the LC's private signal and conditional on her rolling over early. Hence, when choosing transparency or dissemination of information as a policy-measure to avert crises, it is not always be preferable to provide the most precise of information.

Result 3.2c) can be explained by the same commitment-argument made in explaining Result 3.2a): By increasing c_2, the opportunity costs for rolling over early increase. Hence, if the LC rolls over early, SCs interpret this as if she must have received a very good private signal because otherwise she would not take the risk of bearing these costs. Hence, they roll over more often and the probability of coordination failure decreases. Conversely, if she waits, this is seen as a less pessimistic signal because if the potential payoff for a late rollover is sufficiently high, it may be preferable to wait even for a good initial private signal. Hence, SCs are less willing to withdraw if the LC waits, so that coordination failure is less likely.

A similar argument holds for thresholds θ^* and $\overline{\theta}^{**}$ in Result 3.2d): As λ increases, the LC's incentive to rollover early decreases (see Corollary 3.2). So, when comparing a larger with a smaller LC, SCs consider an actual early rollover a more optimistic signal and waiting a less pessimistic signal about θ. It follows that they are more inclined to roll over and coordination failure is less likely. For the game without option to wait, this mechanism does not apply. In this case, a larger LC who rolls over means that fewer SCs have to roll over to render the project successful so that the probability of coordination failure decreases (θ^*_{NW} decreases). At the same time, if a larger LC does not roll over, more SCs are needed for the project to be successful so that coordination failure becomes more likely (θ^{**}_{NW} increases).

3.7 Conclusion

In this chapter I investigate how a large player's choice between moving before or after a group of small players affects the outcome of a global coordination game. Using a simplified credit rollover setting, the large creditor is asked to either roll over her investment before small creditors or wait and observe their aggregate decision about rolling over or withdrawing. In the comparative statics exercises, I shed light on the large player's behavior from three perspectives: 1) I analyze the factors that influence her decision to roll over early or not. The two central themes of this decision are her information about the fundamental state of the investment project and her influence on small players, which she trades off against one another. The *fundamental information* and the *herd behavior channel* highlight how this trade-off changes if her information precision changes. Moreover, while giving the large player the option to wait decreases the probability that she moves first, better private information increases this probability as long as it sufficiently increases the number of small players who follow her. In the limit, i.e., if the large player's signal is completely revealing with respect to the fundamental state, the option to wait has no effect on the players' equilibrium behavior in the sense that the strategies are identical in both games. On the other hand, if it is completely uninformative, the difference in equilibrium strategies amounts to the LC's size, but not beyond. 2) Through the *strategic complementary channel* and the *signaling channel*, I look at how the large player's choice affects small players. In particular the signaling channel, which highlights the transmission of fundamental information through the large player's action, is an innovative feature of my analysis. So far the literature about the effect of a lender-of-last-resort in self-fulfilling debt crises has focused on the importance of strategic complements for catalytic finance, while a bail-out as a means of signaling endorsement is largely ignored. 3) Finally, I investigate how the large player's action, together with its influence on small players affects the probability of a positive outcome for the game. For policy-makers this perspective is particularly interesting as it shows potential approaches to reducing the probability of coordination failure. I show that if the large player moves early, SCs always interpret this as a positive signal and tend to follow her in her decision. In the example of a credit rollover game, this means that the probability of investment project success increases if the large player rolls over first. Policy-makers can make use of this relation e.g. by setting incentives or providing the large player with better information to make her roll over early more often, given that fundamentals are not too bad. Curiously, this result holds even if the large player is no larger than small players, which further emphasizes the importance of the signaling channel and may be interesting to apply to the role of credit rating agencies in such settings.

The two models that I compare in this chapter are highly stylized and abstract and leave opportunities for extension. One obvious simplification is the assumption that the large player observes the fundamental state perfectly after waiting in the first stage of the game. Even though this assumption is unrealistic, it allows to simplify equilibrium derivation and comparative statics analysis. In Chapter 4 of this thesis, I analyze a variation of this model where the large player only observes a noisy signal about the aggregate action of small players. I show that the qualitative results derived in this chapter are robust with regard to the assumption. Moreover, as is common in the global games literature, a pervasive issue is the possible existence of multiple equilibria. I use numerical simulation methods to check for this problem, however they can never completely rule out that multiple solutions exist in some scenarios. For future research it would be interesting to see if simple conditions for the relative information precision can be derived, which determine the existence of multiple equilibria, similar to the famous result by Morris and Shin (2004). Finally, my models completely abstract from the strategic motives of the debtor. I employ a mechanical approach where the project fails if the amount of funds that are withdrawn is too large. However, a strategic debtor might want to signal solid fundamentals e.g. through reporting or the provision of collateral to make creditors roll over their loans. How a change in this assumption affects the results is left for future research.

To summarize, the analysis in this chapter contributes to a better understanding of coordination problems in financial markets and in particular, how heterogeneity of players (i.e. large vs. small) can affect equilibrium outcomes. It connects to the existing literature on lenders-of-last-resort and the effects of catalytic finance by exploring signaling as a means of creating a private bail-in in self-fulfilling debt crises. Moreover, it highlights the role of large players in financial markets in general. In particular, understanding the large players' motives and how they affect small players through certification or endorsement (herd behavior) opens new policy perspectives on how policy-makers can guide market expectations and potentially avert financial crises.

B.1 Appendix: Figures and tables

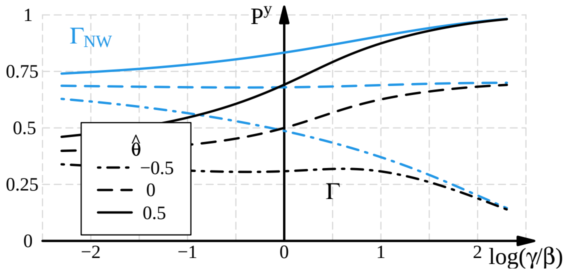
Table B.1: Equilibrium thresholds of Γ and Γ_{NW} and their interpretation.

Eqm. Threshold	Game	Interpretation
y^*, y^*_{NW}	Γ, Γ_{NW}	In stage 1, the LC rolls over early if $y > y^*$ (or $y > y^*_{NW}$) and waits if $y \leq y^*$ (or $y \leq y^*_{NW}$).
x^*, x^*_{NW}	Γ, Γ_{NW}	In stage 2, conditional on the LC rolling over early in stage 1, an SC i rolls over if $x_i > x^*$ (or $x_i > x^*_{NW}$) and withdraws if $x \leq x^*$ (or $x_i \leq x^*_{NW}$).
x^{**}, x^{**}_{NW}	Γ, Γ_{NW}	In stage 2, conditional on the LC waiting in stage 1, an SC i rolls over if $x_i > x^{**}$ (or $x_i > x^{**}_{NW}$) and withdraws if $x \leq x^{**}$ (or $x_i \leq x^{**}_{NW}$).
θ^*, θ^*_{NW}	Γ, Γ_{NW}	Conditional on the LC rolling over early in stage 1, the investment project succeeds if $\theta > \theta^*$ (or $\theta > \theta^*_{NW}$) and fails if $\theta \leq \theta^*$ (or $\theta \leq \theta^*_{NW}$).
$\overline{\theta}^{**}$	Γ	Conditional on the LC waiting in stage 1 and rolling over late in stage 3, the investment project succeeds if $\theta > \overline{\theta}^{**}$ and fails if $\theta \leq \overline{\theta}^{**}$. Moreover, in stage 3 the LC rolls over late if $s = \theta > \overline{\theta}^{**}$ and withdraws if $s = \theta \leq \overline{\theta}^{**}$.
$\underline{\theta}^{**}$	Γ	Conditional on the LC waiting in stage 1 and withdrawing in stage 3, the investment project succeeds if $\theta > \underline{\theta}^{**}$ and fails if $\theta \leq \underline{\theta}^{**}$.
θ^{**}_{NW}	Γ_{NW}	Conditional on the LC withdrawing in stage 1, the investment project succeeds if $\theta > \theta^{**}_{NW}$ and fails if $\theta \leq \theta^{**}_{NW}$.

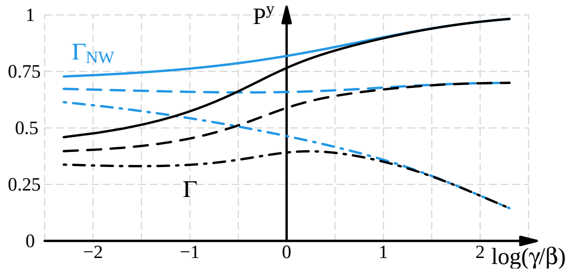
Table B.2: Parameter specification for numerical simulations.

Parameter	Defined on	Values used in Simulation	Interpretation
β	$(0, \infty)$	1	Precision of SC i's private signal x_i.
γ	$(0, \infty)$	$[0.1, 10]$	Precision of LC's private signal y.
λ	$(0, 1)$	$\{0, 0.5\}$	LC's size.
c_1	$(0, \infty)$	1	LC's and SCs' payoff for (early) roll over if project is successful.
c_2	$(0, \infty)$	$\{0.6, 0.8, 0.9999\}$	LC's payoff for late roll over if project is successful.
t	$(0, \infty)$	0.3	Costs of rolling over.

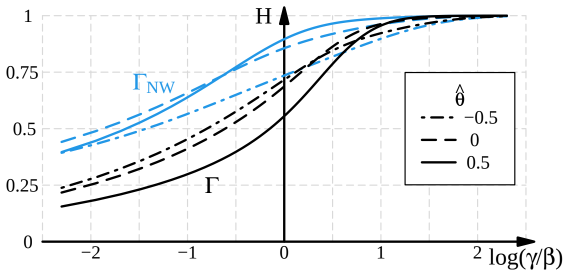
Figure B.1: Effect of change in $\log(\gamma/\beta)$ on probability of early rollover and herd-index for $c_2 = 0.8$.



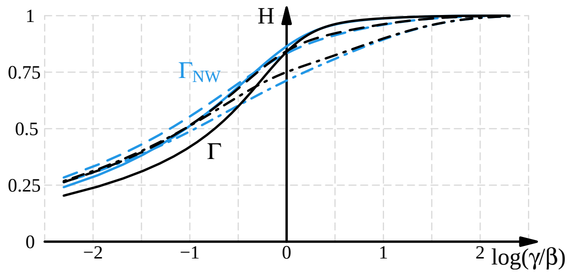
(a) $\lambda = 0.5$, $c_2 = 0.8$.



(b) $\lambda = 0$, $c_2 = 0.8$.



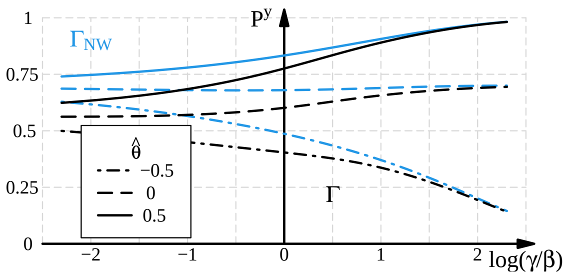
(c) $\lambda = 0.5$, $c_2 = 0.8$.



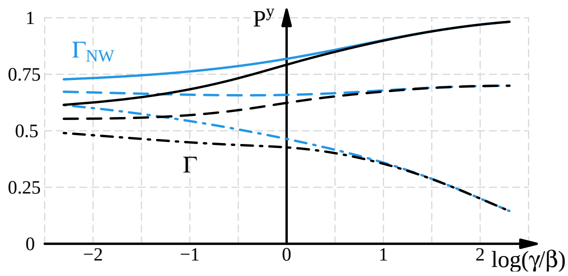
(d) $\lambda = 0$, $c_2 = 0.8$.

Note: Panels (a) and (b): Black = P^y and blue = P^y_{NW}. Panels (c) and (d): Black = H and blue = H_{NW}.

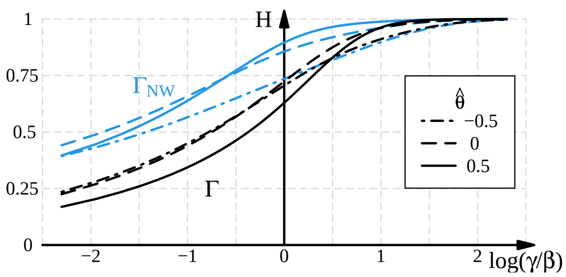
Figure B.2: Effect of change in $\log(\gamma/\beta)$ on probability of early rollover and herd-index for $c_2 = 0.6$.



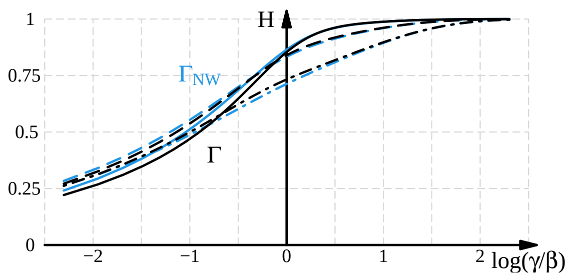
(a) $\lambda = 0.5$, $c_2 = 0.6$.



(b) $\lambda = 0$, $c_2 = 0.6$.



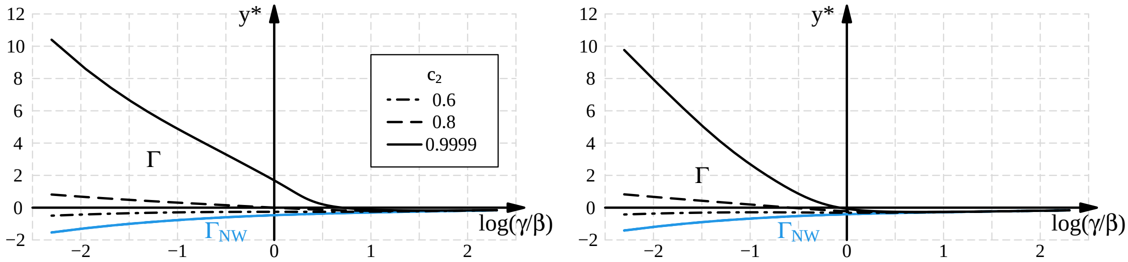
(c) $\lambda = 0.5$, $c_2 = 0.6$.



(d) $\lambda = 0$, $c_2 = 0.6$.

Note: Panels (a) and (b): Black = P^y and blue = P^y_{NW}. Panels (c) and (d): Black = H and blue = H_{NW}.

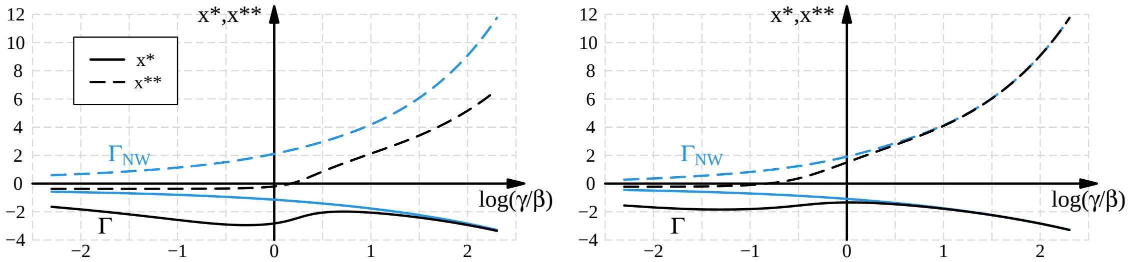
Figure B.3: Effect of change in $\log(\gamma/\beta)$ on y^* and y^*_{NW}.



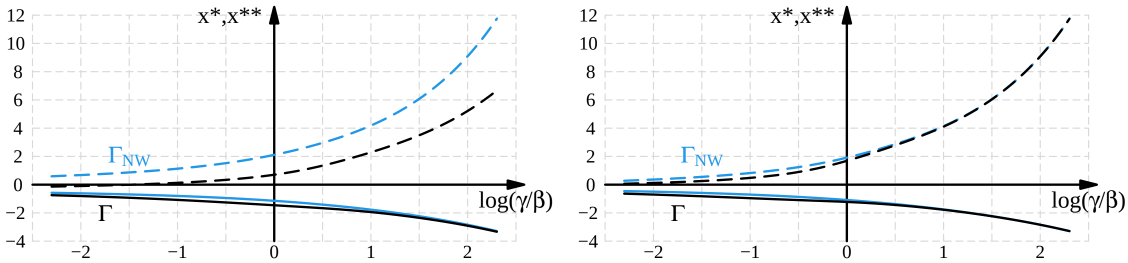
(a) $\lambda = 0.5$. (b) $\lambda = 0$.

Note: Black = y^*. Blue = y^*_{NW}.

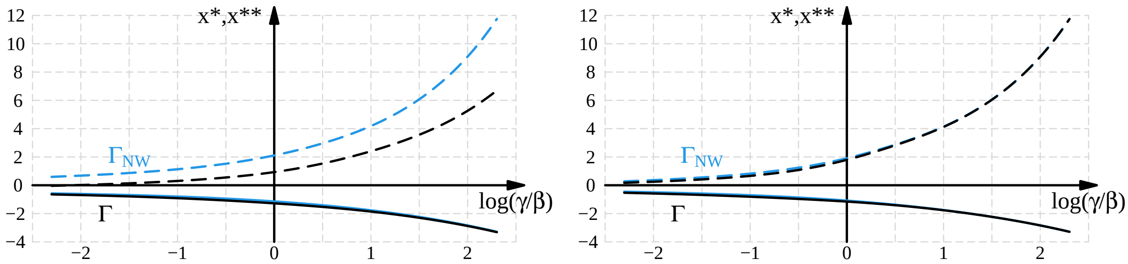
Figure B.4: Effect of change in $\log(\gamma/\beta)$ on x^*, x^{**}, x^*_{NW} and x^{**}_{NW}.



(a) $\lambda = 0.5$, $c_2 = 0.9999$. (b) $\lambda = 0$, $c_2 = 0.9999$.



(c) $\lambda = 0.5$, $c_2 = 0.8$. (d) $\lambda = 0$, $c_2 = 0.8$.



(e) $\lambda = 0.5$, $c_2 = 0.6$. (f) $\lambda = 0$, $c_2 = 0.6$.

Note: Solid = x^* or x^*_{NW}. Dashed = x^{**} or x^{**}_{NW}.

B.2 Appendix: Derivation of equilibrium beliefs

Notation: Let $F(a \mid \tilde{b} \leq b, c) = Pr(\tilde{a} \leq a \mid \tilde{b} \leq b, \tilde{c} = c)$ for some random variables $\tilde{a}, \tilde{b}, \tilde{c}$. Let f denote the corresponding probability density/mass function. The equilibrium conditions in both games dictate that the following conditional distributions are needed to parameterize the probabilities in the indifference equations:

$$F(\theta \mid y) \tag{3.45}$$
$$F(\theta \mid \tilde{y} > y, x) \tag{3.46}$$
$$F(\theta \mid \tilde{y} \leq y, x) \tag{3.47}$$
$$F(\theta \mid x). \tag{3.48}$$

Mind that by using Bayes' Rule, $F(\theta \mid \tilde{y} > y, \tilde{x})$ and $F(\theta \mid \tilde{y} \leq y, x)$ can be written as

$$F(\theta \mid \tilde{y} > y, x) = \frac{F(\theta \mid x) - F(\theta, y \mid x)}{1 - F(y \mid x)} \quad \text{and} \quad F(\theta \mid \tilde{y} \leq y, x) = \frac{F(\theta, y \mid x)}{F(y \mid x)}. \tag{3.49}$$

Hence, in order to find the above distributions, one also needs to find

$$F(\theta, y \mid x) \quad \text{and} \quad F(y \mid x). \tag{3.50}$$

Recall that by assumption

$$\tilde{x} \mid \theta \sim N\left(\theta, \beta^{-1}\right) \quad \text{and} \quad \tilde{y} \mid \theta \sim N\left(\theta, \gamma^{-1}\right) \tag{3.51}$$

and $\tilde{x} \mid \theta$ and $\tilde{y} \mid \theta$ independent. From Bayesian updating using normal random variables (e.g Veldkamp, 2011, p. 11ff.) it follows directly that

$$\tilde{\theta} \mid x \sim N\left(x, \beta^{-1}\right) \quad \text{and} \quad \tilde{\theta} \mid y \sim N\left(y, \gamma^{-1}\right). \tag{3.52}$$

Hence, the only distributions left to be found are $F(\theta, y \mid x)$ and $F(y \mid x)$. First, derive $F(y \mid x)$. Mind that $f(y \mid x)$ is the so-called posterior predictive distribution:

$$f(y \mid x) = \int f(y \mid \theta, x) f(\theta \mid x)\, d\theta = \int f(y \mid \theta) f(\theta \mid x)\, d\theta \tag{3.53}$$

where the last equality follows from the fact that conditional on θ, $\tilde{y}$ is independent from $\tilde{x}$. Since both distributions under the integral are normal, we can write it as

$$f(y \mid x) = \frac{\sqrt{\gamma\beta}}{2\pi} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left[\gamma (y-\theta)^2 + \beta(\theta - x)^2\right]\right) d\theta. \tag{3.54}$$

Using some algebra, one can show that

$$\gamma (y-\theta)^2 + \beta (\theta - x)^2 = \frac{\gamma\beta}{\gamma+\beta}(y-x)^2 + (\gamma+\beta)\left(\theta - \frac{\gamma y + \beta x}{\gamma + \beta}\right)^2. \tag{3.55}$$

It follows that

$$f(y \mid x) = \sqrt{\frac{\gamma\beta}{(\gamma+\beta)2\pi}} \exp\left(-\frac{1}{2}\frac{\gamma\beta}{\gamma+\beta}(y-x)^2\right)$$

$$\cdot \underbrace{\sqrt{\frac{\gamma+\beta}{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}(\beta+\gamma)\left(\theta - \frac{\gamma y + \beta x}{\gamma+\beta}\right)^2\right) d\theta}_{=1} \tag{3.56}$$

and

$$\tilde{y} \mid x \sim N\left(x, \frac{\beta+\gamma}{\beta\gamma}\right). \tag{3.57}$$

To derive $F(\theta, y \mid x)$, recognize that

$$f(\theta, y \mid x) = f(\theta \mid y, x) f(y \mid x). \tag{3.58}$$

From standard Bayesian updating using normal random variables (e.g Veldkamp, 2011, p. 11ff.) it follows that the posterior for θ can be written as

$$\tilde{\theta} \mid y, x \sim N\left(\frac{\gamma y + \beta x}{\gamma+\beta}, \frac{1}{\beta+\gamma}\right).$$

Hence, we can write

$$\begin{aligned} f(\theta, y \mid x) &= \sqrt{\frac{\gamma+\beta}{2\pi}} \exp\left(-\frac{1}{2}(\beta+\gamma)\left(\theta - \frac{\gamma y + \beta x}{\gamma+\beta}\right)^2\right) \\ &\qquad \cdot \sqrt{\frac{\gamma\beta}{(\gamma+\beta)2\pi}} \exp\left(-\frac{1}{2}\frac{\gamma\beta}{\gamma+\beta}(y-x)^2\right) \qquad (3.59) \\ &= \frac{\sqrt{\beta\gamma}}{2\pi} \exp\left(-\frac{1}{2}\left[(\beta+\gamma)\left(\theta - \frac{\gamma y + \beta x}{\gamma+\beta}\right)^2 + \frac{\gamma\beta}{\gamma+\beta}(y-x)^2\right]\right). \qquad (3.60) \end{aligned}$$

Using some algebra, one can show that

$$\begin{aligned} &(\beta+\gamma)\left(\theta - \frac{\gamma y + \beta x}{\gamma+\beta}\right)^2 + \frac{\gamma\beta}{\gamma+\beta}(y-x)^2 \\ &= \frac{\beta+\gamma}{\beta}\left[\beta(\theta-x)^2 + \frac{\beta\gamma}{\beta+\gamma}(y-x)^2 - 2\sqrt{\frac{\gamma}{\beta+\gamma}}\sqrt{\beta}\sqrt{\frac{\beta\gamma}{\beta+\gamma}}(\theta-x)(y-x)\right]. \qquad (3.61) \end{aligned}$$

Using the formula for the density of the multi-variate normal distribution, one can see that $f(\theta, y \mid x)$ corresponds to a bi-variate normal distribution with mean vector $(x, x)^\intercal$, variance-covariance matrix

$$\begin{pmatrix} \beta^{-1} & \beta^{-1} \\ \beta^{-1} & \frac{\beta+\gamma}{\beta\gamma} \end{pmatrix}, \tag{3.62}$$

and correlation coefficient $\sqrt{\frac{\gamma}{\beta+\gamma}}$. Finally, in order to derive probabilities from these densities, one simply needs to integrate over the relevant intervals.

B.3 Appendix: Proofs

For the sake of proofs, I restate the indifference and critical mass conditions for both games: For game Γ_{NW}

$$c_1 \Phi\left(\sqrt{\gamma}\left(y^*_{NW} - \theta^*_{NW}\right)\right) = t, \tag{3.63}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^*_{NW}-\theta^*_{NW})\,,\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^*_{NW}-y^*_{NW})\,;\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^*_{NW}-y^*_{NW})\right)} = t, \tag{3.64}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^{**}_{NW}-\theta^{**}_{NW})\,,\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y^*_{NW}-x^{**}_{NW})\,;-\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y^*_{NW}-x^{**}_{NW})\right)} = t, \tag{3.65}$$

$$\theta^*_{NW} = 1-\lambda-(1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^*_{NW}-x^*_{NW})\right), \tag{3.66}$$

$$\theta^{**}_{NW} = 1-(1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^{**}_{NW}-x^{**}_{NW})\right) \tag{3.67}$$

and for game Γ

$$c_1 \cdot \Phi\left(\sqrt{\gamma}\,(y^*-\theta^*)\right) = c_2 \cdot \Phi\left(\sqrt{\gamma}\left(y^*-\overline{\theta}^{**}\right)\right) + t \cdot \Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**}-y^*\right)\right), \tag{3.68}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^*-\theta^*)\,,\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^*-y^*)\,;\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^*-y^*)\right)} = t, \tag{3.69}$$

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}-\overline{\theta}^{**}\right),\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y^*-x^{**})\,;-\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y^*-x^{**})\right)} = t, \tag{3.70}$$

$$\theta^* = 1-\lambda-(1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^*-x^*)\right), \tag{3.71}$$

$$\overline{\theta}^{**} = 1-\lambda-(1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**}-x^{**}\right)\right), \tag{3.72}$$

$$\underline{\theta}^{**} = 1-(1-\lambda)\Phi\left(\sqrt{\beta}\,(\underline{\theta}^{**}-x^{**})\right). \tag{3.73}$$

The following lemma will be useful for future proofs:

Lemma 3.1. *If random variables X_1 and X_2 are jointly normally distributed with the correlation coefficient $\rho > 0$, then*

$$Pr(X_1 > x_1 \mid X_2 > x_2) \geq Pr(X_1 > x_1) \geq Pr(X_1 > x_1 \mid X_2 \leq x_2) \tag{3.74}$$

for arbitrary values $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$.

Proof. Positive Quadrant Dependence (Tong, 1990, p. 20) states that if X_1 and X_2 are jointly normal with correlation $\rho > 0$, then

$$Pr(X_1 \leq x_1, X_2 \leq x_2) \geq Pr(X_1 \leq x_1)Pr(X_2 \leq x_2). \tag{3.75}$$

This implies that

$$Pr(X_1 > x_1, X_2 > x_2) \geq Pr(X_1 > x_1)Pr(X_2 > x_2). \tag{3.76}$$

Using Bayes's Law, we can re-write this as

$$Pr(X_1 > x_1 \mid X_2 > x_2) \geq Pr(X_1 > x_1), \tag{3.77}$$

and the first inequality as

$$Pr(X_1 \leq x_1 \mid X_2 \leq x_2) \geq Pr(X_1 \leq x_1) \tag{3.78}$$

$$\Leftrightarrow \quad 1 - Pr(X_1 \leq x_1) \geq 1 - Pr(X_1 \leq x_1 \mid X_2 \leq x_2) \tag{3.79}$$

$$\Leftrightarrow \quad Pr(X_1 > x_1) \geq Pr(X_1 > x_1 \mid X_2 \leq x_2). \tag{3.80}$$

It follows that

$$Pr(X_1 > x_1 \mid X_2 > x_2) \geq Pr(X_1 > x_1) \geq Pr(X_1 > x_1 \mid X_2 \leq x_2). \tag{3.81}$$

□

Proof of Proposition 3.1:

Proof. Suppose that $\gamma \to \infty$ and $\beta < \infty$ is constant. The LC's private signal becomes perfectly revealing with regards to θ, so that she wants to roll over early iff $y > \theta^*$ in Γ and iff $y > \theta^*_{NW}$ in Γ_{NW} and wait/withdraw otherwise. Hence, $y^* \to \theta^*$ and $y^*_{NW} \to \theta^*_{NW}$. Moreover, the conditional correlation $Corr(\theta, y \mid x_i) = \sqrt{\frac{\gamma}{\beta+\gamma}} \to 1$. It follows that SCs will completely ignore their private signal and always roll over if the LC rolls over and always withdraw if she withdraws/waits, i.e. $x^*, x^*_{NW} \to -\infty$ and $x^{**}, x^{**}_{NW} \to \infty$. As a result, if the LC rolls over in stage 1, all creditors roll over so that $\theta^*, \theta^*_{NW} \to 0$. In Γ, if she waits and does not roll over, no creditor rolls over and $\underline{\theta}^{**} \to 1$. If she waits and rolls over late, the project will only succeed if fundamentals are larger than the fraction of creditors who have withdrawn, i.e. if $\theta > 1 - \lambda$, so that $\overline{\theta}^{**} \to 1 - \lambda$. In Γ_{NW}, if the LC does not roll over, no creditor rolls over and $\theta^{**}_{NW} \to 1$. □

Proof of Proposition 3.2:

Proof. Suppose that $\gamma \to 0$ and $\beta > 0$ is constant. The LC's private signal is completely uninformative about θ and the LC is agnostic about the outcome of the project. Hence, in stage 1 she assumes a probability 50% of success and failure. Consequently, in Γ_{NW} she always rolls over ($y^*_{NW} \to -\infty$) if $0.5c_1 > t$ and always withdraws ($y^*_{NW} \to \infty$) if $0.5c_1 < t$. From her indifference condition

$$c_1 \Phi(\sqrt{\gamma}(y^*_{NW} - \theta^*_{NW})) = t \quad \Leftrightarrow \quad y^*_{NW} = \theta^*_{NW} + \frac{1}{\sqrt{\gamma}} \Phi^{-1}\left(\frac{t}{c_1}\right) \tag{3.82}$$

it follows that for $0.5c_1 = t$ she rolls over whenever $y > \theta^*_{NW}$ and withdraws otherwise, so that $y^*_{NW} \to \theta^*_{NW}$. Moreover, since an early rollover by the LC is uninformative about θ, SCs rely only on their private signal so that their indifference condition approaches

$$c_1 \Phi(\sqrt{\beta}(x^*_{NW} - \theta^*_{NW})) = t \quad \Leftrightarrow \quad x^*_{NW} = \theta^*_{NW} + \frac{1}{\sqrt{\beta}} \Phi^{-1}\left(\frac{t}{c_1}\right), \tag{3.83}$$

$$c_1 \Phi(\sqrt{\beta}(x^{**}_{NW} - \theta^{**}_{NW})) = t \quad \Leftrightarrow \quad x^{**}_{NW} = \theta^{**}_{NW} + \frac{1}{\sqrt{\beta}} \Phi^{-1}\left(\frac{t}{c_1}\right). \tag{3.84}$$

Using these expressions in the critical mass conditions and solving for θ^*_{NW} and θ^{**}_{NW} gives

$$\theta^*_{NW} = (1 - \lambda)\frac{t}{c_1} \qquad \text{and} \qquad \theta^{**}_{NW} = \lambda + (1 - \lambda)\frac{t}{c_1}. \tag{3.85}$$

Hence, for $\gamma \to 0$ we have

$$\theta^*_{NW} \to (1 - \lambda)\frac{t}{c_1} \tag{3.86}$$

$$\theta^{**}_{NW} \to \lambda + (1 - \lambda)\frac{t}{c_1}, \tag{3.87}$$

$$x^*_{NW} \to (1-\lambda)\frac{t}{c_1} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{t}{c_1}\right), \tag{3.88}$$

$$x^{**}_{NW} \to \lambda + (1-\lambda)\frac{t}{c_1} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{t}{c_1}\right) \tag{3.89}$$

For Γ, basically the same arguments as for Γ_{NW} apply, except that the LC compares c_1 with $c_2 + t$ instead of $0.5c_1$ with t, which alters her threshold y^*. □

Proof of Proposition 3.3:

Proof. The corresponding critical mass conditions for Γ are

$$h(\theta^*, x^*) - \lambda = \theta^* \qquad \text{and} \qquad h(\overline{\theta}^{**}, x^{**}) - \lambda = \overline{\theta}^{**}, \tag{3.90}$$

where

$$h(\theta, x) \equiv 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta - x)\right). \tag{3.91}$$

Given that θ^* and $\overline{\theta}^{**}$ are unique and under Assumption 3.2, the two equations define the implicit functions $\theta^*(x^*)$ and $\overline{\theta}^{**}(x^{**})$. Notice, that the two functions are actually identical and the difference between θ^* and $\overline{\theta}^{**}$ results because it is evaluated at different values of its argument, namely x^* and x^{**}. Hence we can write $\theta^* = \tilde{\theta}(x^*)$ and $\overline{\theta}^{**} = \tilde{\theta}(x^{**})$, where $\tilde{\theta}$ denotes the corresponding implicit function. From the Implicit Function Theorem it follows that

$$\frac{\partial \tilde{\theta}}{\partial x}(x) = \frac{\sqrt{\beta}\phi(\sqrt{\beta}(\theta - x))}{\sqrt{\beta}\phi(\sqrt{\beta}(\theta - x)) + \frac{1}{1-\lambda}} > 0. \tag{3.92}$$

Therefore, $\overline{\theta}^{**} \geq \theta^*$ holds iff $x^{**} \geq x^*$. To show that $x^{**} \geq x^*$, we first plug $\tilde{\theta}(x)$ into the SCs' indifference conditions:

$$Pr(\theta > \tilde{\theta}(x^*) \mid y > y^*, x^*) = \frac{t}{c_1} \tag{3.93}$$

$$Pr(\theta > \tilde{\theta}(x^{**}) \mid y \leq y^*, x^{**}) = \frac{t}{c_1}. \tag{3.94}$$

The statement $x^{**} \geq x^*$ holds if the following three conditions are met: a) x^* is the unique solution to (3.93), b) $Pr(\theta > \tilde{\theta}(x) \mid y > y^*, x)$ as a function of x crosses t/c_1 from below and c) $Pr(\theta > \tilde{\theta}(x) \mid y > y^*, x) \geq Pr(\theta > \tilde{\theta}(x) \mid y \leq y^*, x)$ for all x. The reason is that if all three conditions are true, then the unique intersection of $Pr(\theta > \tilde{\theta}(x) \mid y > y^*, x)$ with t/c_1 (which constitutes the solution x^*) always lies to the left of any intersection of $Pr(\theta > \tilde{\theta}(x) \mid y \leq y^*, x)$ with t/c_1 (which constitutes the solution x^{**}), so that $x^* \leq x^{**}$. Condition a) requires an addition to Assumption 3.2: It requires that x^* is unique, given that $\theta^* = \tilde{\theta}(x^*)$. As with Assumption 3.2, I was also not able to prove uniqueness in this case, which is why I again resort to numerical simulations. The results suggest that it is indeed the unique solution to equation 3.93. Hence, for this proof I assume that condition a) holds. Moreover, to show that b) holds, we look at the limits. To simplify notation, define

$$\psi(x) \equiv Pr(\theta > \tilde{\theta}(x) \mid y > y^*, x) = \frac{\Phi_2\left(\sqrt{\beta}\left(x - \tilde{\theta}(x)\right), \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x - y^*)\,; \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x - y^*)\right)}. \tag{3.95}$$

The limit is $\lim_{x\to\infty}\psi = 1$ because numerator and denominator converge to 1. The limit for $x \to -\infty$ is $0/0$, hence we need to invoke L'Hôpital's Rule. Here we have

$$\lim_{x\to-\infty}\psi = \lim_{x\to-\infty}\sqrt{\frac{\beta+\gamma}{\gamma}}\frac{\phi(m_1)}{\phi(m_2)}\frac{\Phi\left(\sqrt{\gamma}(\tilde{\theta}(x)-y^*)\right)}{1+(1+\lambda)\sqrt{\beta}\phi(-m_1)} + \lim_{x\to-\infty}\Phi\left(-\sqrt{\beta+\gamma}\left(\tilde{\theta}(x)-\frac{\gamma y+\beta x}{\beta+\gamma}\right)\right), \tag{3.96}$$

where

$$m_1 = \sqrt{\beta}(x-\tilde{\theta}(x)), \qquad \text{and} \qquad m_2 = \sqrt{\frac{\beta\gamma}{\beta+\gamma}}(x-y^*). \tag{3.97}$$

Notice that

$$\frac{\phi(m_1)}{\phi(m_2)} = \exp\left(-\frac{\beta}{2}\left(\frac{\beta x^2}{\beta+\gamma} - 2x\left(\tilde{\theta}(x)-\frac{\gamma y^*}{\beta+\gamma}\right)+\tilde{\theta}(x)^2-\frac{\gamma y^{*2}}{\beta+\gamma}\right)\right), \tag{3.98}$$

which converges to 0 if $x\to\infty$ or $x\to-\infty$. Hence,

$$\lim_{x\to-\infty}\psi = 0. \tag{3.99}$$

It follows that if x^* is unique, $\psi(x)$ must cross t/c_1 from below. Finally, for c) recall Lemma 3.1: It says that

$$Pr(\theta > \tilde{\theta}(x) \mid y > y^*, x) \geq Pr(\theta > \tilde{\theta}(x) \mid y \leq y^*, x) \tag{3.100}$$

for all x, as long as $Corr(\theta, y \mid x) > 0$. The correlation is $\sqrt{\gamma/(\gamma+\beta)} > 0$, so requirement c) also holds. Hence, x^* is always smaller than (or equal to) x^{**} and therefore $\overline{\theta}^{**} \geq \theta^*$ holds.

The proof that $\theta^{**}_{NW} \geq \theta^*_{NW}$ follows the same idea. We can write the critical mass conditions

$$h(\theta^*_{NW}, x^*_{NW}) - \lambda = \theta^*_{NW} \qquad \text{and} \qquad h(\theta^{**}_{NW}, x^{**}_{NW}) = \theta^{**}_{NW}, \tag{3.101}$$

where again

$$h(\theta, x) \equiv 1-(1-\lambda)\Phi\left(\sqrt{\beta}(\theta-x)\right). \tag{3.102}$$

The difference to game Γ is that for θ^{**}_{NW}, the function h is not shifted downward by λ. It follows that if $\lambda > 0$, $\theta^{**}_{NW} > \theta^*_{NW}$ even if $x^*_{NW} = x^{**}_{NW}$. In order to include the case where $\lambda \to 0$, the proof follows the same steps as in Γ, using the same conditions a)-c) to show that $x^*_{NW} \leq x^{**}_{NW}$. Again, I resort to numerical simulations to ensure uniqueness of x^*_{NW} in condition a), while the remaining conditions b) and c) follow the same procedure as before and the results are also the same. □

Proof of Corollary 3.1:

Proof. Recall from Propositions 3.1 and 3.2 that

$$\lim_{\gamma\to 0} x^* = \lim_{\gamma\to 0} x^{**} = \lim_{\gamma\to 0} x^*_{NW} = (1-\lambda)\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}}, \tag{3.103}$$

$$\lim_{\gamma\to 0} x_{NW}^{**} = \lambda + (1-\lambda)\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}}, \tag{3.104}$$

$$\lim_{\gamma\to\infty} x^{*} = \lim_{\gamma\to\infty} x_{NW}^{*} = -\infty, \tag{3.105}$$

$$\lim_{\gamma\to\infty} x^{**} = \lim_{\gamma\to\infty} x_{NW}^{**} = \infty. \tag{3.106}$$

Moreover, the herd-index is defined by

$$H = \Phi\left(\sqrt{\beta}\left(\theta - x^{*}\right)\right) - \Phi\left(\sqrt{\beta}\left(\theta - x^{**}\right)\right), \tag{3.107}$$

$$H_{NW} = \Phi\left(\sqrt{\beta}\left(\theta - x_{NW}^{*}\right)\right) - \Phi\left(\sqrt{\beta}\left(\theta - x_{NW}^{**}\right)\right). \tag{3.108}$$

Hence, for $\gamma \to 0$ we have

$$\lim_{\gamma\to 0} H = 0 \qquad \text{and} \qquad \lim_{\gamma\to 0} H_{NW} = \Phi\left(M\right) - \Phi\left(M - \sqrt{\beta}\lambda\right). \tag{3.109}$$

where

$$M = \sqrt{\beta}\left(\theta - (1-\lambda)\frac{t}{c_1} - \Phi^{-1}\left(\frac{t}{c_1}\right)\right) \tag{3.110}$$

and for $\gamma \to \infty$

$$\lim_{\gamma\to\infty} H = \lim_{\gamma\to\infty} H_{NW} = 1. \tag{3.111}$$

Moreover, in the proof of Proposition 3.3 it is shown that under the assumption that x^* is the unique solution to

$$Pr(\theta > \tilde{\theta}(x^*) \mid y > y^*, x^*) = \frac{t}{c_1}, \tag{3.112}$$

$x^* \leq x^{**}$ so that $H \geq 0$. From the proof it also becomes clear that the same holds for $x_{NW}^* \leq x_{NW}^{**}$, so that here also $H_{NW} \geq 0$. □

Proof that y^* is the unique solution to the LC's indifference condition. For every $\theta_1, \theta_2 \in \mathbb{R}$ fixed and $c_1 > c_2 > t > 0$, the indifference condition

$$f(y,\theta_1,\theta_2) \equiv c_1 \cdot \Phi\left(\sqrt{\gamma}\left(y-\theta_1\right)\right) - c_2 \cdot \Phi\left(\sqrt{\gamma}\left(y-\theta_2\right)\right) - t \cdot \Phi\left(\sqrt{\gamma}\left(\theta_2 - y\right)\right) = 0 \tag{3.113}$$

has a unique solution $y = y^*$, so that y^* can be written as a function $\tilde{y}(\theta_1,\theta_2)$. To see that, fix θ_1 and θ_2 and assume $c_1 > c_2 > t > 0$. Define

$$g_1 \equiv \sqrt{\gamma}\left(y - \theta_1\right) \qquad \text{and} \qquad g_2 \equiv \sqrt{\gamma}\left(y-\theta_2\right). \tag{3.114}$$

Then

$$\frac{\partial f}{\partial y} = c_1\sqrt{\gamma}\phi\left(g_1\right) - \left(c_2 - t\right)\sqrt{\gamma}\phi\left(g_2\right) = c_1\sqrt{\gamma}\phi\left(g_1\right)\left(1 - \frac{c_2 - t}{c_1}\frac{\phi\left(g_2\right)}{\phi\left(g_1\right)}\right). \tag{3.115}$$

For given θ_1 and θ_2, the equation $f\left(y,\theta_1,\theta_2\right) = 0$ has a unique solution y^* if the following conditions are met:

1. f is continuous in y.

2. $f(\cdot) > 0$ for some y and $f(\cdot) < 0$ for some other y. Then, together with (1.), the conditions for the Intermediate Value Theorem (IVT) are fulfilled and $f(\cdot) = 0$ has at least one solution.

3. $\frac{\partial f}{\partial y}(y, \theta_1, \theta_2) = 0$ for exactly one value $y = y_0$. Then f has either one maximum, one minimum, or a saddle point at y_0, and thus there are at most two solutions for $f(\cdot) = 0$.

4. $sign\left[\lim_{y\to-\infty} f(y, \theta_1, \theta_2)\right] \neq sign\left[\lim_{y\to\infty} f(y, \theta_1, \theta_2)\right]$. Given conditions 2 and 3 hold, if the signs of the two limits were equal, f would have to cross 0 twice and thus would have two solutions. If the signs are different, it crosses 0 at most once and thus there is only one $y = y^*$ solving $f(y, \theta_1, \theta_2) = 0$.

Condition 1 is clearly satisfied. Moreover, note that

$$\lim_{y\to-\infty} f(y, \theta_1, \theta_2) = c_1 \cdot 0 - c_2 \cdot 0 - t \cdot 1 = -t < 0 \tag{3.116}$$

$$\lim_{y\to\infty} f(y, \theta_1, \theta_2) = c_1 \cdot 1 - c_2 \cdot 1 - t \cdot 0 = c_1 - c_2 > 0, \tag{3.117}$$

so that conditions 2 and 4 are fulfilled as well. From IVT it follows that there is at least one solution y^*. Condition 3 requires

$$\frac{\partial f}{\partial y} = c_1\sqrt{\gamma}\phi\left(g_1\right)\left(1 - \frac{c_2 - t}{c_1}\frac{\phi\left(g_2\right)}{\phi\left(g_1\right)}\right) = 0 \tag{3.118}$$

to have one solution $y = y_0$. This is equivalent to

$$1 - \frac{c_2 - t}{c_1}\frac{\phi\left(g_2\right)}{\phi\left(g_1\right)} = 0 \tag{3.119}$$

$$\Leftrightarrow \quad \frac{\phi\left(g_1\right)}{\phi\left(g_2\right)} = \frac{c_2 - t}{c_1} \tag{3.120}$$

$$\Leftrightarrow \quad \exp\left(-\frac{1}{2}\gamma\left[\left(\theta_1 - y_0\right)^2 - \left(\theta_2 - y_0\right)^2\right]\right) = \frac{c_2 - t}{c_1} \tag{3.121}$$

$$\Leftrightarrow \quad \left(\theta_2 - y_0\right)^2 - \left(\theta_1 - y_0\right)^2 = \frac{2}{\gamma}\log\left(\frac{c_2 - t}{c_1}\right) \tag{3.122}$$

$$\Leftrightarrow \quad \theta_2^2 - \theta_1^2 - 2y_0\left(\theta_2 - \theta_1\right) = \frac{2}{\gamma}\log\left(\frac{c_2 - t}{c_1}\right) \tag{3.123}$$

$$\Leftrightarrow \quad y_0 = \frac{\theta_2 + \theta_1}{2} - \frac{\log\left(\frac{c_2 - t}{c_1}\right)}{\gamma\left(\theta_2 - \theta_1\right)} \qquad \text{for} \quad \theta_2 \neq \theta_1. \tag{3.124}$$

For every $\theta_1 \neq \theta_2$, the equation $\frac{\partial f}{\partial y}(y, \theta_1, \theta_2) = 0$ has a unique solution y_0 given by equation (3.124). Hence, condition 3 is fulfilled. There is at most one solution y^* to equation $f(y, \theta_1, \theta_2) = 0$, and y^* can be written as a function $\tilde{y}(\theta_1, \theta_2)$ for $\theta_1 \neq \theta_2$. Notice that the solution y^* is also unique for $\theta_1 = \theta_2$, since in this case $f(y, \theta_1, \theta_2) = 0$ simplifies to

$$y^* = \theta_1 - \frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{c_1 - c_2}{c_1 - c_2 + t}\right), \tag{3.125}$$

which also defines $\tilde{y}(\theta_1, \theta_2)$ uniquely. □

Proof that $\underline{\theta}^{**} \geq \overline{\theta}^{**}$. Notice that we can write the critical mass conditions as

$$\overline{\theta}^{**} = h\left(\overline{\theta}^{**}, x^{**}\right) - \lambda, \qquad \text{and} \qquad \underline{\theta}^{**} = h\left(\underline{\theta}^{**}, x^{**}\right), \tag{3.126}$$

where

$$h\left(\theta, x\right) \equiv 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta - x\right)\right). \tag{3.127}$$

Hence, given x^{**}, $\underline{\theta}^{**}$ is defined by the intersection of h with the 45-degree line through the origin, while $\overline{\theta}^{**}$ is the intersection of h shifted downwards by λ with the 45-degree line. The function h of θ resembles a monotonically decreasing sigmoid function, which approaches 1 as $\theta \to -\infty$ and λ as $\theta \to \infty$. It follows that if h is shifted downward by $\lambda \geq 0$, its intersection with the 45-degree lines through the origin lies to the left of its intersection if it is not shifted. Hence, given x^{**}, $\underline{\theta}^{**} \geq \overline{\theta}^{**}$ must hold. □

B.4 Appendix: Numerical stability and equilibrium uniqueness

Numerical solution: For the comparative statics results, the R-command `nleqslv` from the package of the same name is used (Version 3.3.4) (Hasselman, 2023). In general, I used the following set of arguments:

- `method="Newton"`
- `global="dbldog"`
- `xscalm="fixed"`
- `allowSingular=FALSE`
- `xtol=1e-14`

CDF of multivariate normal distribution: To solve the CDF of the bi-variate normal distribution, I use the Genz-Bretz-algorithm as implemented in the R-command `pmvnorm` of the R-package `mvtnorm` (Version 1.2-4) (Genz and Bretz, 2009).

Algorithm to determine equilibrium uniqueness: If there are multiple equilibria, the Newton-algorithm should converge to any of them if the initial conditions are sufficiently close. Therefore, I compute the solution to the equation system for a set of different initial conditions in the set of possible solutions, for a subset of the parameter space. In R, the package `BB` (Version 2019.10-1) (Varadhan and Gilbert, 2009) offers the command `multiStart` to check for multiple solutions in this manner. I employ the following algorithm:

Algorithm:

A. Define a parameter-vector Ω with values $(\beta, \gamma, \lambda, c_1, c_2, t)$ and a matrix M with 5 columns and N rows.

B. Get a set of different initial values and save them in M: For each n in N repeat:

1. Draw randomly y^* from $[-4.25, 15.25]$ and save in column 1, row n of matrix M.
2. Draw randomly x^* from $[-5, 1]$ and save in column 2, row n of matrix M.
3. Draw randomly x^{**} from $[-4.5, 16.5]$ and save in column 3, row n of matrix M.
4. Draw randomly θ^* from $(0, 1)$ and save in column 4, row n of matrix M.
5. Draw randomly θ^{**} from $(0, 1)$ and save in column 5, row n of matrix M.

C. Use a numerical algorithm (see documentation of `multiStart` (Varadhan and Gilbert, 2009)) to solve the model N times given Ω and using a new row of M as initial values in each iteration. For each iteration, document whether the algorithm converged or not and store the solution, if one is found.

The idea of determining the set of initial values in steps B.1. through B.5. is as follows: From the results in this chapter (see Figures B.3 and B.4) one can see that for the given sets of parameters the solutions y^*, x^* and x^{**} usually lie inside the intervals $[-1, 12]$, $[-4, 0]$ and $[-1, 13]$ respectively. In order to cover cases where additional equilibria may lie outside of these intervals, I extend both of their borders by 25% of the range of the interval and draw initial values for y^*, x^*, and x^{**} from these extended intervals. Of course, I cannot exclude the fact that solutions lie outside of these intervals as well. However, in order to have the numerical algorithm converge in a sufficient number of cases, extending the interval would require one to increase the number of initial values drawn, also, which would in turn overreach the limits of computational capacity available. Hence, to keep a balance, I limit my analysis to these intervals. With respect to the fundamental thresholds, they are naturally bounded between 0 and 1. Therefore, I draw initial values from these intervals. Given Ω, the numerical algorithm then tries to find a solution to the system of equations for each set of initial values. The results of these attempts for both models are stored and evaluated with regard to the multiplicity of solutions. Using the parameter values described in the text for the vector Ω, I could not find evidence for multiple solutions for any of the two games.

Uniqueness of x^* and x^{**} (Assumptions 3.1 and 3.2): The simulations to determine the uniqueness of the solution x^* and x^{**} is essentially the same as for the whole system of equilibrium equations. I use the same algorithm as before, however, I only solve equations (3.19) and (3.22) (for game Γ and equations (3.6) and (3.8) for game Γ_{NW}) separately. Again I find no evidence of multiple solutions for a large set of parameters and initial values. Therefore, the results indicate that Assumptions 3.1 and 3.2 hold.

Uniqueness of x^* given θ^* as an implicit function of x^* (Proposition 3.3): Using simulations, I check for requirement a) in the proof of Proposition 3.3. For that purpose, I numerically solve the following system of equations

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^* - \theta^*)\,,\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*)\,;\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*)\right)} = t, \tag{3.128}$$

$$\theta^* = 1 - \lambda - (1 - \lambda)\Phi\left(\sqrt{\beta}\,(\theta^* - x^*)\right) \tag{3.129}$$

for x^* and θ^*, and y^* being given. Using the same algorithm and parameter sets as in the other simulations in this chapter, I cannot find evidence for multiple solutions. Hence, the results support the assumption that requirement a) in the proof of Proposition 3.3 is satisfied.

4

Signaling and Imperfect Learning

Abstract

In this chapter I extend the findings from the previous chapter by lifting the assumption that the large creditor observes the fraction of small creditors who roll over perfectly, resulting in a game with imperfect learning. I investigate the sensitivity of the equilibrium with respect to information precision and two central results emerge: 1) I show that the game with perfect learning and the game without the option to wait (both introduced in the previous chapter) are special cases of the game with imperfect learning. Thereby the new parameter for signal precision governs how close the game with imperfect learning is to either of the other two games. 2) I confirm that the qualitative results from the previous chapter still hold, even if the large creditor is imperfectly informed. Hence, these results are robust towards the assumption that the large creditor observes small creditors perfectly.

4.1 Introduction

In the previous chapter I investigated how the option to wait affects the large creditor's choice to roll over early or wait and the small creditors' reaction in a credit rollover game.[38] The central mechanism that distinguishes the model with the option to wait from the one without is the large creditor's trade-off between signaling to small players by moving first or learning from them by moving last. If payoffs do not differ between the two actions, the precision of information about the fundamental state of the investment project is the central driver of the large player's decision. In order to keep the equilibrium derivation and analysis simple, I assumed that if the large creditor waits, she perfectly observes the proportion of small creditors who roll over their loan. However, in reality this is hardly the case. Not only might creditors be dispersed and it would be difficult to acquire information about the choices of every one of them, but most likely they also do not decide instantaneously and at the same time, which complicates the observation of an aggregate action. Hence, to approach a more realistic interpretation of the model, in this chapter I lift the assumption of perfect learning, resulting in a game with imperfect learning. By introducing an additional source of noise, I am able to shed light on the large creditor's learning process an how it affects the small players' decisions.

Two central results emerge: 1) I show that the game with perfect learning and the one without the option to wait are special cases of the game with imperfect learning. Thereby the new parameter for signal precision governs how close the game with imperfect learning is to either

[38]For better understanding, I refer to the large player as "she" and "her" and an individual small player as "he" and "him" throughout the rest of this chapter. I use the terms "loan" and "credit" interchangeably to refer to liquidity provided by a lender or creditor for a limited period of time.

of the other two games. 2) I confirm that the qualitative results from the previous chapter still hold, even if the large creditor is imperfectly informed. Hence, these results are robust towards the assumption that the large creditor observes small creditors perfectly. Overall, the analysis in this chapter completes the picture drawn in the previous chapter as it generalizes the games with and without the option to wait and shows how they are related. Moreover, it offers a new interpretation of the game without the option to wait as a game without learning, categorizing all three models on the spectrum of the large creditor's ability to learn from small creditors: no learning, imperfect learning and perfect learning.

In the context of the literature, this chapter contributes to the existing research on global coordination games as introduced by Carlsson and Van Damme (1993) and Morris and Shin (1998, 2003). Since the large player's optimal choice is an endogenous public signal, it also falls in line with papers that investigate the effects of public information on the players' ability to coordinate their actions (Morris and Shin, 2004; Angeletos et al., 2006, 2007). My analysis complements the contributions on the role of large players in global games (Corsetti et al., 2004; Bannier, 2005) and I show how signaling reinforces the traditional catalytic finance mechanism based on strategic complements between a lender-of-last-resort and private investors (Rochet and Vives, 2004; Morris and Shin, 2006; Corsetti et al., 2006). In a more general sense, my analysis can also be considered as a specific implementation of Bayesian Persuasion in a global game (Kamenica and Gentzkow, 2011; Goldstein and Huang, 2016), as the large creditor aims to convince small creditors to roll over by rolling over early and thereby provides substance to the claim that fundamentals are good. Finally, the idea of strategic waiting and the option to learn from others in the global games literature was first introduced by Dasgupta (2007) and Angeletos et al. (2006). Even though they consider only homogeneous small players instead of different sizes, the specific noise and learning technology that I assume for the large player is borrowed from their contribution.

The rest of this chapter is organized as follows: Section 4.2 briefly states the setup of the game with imperfect learning and derives the equilibrium conditions. Section 4.3 explains how the model is solved numerically and how the comparative statics analysis is conducted. The subsequent section 4.4 describes and interprets the comparative statics results, while the final section 4.5 concludes and discusses the findings.

4.2 Game Γ_{IL} with imperfect learning

4.2.1 Setup

In order to distinguish the analysis in this chapter from the previous chapter, I denote the game with imperfect learning as Γ_{IL}, in contrast to Γ for the game with perfect learning (and the option to wait) and Γ_{NW} for the game without the option to wait.[39] The corresponding subscripts also indicate the game from which the equilibrium variables are derived: *IL* for game Γ_{IL}, *NW* for game Γ_{NW} and no subscript for game Γ.

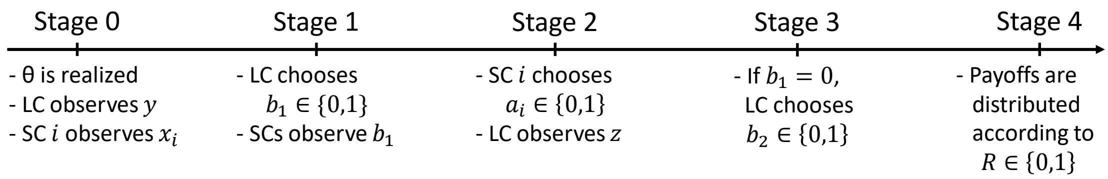
Table 4.1: Payoffs in game Γ_{IL} with imperfect learning.

Player	SC i		LC		
Action	$a_i = 0$	$a_i = 1$	$b_1 = 0$ $b_2 = 0$	$b_1 = 0$ $b_2 = 1$	$b_1 = 1$
$R = 0$	0	$-t$	0	$-t$	$-t$
$R = 1$	0	$c_1 - t$	0	$c_2 - t$	$c_1 - t$

[39] NW as an abbreviation for *no waiting*.

The setup in Γ_{IL} is essentially identical to game Γ, with the exception of an additional source of noise in the LC's learning process. Hence, I only briefly restate the basic assumptions and the setting of game Γ_{IL}: A large risk-neutral creditor (LC) of size $\lambda \in (0,1)$ and a mass of small risk-neutral creditors (SCs) indexed by $i \in (0, 1-\lambda)$ are invested in a risky investment project. At an interim stage of the project, each creditor is asked whether they want to roll over their investment or withdraw. Rolling over is risky because the project can succeed or fail, depending on the fundamental state and the proportion of creditors who withdraw their funds. If too many creditors withdraw relative to the fundamental state, the project fails because it has to be liquidated prematurely in order to repay the creditors. Accordingly, individual payoffs depend on the players' decisions and the outcome of the project: If an SC i rolls over and the project succeeds ($R = 1$), he receives a positive payoff of c_1. If the LC rolls over and the project is successful, she receives c_1 or $c_2 (\leq c_1)$, depending on the timing of her decision. However, for each creditor, rolling over comes at fixed costs $t(< c_2)$, so that the net payoffs are $c_1 - t > 0$ and $c_2 - t > 0$.[40] In case of project failure ($R = 0$), rolling over always pays 0, so that net payoffs are $-t$ for any player who rolls over. Withdrawing pays 0 without any additional costs. Table 4.1 provides an overview of individual payoffs, depending on actions and project outcome.

Figure 4.1: Sequence of actions in game Γ_{IL}.



Actions are sequential, as depicted in Figure 4.1: In stage 0, nature draws the fundamental state of the project θ from an improper uniform distribution on the real line. It represents the economic outlook of the project, i.e., a larger θ represents a higher chance that the project will be successful, while lower values represent a lower chance. It also captures external factors that may affect the project success, as well as project management quality. The creditors do not observe the fundamental state; however, they observe private signals about it. In particular, the LC observes a private signal

$$y = \theta + \varepsilon_y, \tag{4.1}$$

where noise $\varepsilon_y \sim N(0, \gamma^{-1})$. Each SC observes a private private signal

$$x_i = \theta + \varepsilon_i, \tag{4.2}$$

where noise $\varepsilon_i \overset{iid}{\sim} N(0, \beta^{-1})$ and independent from ε_y. The parameters γ and β represent the precision of the signals. Noise distributions are common knowledge. In stage 1 the LC decides whether she wants to roll over early (denote $b_1 = 1$) her loan or wait (denote $b_1 = 0$) until the SCs have made their decisions.[41] The SCs observe the action of the LC in stage 1 and decide whether

[40] This is a common assumption in the global games literature (cf. Morris and Shin, 2004).

[41] For the LC it is never optimal to withdraw early because withdrawing early or late will always result in zero payoff. Since waiting allows the LC to observe SCs, there is an information advantage in waiting and withdrawing in stage 3 instead of stage 1. It follows, that it is always better to wait than to withdraw in stage 1.

they want to roll over (denote $a_i = 1$) or withdraw (denote $a_i = 0$) in stage 2. The LC observes a noisy signal

$$z = \Phi^{-1}\left(\frac{A}{1-\lambda}\right) + \varepsilon_z \tag{4.3}$$

about the mass of SCs' who roll over A, which I call the SCs' aggregate (or average) action.[42] The noise distribution is $\varepsilon_z \sim N(0, \eta^{-1})$ and independent from all other noise. As before, η can be interpreted as the precision of signal z. Given that the LC chooses to wait in stage 1, in stage 3 the LC chooses between rolling over late (denote $b_2 = 1$) or withdrawing from the project for good (denote $b_2 = 0$). In stage 4, the outcome of the project is determined. It fails if the fundamental state is weak, relative to the proportion of creditors who withdraw. It succeeds if the fundamental state is sufficiently large. Payoffs are distributed according to individual decisions and project outcome.

4.2.2 Equilibrium

As in the previous chapter, I focus on equilibria in threshold strategies. In fact, the equilibrium definition (Definition 4.2 below) used for game Γ_{IL} is identical to the one used for Γ (Definition 3.2 in Chapter 3). The reason is that the two games are conceptually identical, in particular in that in stage 3 the LC observes a signal about the fraction of SCs who roll over. The difference, however, is that the signal is infinitely precise in Γ whereas it is observed with random noise in Γ_{IL}. Therefore, I can use the same definition of equilibrium, while only adapting the creditors' beliefs for the additional uncertainty using Bayes' Law.

To derive the equilibrium, I proceed as follows: First, I provide definitions for the necessary terminology related to the equilibrium and its elements. I then explain step by step how the equilibrium conditions are derived from the optimization problems of the various actors.

Definition 4.1 (Project failure and success)**.** The investment project is said to fail if $\theta \leq 1 - \ell$ and to succeed if $\theta > 1 - \ell$.

I define

$$\ell \equiv \lambda\,(b_1 + b_2) + A, \tag{4.4}$$

to be the fraction of creditors who roll over their loan. Accordingly, $1 - \ell$ is the fraction which withdraws. This type of condition for success and failure is common in the literature about global games as it allows for cases where crises occur due to both bad fundamentals as well as self-fulfilling expectations.[43]

The definition of a Perfect Bayesian Equilibrium in threshold strategies is stated as follows:

[42] In the definition of signal z I divide A by the SCs' aggregate size $1 - \lambda$ in order to get the proportion of SCs who roll over instead of the overall mass. This is merely a technical assumption that simplifies later expressions but has no impact on the overall results. Using the quantile function Φ^{-1} of the standard normal distribution evaluated at the aggregate action A allows us to simplify the expression for the LC's equilibrium beliefs, as will be shown below. At the same time, since Φ^{-1} is strictly monotone in its argument, it retains the idea that the signal z increases monotonously in the aggregate action A. This particular representation of how players learn about the aggregate action of others was first proposed by Dasgupta (2007).

[43] I provide an explicit definition of project failure because for the interpretation of results it is important to distinguish between project failure due to bad fundamentals and project failure due to coordination failure. In the equilibrium derivation below I explain the difference between these two types of project failure.

Definition 4.2 (Equilibrium in Γ_{IL})**.** The PBE in threshold strategies in Γ_{IL} consists of the following elements:

1. Threshold strategies:
   - A threshold strategy around y^*_{IL}, such that rolling over early ($b_1 = 1$) maximizes the LC's expected payoff if $y > y^*_{IL}$ and waiting ($b_1 = 0$) otherwise.
   - A threshold strategy around s^*_{IL}, such that rolling over late ($b_2 = 1$) maximizes the LC's expected payoff if $s > s^*_{IL}$ and withdrawing ($b_2 = 0$) otherwise. $s = s(y, A)$ is a sufficient statistic with respect to θ, that summarizes all information that are relevant for the LC's decision in stage 3.
   - A threshold strategy around x^*_{IL} and x^{**}_{IL}, such that rolling over ($a_i = 1$) maximizes SC i's expected payoff if a) $b_1 = 1$ and $x_i > x^*_{IL}$, or b) $b_1 = 0$ and $x_i > x^{**}_{IL}$ and withdrawing ($a_i = 0$) otherwise.
2. Creditors' aggregate action: The mass of creditors who roll over is given by $\ell = \lambda(b_1 + b_2) + A$, where $A = \int_0^{1-\lambda} a_i di$ is the mass of SCs who roll over. The mass of creditors who withdraw is $1 - \ell$.
3. Critical mass conditions:
   - A fundamental threshold θ^*_{IL}, such that conditional on $b_1 = 1$, the project succeeds ($R = 1$) iff $\theta > \theta^*_{IL}$.
   - A fundamental threshold $\overline{\theta}^{**}_{IL}$, such that conditional on $b_2 = 1$, the project succeeds ($R = 1$) iff $\theta > \overline{\theta}^{**}_{IL}$.
   - A fundamental threshold $\underline{\theta}^{**}_{IL}$, such that conditional on $b_2 = 0$, the project succeeds ($R = 1$) iff $\theta > \underline{\theta}^{**}_{IL}$.
4. Beliefs:
   - The LC forms beliefs $f(R \mid y)$ and $f(R \mid s)$ using Bayes' Law.
   - SC i forms beliefs $f(R \mid x_i, y > y^*_{IL})$ and $f(R \mid x_i, y \leq y^*_{IL})$ using Bayes' Law.

Large Creditor: To derive equilibrium conditions, I start in stage 3, given that the LC waited in stage 1. At this point of the game, she has observed her private signals y and z. Notice from the definition of z that the mass of SCs who roll over A is an equilibrium quantity. That means, it can only be determined once the SCs' equilibrium strategy is known. As in Γ, I will show that the SCs follow a threshold strategy around their thresholds x^*_{IL} and x^{**}_{IL}. Hence, conditional on the LC waiting in stage 1, the mass of SCs who roll over is given by $(1-\lambda)Pr(x > x^{**}_{IL} \mid \theta) = (1-\lambda)\Phi(\sqrt{\beta}(\theta - x^{**}_{IL}))$, where Φ is the CDF of the standard normal distribution. If we plug this expression into z, we get[44]

$$z = \sqrt{\beta}(\theta - x^{**}_{IL}) + \varepsilon_z. \tag{4.5}$$

[44] Here it becomes clear why the transformation of A with the standard normal quantile function Φ^{-1} and the division by $1 - \lambda$ in the definition of z are useful. In equilibrium z is a linear combination of θ, a constant (x^{**}_{IL}) and normal random noise, so that z conditional on θ and thereby θ conditional on z are normal again.

The distribution of θ conditional on z is given by $\theta \mid z \sim N(x_{IL}^{**} + z/\sqrt{\beta}, (\eta\beta)^{-1})$. Hence, using standard Bayesian updating for normal random variables (cf. Veldkamp, 2011, p. 11ff.), the posterior of θ given y and z is

$$\theta \mid y, z \sim N\left(\frac{\gamma y + \eta\sqrt{\beta}z + \eta\beta x_{IL}^{**}}{\gamma + \eta\beta}, \frac{1}{\gamma + \eta\beta}\right) \tag{4.6}$$

As demonstrated in Dasgupta (2007) and Angeletos and Werning (2006), we can define a sufficient statistic

$$s \equiv \frac{\gamma y + \eta\sqrt{\beta}z + \eta\beta x_{IL}^{**}}{\gamma + \eta\beta} \tag{4.7}$$

so that we can express the LC's posterior belief of θ in stage 3 in terms of s:

$$\theta \mid y, z \quad = \quad \theta \mid s \sim N\left(s, \frac{1}{\tau}\right), \tag{4.8}$$

where for ease of notation I define $\tau \equiv \gamma + \beta\eta$. This notation allows us to formulate the LC's expected payoff in terms of the statistic s and derive a threshold s_{IL}^{*}, such that she wants to roll over late if $s > s_{IL}^{*}$ and withdraw if $s \leq s_{IL}^{*}$. In particular, given that the LC rolls over late, at this stage of the game, the project succeeds if $\theta > \overline{\theta}_{IL}^{**}$. Hence, her expected payoff for rolling over is

$$(c_2 - t)Pr\left(\theta > \overline{\theta}_{IL}^{**} \mid s\right) - tPr\left(\theta \leq \overline{\theta}_{IL}^{**} \mid s\right) = c_2\Phi\left(\sqrt{\tau}\left(s - \overline{\theta}_{IL}^{**}\right)\right) - t, \tag{4.9}$$

whereas for withdrawing it is 0. Her indifference equation is

$$c_2\Phi\left(\sqrt{\tau}\left(s_{IL}^{*} - \overline{\theta}_{IL}^{**}\right)\right) - t = 0 \quad \Leftrightarrow \quad s_{IL}^{*} = \overline{\theta}_{IL}^{**} + \frac{1}{\sqrt{\tau}}\Phi^{-1}\left(\frac{t}{c_2}\right), \tag{4.10}$$

and its unique solution s_{IL}^{*} constitutes the equilibrium threshold. Notice that in comparison to Γ, due to the additional noise ε_z, observing s is not equivalent to θ and thus $s_{IL}^{*} \neq \overline{\theta}_{IL}^{**}$. It follows that the outcome of the project and the LC's decision in stage 3 are no longer perfectly congruent and there may be cases where the LC, despite observing the aggregate action A, may wrongly choose to roll over late.

In stage 1, the LC only knows y. If she rolls over early, the project succeeds if $\theta > \theta_{IL}^{*}$, so her expected payoff is

$$(c_1 - t)Pr\left(\theta > \theta_{IL}^{*} \mid y\right) - tPr\left(\theta \leq \theta_{IL}^{*} \mid y\right) = c_1\Phi\left(\sqrt{\gamma}\left(y - \theta_{IL}^{*}\right)\right) - t. \tag{4.11}$$

If she decides to wait, her expected payoff depends on her action in stage 3, which in turn depends on z (through s). Since z is not realized yet, she not only has to predict the fundamental state, but also her own behavior in stage 3, given her initial private signal y. Accordingly, the expected payoff for waiting is given by

$$(c_2 - t)Pr\left(\theta > \overline{\theta}_{IL}^{**}, s > s_{IL}^{*} \mid y\right) - tPr\left(\theta \leq \overline{\theta}_{IL}^{**}, s > s_{IL}^{*} \mid y\right). \tag{4.12}$$

The first sumand represents the joint probability that the LC rolls over late ($s > s_{IL}^{*}$) and the project being successful ($\theta > \overline{\theta}_{IL}^{**}$), conditional on y. The second sumand captures the event that the LC rolls over late, but the project is not successful ($\theta \leq \overline{\theta}^{**}_{IL}$). Using the normality of private signals, we can write this expected payoff as

$$c_2\Phi_2\left(\sqrt{\gamma}\left(y-\overline{\theta}^{**}_{IL}\right), \sqrt{\nu}\left(y-s^*_{IL}\right); \sqrt{\frac{\gamma}{\nu}}\right) + t\Phi\left(\sqrt{\nu}\left(s^*_{IL}-y\right)\right) - t, \tag{4.13}$$

where $\nu \equiv \gamma\left(\gamma+\beta\eta\right)/(\beta\eta)$ is the precision of s, conditional on y. The function Φ_2 is the CDF of the bivariate normal distribution with the third argument being the correlation coefficient, the mean a vector of zeros and variances standardized to 1. By equating expressions (4.11) and (4.13) we get the indifference equation

$$\begin{aligned} c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\theta^*_{IL}\right)\right) = \\ c_2\Phi_2\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right), \sqrt{\nu}\left(y^*_{IL}-s^*_{IL}\right); \sqrt{\frac{\gamma}{\nu}}\right) + t\Phi\left(\sqrt{\nu}\left(s^*_{IL}-y^*_{IL}\right)\right). \end{aligned} \tag{4.14}$$

The solution y^*_{IL} constitutes the threshold for the LC's threshold strategy in stage 1, so that in equilibrium she rolls over early if $y > y^*_{IL}$ and waits if $y \leq y^*_{IL}$. For y^*_{IL} to be a threshold in a threshold strategy, it must be the unique solution to (4.14). However, I was unable to show analytically that the indifference equation has a unique solution for every s^*_{IL}, θ^*_{IL} and $\overline{\theta}^{**}_{IL}$. Numerical simulations suggest that it is unique for the different scenarios considered in the analysis of this chapter, however multiplicity cannot be ruled out in general.[45] Hence, in order to close the model and derive further analytical results, I assume that y^*_{IL} is the only solution to indifference equation (4.14):

Assumption 4.1 (Uniqueness of y^*)**.** The indifference equation (4.14) has a unique solution y^*_{IL}, which constitutes a threshold in the LC's equilibrium threshold strategy.

Small Creditors: In stage 2, SCs have two pieces of information that are relevant for their decision: Their private signal x_i and the initial action of the LC in stage 1. SCs know that in equilibrium the LC rolls over early if $y > y^*_{IL}$ and that conditional on an early rollover, the project is successful if $\theta > \theta^*_{IL}$. Hence, an SC's expected payoff for rolling over in this case is

$$\begin{aligned} &(c_1-t)Pr\left(\theta > \theta^*_{IL} \mid y > y^*_{IL}, x_i\right) - tPr\left(\theta \leq \theta^*_{IL} \mid y > y^*_{IL}, x_i\right) \\ &= c_1\frac{Pr\left(\theta > \theta^*_{IL}, y > y^*_{IL} \mid x_i\right)}{Pr\left(y > y^*_{IL} \mid x_i\right)} - t = c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x_i-\theta^*_{IL}\right), \sqrt{\alpha}\left(x_i-y^*_{IL}\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x_i-y^*_{IL}\right)\right)} - t, \end{aligned} \tag{4.15}$$

where for simplicity I define $\alpha \equiv \beta\gamma/(\beta+\gamma)$ to be the precision of y conditional on x_i. If he withdraws, his expected payoff is 0. Therefore, the indifference equation is

$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{IL}-\theta^*_{IL}\right), \sqrt{\alpha}\left(x^*_{IL}-y^*_{IL}\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x^*_{IL}-y^*_{IL}\right)\right)} - t = 0, \tag{4.16}$$

where the solution x^*_{IL} constitutes one of the SCs' equilibrium thresholds. It is interpreted as usual: If $x_i > x^*_{IL}$, the expected payoff for rolling over exceeds the payoff for withdrawing and the SC i wants to roll over as well. On the other hand, if $x_i \leq x^*_{IL}$, the expected payoff for rolling over is not greater than t and withdrawing is preferable.

[45] Further information about the simulation can be found in Appendix C.4.

The most profound difference between the equilibria in Γ and Γ_{IL} can be seen in the indifference condition for SCs in case that the LC waits in stage 1. The reason is that now the outcome of the project and the LC's choice in stage 3 are no longer equivalent, i.e., the LC may choose to withdraw even though θ is sufficiently large to make the project succeed. Due to her observing an imprecise signal about A (and thus θ), she may observe a bad signal z simply by bad luck and thus decide to withdraw, despite θ being large. Hence, given their private signal x_i and the information that $y \leq y^*_{IL}$, SCs not only have to guess whether θ is actually sufficiently large, they also have to guess whether the LC is likely to make a mistake in her stage 3-decision. These considerations also reflect in their expected payoff for rolling over, which is given by[46]

$$\begin{aligned}(c_1 - t)&\left[Pr\left(\theta > \underline{\theta}^{**}_{IL} \mid y \leq y^*_{IL}, x_i\right) + Pr\left(\underline{\theta}^{**}_{IL} \geq \theta > \overline{\theta}^{**}_{IL}, s > s^*_{IL} \mid y \leq y^*_{IL}, x_i\right)\right] \\ &\quad - t\left[Pr\left(\theta \leq \overline{\theta}^{**}_{IL} \mid y \leq y^*_{IL}, x_i\right) + Pr\left(\underline{\theta}^{**}_{IL} \geq \theta > \overline{\theta}^{**}_{IL}, s \leq s^*_{IL} \mid y \leq y^*_{IL}, x_i\right)\right] \\ = c_1&\left[\frac{\Phi_2\left(\sqrt{\beta}\left(x_i - \underline{\theta}^{**}_{IL}\right), \sqrt{\alpha}\left(y^*_{IL} - x_i\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x_i\right)\right)} + \right. \\ &\left. \frac{\Psi_3\left(\underline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x_i}, \Sigma\right) - \Psi_3\left(\overline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x_i}, \Sigma\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x_i\right)\right)}\right] - t, \quad (4.17)\end{aligned}$$

where Ψ_3 is the CDF[47] of the trivariate normal distribution with mean vector $\mathbf{x_i} = (x_i, -x_i, x_i)^\top$ and variance-covariance matrix

$$\Sigma = \begin{bmatrix} \beta^{-1} & -\beta^{-1} & \beta^{-1} \\ -\beta^{-1} & \frac{\beta+\tau}{\beta\tau} & -\frac{\beta+\tau}{\beta\tau} \\ \beta^{-1} & -\frac{\beta+\tau}{\beta\tau} & \alpha^{-1} \end{bmatrix}. \quad (4.18)$$

Given that the LC waits in stage 1 ($y \leq y^*_{IL}$), the expected payoff captures two cases in which the project will be successful and two in which it fails: The project will be successful regardless of the LC's action if $\theta > \underline{\theta}^{**}_{IL}$ because the mass of SCs who roll over is already sufficient to keep the project liquid. Additionally, it succeeds if θ is in the range $(\overline{\theta}^{**}_{IL}, \underline{\theta}^{**}_{IL}]$ and the LC rolls over late ($s > s^*_{IL}$). The project fails if $\theta \leq \overline{\theta}^{**}_{IL}$, even if the LC rolls over late, or if θ is in the range $(\overline{\theta}^{**}_{IL}, \underline{\theta}^{**}_{IL}]$ and the LC withdraws ($s \leq s^*_{IL}$).

However, if the SC decides to withdraw, his expected payoff is 0 and therefore the indifference equation is given by

$$\begin{aligned}c_1&\left[\frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL} - \underline{\theta}^{**}_{IL}\right), \sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right)\right)} + \right. \\ &\left. \frac{\Psi_3\left(\underline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right) - \Psi_3\left(\overline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right)\right)}\right] - t = 0, \quad (4.19)\end{aligned}$$

where $\mathbf{x}^{**}_{IL} = (x^{**}_{IL}, -x^{**}_{IL}, x^{**}_{IL})^\top$ and the solution x^{**}_{IL} constitutes the second threshold in the SCs' equilibrium strategy.

[46]For a detailed derivation of the parameterized probabilities see the appendix in the previous chapter and Appendix C.2 in this chapter.

[47]I denote the tri-variate normal CDF with Ψ_3 instead of Φ_3 to make clear that this normal distribution is not centered around a zero-mean vector and variances are not normalized to 1. This is in contrast to the previous notation of e.g. Φ_2, which denotes the CDF of a bi-variate normal with zero-mean vector and variances normalized to 1.

As with the LC's indifference equation, I could not find an analytical proof that equations (4.16) and (4.19) always have a unique solution, even though numerical simulations suggest that they do.[48] Hence, to derive further results, I assume that the thresholds x^*_{IL} and x^{**}_{IL} are unique.

Assumption 4.2 (Uniqueness of x^*_{IL} and x^{**}_{IL})**.** Each of the indifference equations (4.16) and (4.19) has a unique solution x^*_{IL} and x^{**}_{IL}, which constitutes a threshold in the SCs' equilibrium threshold strategy.

Fundamental Thresholds: Recall from Definition 4.1 that the project fails if the realized fundamental state is too weak relative to the percentage of creditors who roll over, that is, if $1-\ell \geq \theta$. In contrast, coordination failure describes the case where fundamentals would be strong enough to support a project success, but too many creditors withdraw their funds so that the project fails anyway. In terms of the fundamental state this is the case if $\theta \in (0, 1-\ell]$.

The outcome of the project is determined by fundamental thresholds, similar to those derived for the creditors' threshold strategies. To see this, we can write the mass of SCs who roll over A as and equilibrium quantity:

$$A(\theta, b_1) = \int_0^{1-\lambda} a^*(x_i, b_1)\, di, \tag{4.20}$$

where $a^*(x_i, b_1)$ is the SCs' action according to their threshold strategy around x^*_{IL} and x^{**}_{IL}. Notice that due to the law of large numbers, this quantity can be expressed as the probability that an individual signal x_i is realized above or below a respective threshold x^*_{IL} or x^{**}_{IL}, i.e.,

$$A(\theta, b_1) = \begin{cases} (1-\lambda)\, Pr\left(x_i > x^*_{IL} \mid \theta\right) & \text{if } b_1 = 1, \\ (1-\lambda)\, Pr\left(x_i > x^{**}_{IL} \mid \theta\right) & \text{if } b_1 = 0, \end{cases} = \begin{cases} (1-\lambda)\, \Phi\left(\sqrt{\beta}\left(\theta - x^*_{IL}\right)\right) & \text{if } b_1 = 1, \\ (1-\lambda)\, \Phi\left(\sqrt{\beta}\left(\theta - x^{**}_{IL}\right)\right) & \text{if } b_1 = 0. \end{cases} \tag{4.21}$$

As one can see, conditional on b_1 and b_2, the equilibrium mass of SCs who roll over $A(\theta, b_1)$ and thus the overall proportion of creditors who roll over ℓ are strictly monotonously increasing in θ. Hence, there is always a unique value of θ which solves $\theta = 1 - \lambda(b_1 + b_2) - A(\theta, b_1)$. For $b_1 = 1$ I call this value θ^*_{IL} and it is the solution for the so-called critical mass condition

$$\theta^*_{IL} = 1 - \lambda - (1-\lambda)\, \Phi\left(\sqrt{\beta}\left(\theta^*_{IL} - x^*_{IL}\right)\right). \tag{4.22}$$

If $b_1 = 0$ and $b_2 = 1$, the critical mass condition is

$$\overline{\theta}^{**}_{IL} = 1 - \lambda - (1-\lambda)\, \Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**}_{IL} - x^{**}_{IL}\right)\right), \tag{4.23}$$

and for $b_1 = 0$ and $b_2 = 0$ it is

$$\underline{\theta}^{**}_{IL} = 1 - (1-\lambda)\, \Phi\left(\sqrt{\beta}\left(\underline{\theta}^{**}_{IL} - x^{**}_{IL}\right)\right), \tag{4.24}$$

with $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ denoting the respective solutions. The values θ^*_{IL}, $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ constitute fundamental thresholds in the sense that conditional on the LC's action, the project fails whenever θ is smaller than or equal to the respective threshold and it succeeds otherwise. Figure 4.2 illustrates why this is the case. It plots the quantity $1-\ell$ as a function of θ on the y-axis. Curve (a) represents

[48] Further information about the simulation can be found in Appendix C.4.

$1-\ell$ if $b_1 = 1$, curve (b) if $b_1 = 0$ and $b_2 = 1$ and (c) if $b_1 = 0$ and $b_2 = 0$. The solutions θ^*_{IL}, $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ are depicted as the intersections of the curves with the 45-degree line and it becomes clear that conditional on the LC's actions, θ is smaller than $1-\ell$ for all θ that are smaller than the thresholds and vice versa for all values greater than the thresholds. Hence, the equilibrium outcome of the project is determined by these thresholds.

Figure 4.2: Fraction of creditors who withdraw $1-\ell$ at θ in game Γ_{IL}.

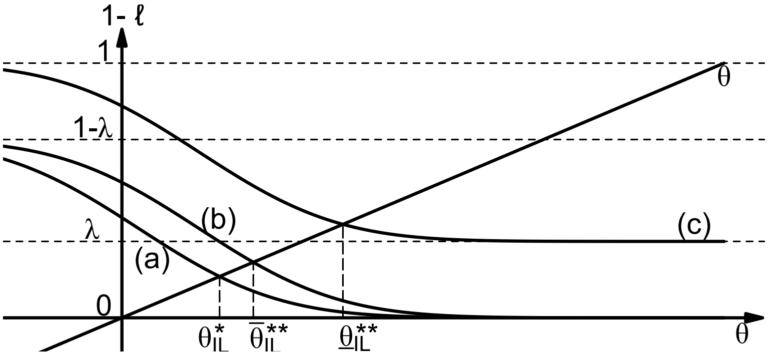


Figure 4.2 looks strikingly similar to Figure 3.4 in the previous chapter for game Γ. In particular the ordering of the thresholds $\theta^*_{IL} \leq \overline{\theta}^{**}_{IL} \leq \underline{\theta}^{**}_{IL}$ is identical. This observation is intuitive because the two games are almost identical with the exception of additional noise in the LC's learning process. However, in comparison to game Γ, in this setup with imperfect learning the threshold $\underline{\theta}^{**}_{IL}$ is more important. Recall that in the setup with perfect learning (game Γ), the LC's decision in stage 3 and the outcome of the project were identical because the LC essentially knows θ. That means, as soon as $\theta > \overline{\theta}^{**}_{IL}$, she would roll over and withdraw otherwise. Hence, conditional on the LC waiting in stage 1, the project could never be successful if the LC does not roll over in stage 3, so that threshold $\underline{\theta}^{**}_{IL}$ has no practical meaning. In game Γ_{IL} this is different. Since the LC observes the mass of SCs who roll over only with noise, her second private signal about θ is also noisy so that the project may succeed, even if she withdraws at the end.

4.3 Solution and parameter specification

The equilibrium of game Γ_{IL} is given by the solution $(y^*_{IL}, s^*_{IL}, x^*_{IL}, x^{**}_{IL}, \theta^*_{IL}, \overline{\theta}^{**}_{IL}, \underline{\theta}^{**}_{IL})$ to the system of equations

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\theta^*_{IL}\right)\right) = c_2\Phi_2\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right), \sqrt{\nu}\left(y^*_{IL}-s^*_{IL}\right); \sqrt{\frac{\gamma}{\nu}}\right) + t\Phi\left(\sqrt{\nu}\left(s^*_{IL}-y^*_{IL}\right)\right), \tag{4.25}$$

$$c_2\Phi\left(\sqrt{\tau}\left(s^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right) - t = 0, \tag{4.26}$$

$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{IL}-\theta^*_{IL}\right), \sqrt{\alpha}\left(x^*_{IL}-y^*_{IL}\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x^*_{IL}-y^*_{IL}\right)\right)} - t = 0, \tag{4.27}$$

$$c_1\left[\frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL}-\underline{\theta}^{**}_{IL}\right), \sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right)\right)} + \frac{\Psi_3\left(\underline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right) - \Psi_3\left(\overline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right)\right)}\right] - t = 0, \tag{4.28}$$

$$\theta^*_{IL} = 1 - \lambda - (1-\lambda)\,\Phi\left(\sqrt{\beta}\,(\theta^*_{IL} - x^*_{IL})\right), \tag{4.29}$$

$$\overline{\theta}^{**}_{IL} = 1 - \lambda - (1-\lambda)\,\Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**}_{IL} - x^{**}_{IL}\right)\right), \tag{4.30}$$

$$\underline{\theta}^{**}_{IL} = 1 - (1-\lambda)\,\Phi\left(\sqrt{\beta}\,(\underline{\theta}^{**}_{IL} - x^{**}_{IL})\right), \tag{4.31}$$

where $\mathbf{x}^{**}_{IL} = (x^{**}_{IL}, -x^{**}_{IL}, x^{**}_{IL})^\top$ and Σ as defined in (4.18). The elements of the solution represent thresholds in the threshold strategies of the players as well as fundamental thresholds from the critical mass conditions.[49]

For the comparative statics analysis I proceed in two steps: First, I analyze how the thresholds behave in the limit as I let the LC's private information precisions γ and η go to 0 and ∞ respectively. We will see that the equilibria in the games Γ with perfect learning and Γ_{NW} without the option to wait emerge as special cases of the game Γ_{IL} with imperfect learning. In a second step I investigate the sensitivity of the equilibrium away from the limit with respect to changes in the parameters of the game. Due to the non-linear structure of the equilibrium equations there is, however, no closed form representation of the solution. Hence, I use numerical methods to find the equilibrium thresholds for the various parameter specifications and compare the results graphically.[50]

The main focus of the numerical equilibrium analysis lies on the impact of changes in the LC's private information precision γ, while all other parameters remain fixed. Thereby I proceed by computing the equilibrium for an evenly distributed number of values of γ in the interval $[0.1, 10]$. The resulting equilibrium values are then plotted on the y-axis against (the log of) γ/η on the x-axis. Investigating the role of the "quality" of the LC's signal is particularly interesting as it is one of the main determinants of the decision to roll over early or wait. For the remaining parameters I use the same specification of values as in the previous chapter, with the additional parameter η set to 1 in order to compare γ in terms of fractions or multiples of η. Table C.2 in Appendix C.1 summarizes the parameter values used for the simulations, as well as their interpretation.

The goal of the analysis in this chapter is to assess the robustness of results from game Γ in the previous chapter with regards to the assumption that the LC observes the mass of SCs who roll over perfectly. Therefore, I compare the equilibrium variables derived from game Γ_{IL} with the equilibrium variables derived from the other two games. In order to evaluate the creditors' equilibrium behavior, I use the ex-ante probabilities of rolling over (early) instead of plain equilibrium thresholds. Probabilities are not only more intuitive to interpret than thresholds, they also account for changes in the distribution of private signals, which in turn affect the creditors' behavior. This is especially relevant for the LC as a change in γ not only changes her threshold y^*_{IL}, but also the dispersion of her private signal y, which in turn has an effect on how often she is willing to roll over early or not. Accordingly, to evaluate her behavior I calculate the probability that she rolls over early

$$P^y_{IL} \equiv Pr\left(y > y^*_{IL} \mid \hat{\theta}\right) = \Phi\left(\sqrt{\gamma}\left(\hat{\theta} - y^*_{IL}\right)\right), \tag{4.32}$$

where $\hat{\theta}$ is the value for the fundamental state under which the signal is realized. In the simulations I use the values -0.5 and 0.5 for the realized fundamental value to represent a bad and a good fundamental state respectively.[51]

[49] Table C.1 in the appendix provides an overview of the equilibrium thresholds together with their interpretation.

[50] As in the previous chapter, I use the (Quasi-)Newton's algorithm (cf. Judd, 1998, Ch. 5) as implemented in the `nleqslv`-command in the R-package of the same name (Hasselman, 2023). See Appendix C.4 for more information on the exact setting of control arguments of the R-command.

[51] In contrast to the previous chapter, I omit the realized fundamental value of 0 in this chapter in order to keep the illustrations of my simulation results simple and clear.

In my analysis I do not explicitly examine how s^*_{IL} or the probability of a late rollover changes if we change parameters. The reason is that a late rollover does not affect the SCs' decision (because they do not observe it) and it only affects the probability of coordination failure to the extent of the LC's size. Hence, in order to study the players' interactions, signaling and learning and effects on the probability of coordination failure, I focus on the LC's decision in stage 1 and omit her decision in stage 3.

With regard to the SCs' equilibrium behavior, I calculate the herd-index H_{IL}, which is defined by

$$H_{IL} \equiv Pr(x > x^*_{IL} \mid \hat{\theta}) - Pr(x > x^{**}_{IL} \mid \hat{\theta}) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*_{IL}\right)\right) - \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}_{IL}\right)\right). \quad (4.33)$$

It is a measure for the proportion of SCs who roll over if and only if the LC rolls over early. Much like for game Γ, this quantity will prove to be useful for understanding the LC's incentive to roll over early.

Finally, in order to evaluate the probability of coordination failure, I use the fundamental thresholds θ^*_{IL}, $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ as approximations for this probability conditional on the LC's behavior. The idea is that if any of these thresholds increases, the range of fundamentals for which the project fails increases and thus the probability of project failure increases. The critical mass conditions (4.22) through (4.24) require that the thresholds lie in the interval $(0, 1)$. Hence, in this regard, a change in any of the thresholds not only describes a change in the probability of project failure, but also more specifically coordination failure.

4.4 Comparative statics results

4.4.1 The effect of information precision in the limit

As a first step of my analysis, I investigate the sensitivity of the equilibrium if the LC's private signals is either uninformative of perfectly revealing with regards to the fundamental state θ. For Proposition 4.1 the signal z about the mass of SCs who roll over is uninformative ($\eta \to 0$), while keeping the precision of signal y constant.[52]

Proposition 4.1. *Suppose $\eta \to 0$ and $\gamma > 0$ is constant. The equilibrium thresholds in game Γ_{IL} converge to the respective thresholds of game Γ_{NW}, i.e., to the solution of the system*

$$y^*_{IL} = \theta^*_{IL} + \frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{t}{c_1}\right),$$

$$s^*_{IL} = \overline{\theta}^{**}_{IL} + \frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{t}{c_2}\right),$$

$$c_1 \frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{IL} - \theta^*_{IL}\right), \sqrt{\alpha}\left(x^*_{IL} - y^*_{IL}\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x^*_{IL} - y^*_{IL}\right)\right)} - t = 0,$$

$$c_1 \frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL} - \underline{\theta}^{**}_{IL}\right), \sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right)\right)} - t = 0,$$

$$\theta^*_{IL} = 1 - \lambda - (1 - \lambda)\Phi\left(\sqrt{\beta}\left(\theta^*_{IL} - x^*_{IL}\right)\right),$$

[52]For proof see Appendix C.3.

$$\overline{\theta}_{IL}^{**} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}_{IL}^{**} - x_{IL}^{**}\right)\right),$$
$$\underline{\theta}_{IL}^{**} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\underline{\theta}_{IL}^{**} - x_{IL}^{**}\right)\right).$$

Notice that s_{IL}^* appears only in one equation. The reason is that as the precision of the second private signal z goes to 0, the statistic s contains no more information about θ than the information given by y. Waiting does not offer any new information for the LC, so any decision by the LC made in stage 1 will not be changed in a later stage. Since the LC rolls over early only if she receives a good private signal ($y > y_{IL}^*$), SCs interpret this as an optimistic signal about the fundamental state. Accordingly, they are more willing to roll over as well, which in turn increases the probability of project success, relative to the case when the LC waits in stage 1. For the LC rolling over early is therefore strictly superior to a late rollover. Hence, she either wants to roll over early or not at all.[53] The threshold s_{IL}^* has therefore no relevance for the LC's decision. This in turn means that the threshold $\overline{\theta}_{IL}^{**}$ is also not relevant, because the LC never rolls over late. The result is a system of equations that is equivalent to the system of indifference and critical mass conditions derived for game Γ_{NW} without the option to wait.

From another perspective, Propositions 4.1 emerges because by letting the signal z about the mass of SCs who roll over become uninformative, we remove the only benefit of waiting over withdrawing, which is the learning aspect. If the LC cannot acquire new information about θ by waiting, there is no reason to wait and roll over late, because rolling over early offers the benefit of sending an optimistic signal to SC. Hence, the only decision left for the LC is to either roll over early or withdraw. SCs incorporate these incentives and the equilibria of Γ_{IL} and Γ_{NW} converge.[54]

Proposition 4.2 deals with the opposite case where we let the signal z be infinitely precise. As expected, in this case the equilibrium of Γ_{IL} converges to the equilibrium of game Γ with perfect learning.[55]

Proposition 4.2. *Suppose $\eta \to \infty$ and $\gamma < \infty$ is constant. The equilibrium thresholds in game Γ_{IL} converge to the respective thresholds of game Γ, i.e., to the solution of the system*

$$c_1\Phi\left(\sqrt{\gamma}\left(y_{IL}^* - \theta_{IL}^*\right)\right) = c_2\Phi\left(\sqrt{\gamma}\left(y_{IL}^* - \overline{\theta}_{IL}^{**}\right)\right) + t\Phi\left(\sqrt{\gamma}\left(\overline{\theta}_{IL}^{**} - y_{IL}^*\right)\right),$$
$$s_{IL}^* = \overline{\theta}_{IL}^{**},$$
$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x_{IL}^* - \theta_{IL}^*\right), \sqrt{\alpha}\left(x_{IL}^* - y_{IL}^*\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x_{IL}^* - y_{IL}^*\right)\right)} - t = 0,$$
$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x_{IL}^{**} - \overline{\theta}_{IL}^{**}\right), \sqrt{\alpha}\left(y_{IL}^* - x_{IL}^{**}\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y_{IL}^* - x_{IL}^{**}\right)\right)} - t = 0,$$
$$\theta_{IL}^* = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta_{IL}^* - x_{IL}^*\right)\right),$$
$$\overline{\theta}_{IL}^{**} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}_{IL}^{**} - x_{IL}^{**}\right)\right),$$
$$\underline{\theta}_{IL}^{**} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\underline{\theta}_{IL}^{**} - x_{IL}^{**}\right)\right).$$

Propositions 4.1 and 4.2 show that game Γ_{IL} with imperfect learning can be interpreted as a generalization of game Γ with perfect learning and game Γ_{NW} without the option to wait

[53] For more detail on this argument see the proof for Proposition 4.1 in Appendix C.3.

[54] Notice that the threshold θ_{NW}^{**} from game Γ_{NW} in the previous chapter corresponds to the threshold $\underline{\theta}_{IL}^{**}$ in game Γ_{IL}.

[55] For proof see Appendix C.3.

(i.e. without learning by the LC) and parameter η governs the extent to which the LC learns throughout the course of the game.

For comparability with the previous chapter, I also consider the cases where the initial signal y becomes infinitely precise or uninformative, while keeping the precision of signal z fixed.[56]

Proposition 4.3. *Suppose $\gamma \to 0$ and $\eta > 0$ is constant. The equilibrium thresholds in game Γ_{IL} converge to the solution of the system of equations*

$$y_{IL}^* = \begin{cases} -\infty & \text{if} \quad c_1 > c_2 + t, \\ \hat{y}^* & \text{if} \quad c_1 = c_2 + t, \\ \infty & \text{if} \quad c_1 < c_2 + t, \end{cases}$$

$$s_{IL}^* = \overline{\theta}_{IL}^{**} + \frac{1}{\sqrt{\beta\eta}} \Phi^{-1}\left(\frac{t}{c_2}\right),$$

$$x_{IL}^* = (1-\lambda)\frac{t}{c_1} + \frac{1}{\sqrt{\beta}} \Phi^{-1}\left(\frac{t}{c_1}\right),$$

$$c_1 \left[\Phi\left(\sqrt{\beta}\,(x_{IL}^{**} - \underline{\theta}_{IL}^{**})\right) + \Phi_2\left(\sqrt{\beta}\,(\underline{\theta}_{IL}^{**} - x_{IL}^{**}), -\sqrt{\frac{\eta}{1+\eta}} s_{IL}^*; -\sqrt{\frac{\eta}{1+\eta}}\right) - \Phi_2\left(\sqrt{\beta}\left(\overline{\theta}_{IL}^{**} - x_{IL}^{**}\right), -\sqrt{\frac{\eta}{1+\eta}} s_{IL}^*; -\sqrt{\frac{\eta}{1+\eta}}\right)\right] - t = 0,$$

$$\theta_{IL}^* = (1-\lambda)\frac{t}{c_1},$$

$$\overline{\theta}_{IL}^{**} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}_{IL}^{**} - x^{**}\right)\right),$$

$$\underline{\theta}_{IL}^{**} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\,(\underline{\theta}_{IL}^{**} - x_{IL}^{**})\right).$$

By letting the initial signal y become uninformative with regard to θ, the LC's choice to roll over early or wait essentially becomes a coin toss because success and failure of the project are equally likely to her. Hence, her optimal choice comes down to payoffs: If $c_1 > c_2 + t$, she always wants to roll over early, while she always waits if $c_1 < c_2 + t$. Merely for $c_1 = c_2 + t$ her threshold converges to a finite value, which I call $\hat{y}^*$. For game Γ, this value is equal to $(1-\lambda)t/c_1$ (see Proposition 3.2 in the previous chapter), while in this setup there is no closed form expression.[57]

In case that the LC rolls over early, the SCs' optimal response is identical to that in game Γ, i.e. the expressions for x_{IL}^* and θ_{IL}^* are identical to the ones from Proposition 3.2 in the previous chapter. However, the major difference between Proposition 4.3 and the previous chapter results if the LC waits. The reason is that from an SC's perspective, the prediction of the LC's action in stage 3 is not equivalent to predicting whether $\theta > \overline{\theta}_{IL}^{**}$. Due to the uncertainty in the LC's signal z, in game Γ_{IL} there is the possibility that the LC chooses to roll over in stage 3, even though the project will not succeed. Hence, the SCs' optimization problem is more complicated and the indifference equation cannot be solved for a closed form expression of x_{IL}^{**}. Consequently, also no closed form expressions for $\overline{\theta}_{IL}^{**}$ and $\underline{\theta}_{IL}^{**}$ can be found.

If we let $\gamma \to \infty$, the results are again identical to the ones from the previous chapter.

[56] For proof see Appendix C.3.
[57] However, numerical simulations suggest that it is finite.

Proposition 4.4. *Suppose $\gamma \to \infty$ and $\eta < \infty$ is constant. The equilibrium thresholds in game Γ_{IL} converge to the following values*

$$y^*_{IL} \to \theta^*_{IL} \to 0, \quad s^*_{IL} \to \infty, \quad x^*_{IL} \to -\infty, \quad x^{**}_{IL} \to \infty, \quad \overline{\theta}^{**}_{IL} \to 1-\lambda, \quad \underline{\theta}^{**}_{IL} \to 1.$$

The LC is perfectly informed about θ in stage 1 and accordingly rolls over early if $y > \theta^*_{IL}$ and waits otherwise. If she rolls over early, all SCs follow her ($x^*_{IL} \to -\infty$), while ignoring their private signals because they know that the LC only rolls over if she is certain that the project will be successful. Since in this case every creditor rolls over, $\theta^*_{IL} \to 0$. If the LC waits, SCs know that $\theta \leq \theta^*_{IL}$. In equilibrium it holds that $\theta^*_{IL} \leq \overline{\theta}^{**}_{IL} \leq \underline{\theta}^{**}_{IL}$, so that that the project can never be successful and every SC withdraws ($x^{**}_{IL} \to \infty$).[58] In stage 3, the LC then also withdraws because she cannot render the project successful herself ($s^*_{IL} \to \infty$). Accordingly, the fundamental thresholds converge to $\overline{\theta}^{**}_{IL} \to 1-\lambda$ and $\underline{\theta}^{**}_{IL} \to 1$.

Moreover, with regard to the herd-index H_{IL} the following result emerges.[59]

Corollary 4.1. *a) In the limit, as $\gamma \to 0$ and $\eta > 0$ is constant, the herd-index converges to*

$$\lim_{\gamma \to 0} H_{IL} = \hat{H} \geq 0,$$

for a realized fundamental state θ.
b) As $\gamma \to \infty$ and $\eta < \infty$ is constant, the herd-index converges to

$$\lim_{\gamma \to \infty} H_{IL} = 1.$$

c) It holds that $H_{IL} \geq 0$ for all $\theta \in \mathbb{R}$.

Unlike for game Γ, in the game with imperfect learning, the herd-index does not converge to 0 as the LC's initial private signal y becomes completely uninformative. The reason is that the two thresholds x^*_{IL} and x^{**}_{IL} do not necessarily converge to the same value. The difference in the SCs' behavior results from the unpredictability of the outcome of the project, once the LC chooses to wait. Additional to the fundamental uncertainty about θ, SCs have to factor in the uncertainty about the LC's second signal z, because even if $\theta > \overline{\theta}^{**}_{IL}$, the project may still fail if the LC happens to observe a bad signal z (and thus a small statistic s) by mere chance. As a consequence, SCs cannot be sure that the LC rolls over late when success is possible, so that x^*_{IL} and x^{**}_{IL} differ and H_{IL} converges to some (finite) value, which I call $\hat{H}$ for illustration.

Moreover, much like in the previous chapter, the herd-index in game Γ_{IL} is always greater (or equal to) zero. This shows that also in this setup, SCs interpret an early rollover by the LC as an optimistic signal, which increases their willingness to roll over relative to the case when the LC waits.

4.4.2 The probability of early rollover and herd behavior

Figure 4.3 shows the results of the numerical simulation of the creditors' behavior for the case that $c_2 = 0.9999$ and λ being equal to 0.5 or 0. In each of the four panels, the equilibrium

[58] See Proposition 4.5.
[59] For proof see Appendix C.3.

variables are plotted against the log of the LC's relative information precision γ/η on the x-axis. Panels (a) and (b) show the probability that the LC rolls over early for the three different games Γ_{NW}, Γ and Γ_{IL} and the realized fundamental value $\hat{\theta}$ equal to -0.5 or 0.5. Panels (c) and (d) depict the herd index for the three games and different values of the realized fundamental state. In general, the qualitative results from the previous chapter still hold, even if the LC is imperfectly informed. Hence, they are robust against the assumption that the LC observes the SCs' behavior perfectly. Accordingly, the following Results 4.1a) through d) restate the results from the previous chapter adjusted for game Γ_{IL}.

Simulation Result 4.1. *Probability of early roll-over P^y_{IL} and herd-index H_{IL}:*

a) $P^y_{NW} > P^y_{IL} > P^y$ holds for all γ.

b) The relation between P^y_{IL} and γ may be non-monotone.

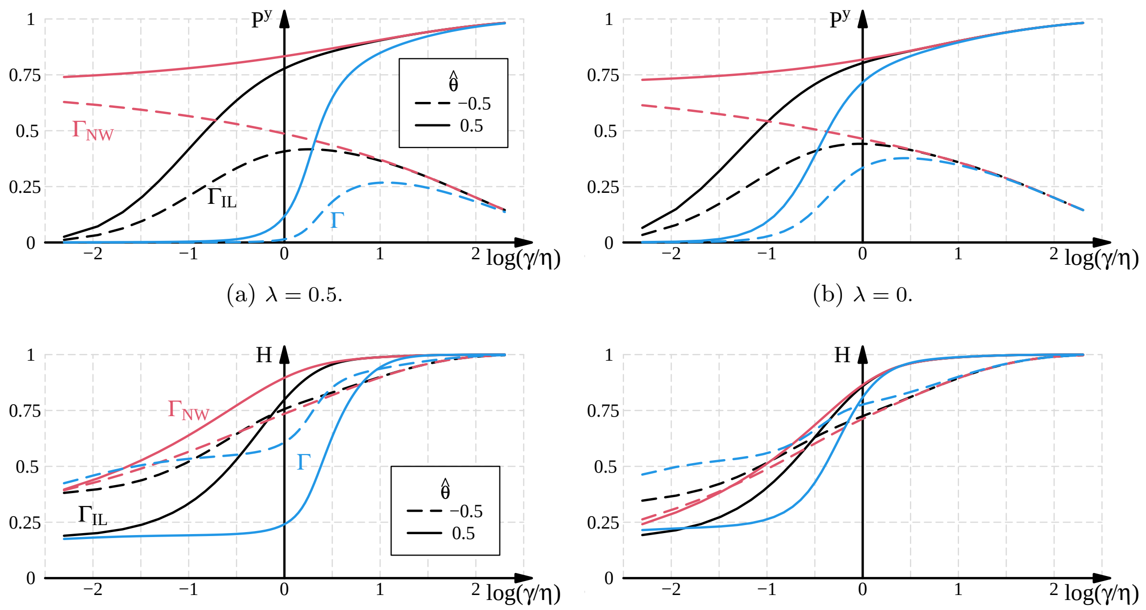
c) If c_2 increases, the probability of an early roll-over P^y_{IL} decreases.

d) If the LC's size λ increases the probability of an early roll-over P^y_{IL} decreases.

e) If γ increases, the herd index H_{IL} increases monotonically.

Notice, however, that there are quantitative differences. In particular, the probability of an early rollover in game Γ_{IL} lies between the probability of early rollover in the other two games (Result 4.1a)). The intuition behind this observation becomes clear when considering Propositions 4.1 and 4.2. Games Γ and Γ_{NW} are special cases of Γ_{IL} if the second private signal which the LC observes either becomes completely uninformative or perfectly precise. The simulations reflect this because for intermediate precision ($\eta = 1$) the probability of early rollover lies in between those two extremes.

Figure 4.3: Effect of change in $\log(\gamma/\eta)$ on probability of early rollover and herd-index for $c_2 = 0.9999$.



(a) $\lambda = 0.5$.

(b) $\lambda = 0$.

(c) $\lambda = 0.5$.

(d) $\lambda = 0$.

Note: Panels (a) and (b): Black = P^y_{IL}, blue = P^y and red = P^y_{NW}. Panels (c) and (d): Black = H_{IL}, blue = H and red = H_{NW}.

Result 4.1b) essentially confirms the previous finding for game Γ. The non-monotonicity of P^y_{IL} for a bad fundamental state is the result of the LC trading off two effects: On the one hand, through the herd behavior channel, an increase in γ increases the fraction of SCs who are willing to roll over if the LC rolls over early.[60] This in turn increases the probability that the project is successful and thus it increases the LC's incentive to roll over early. On the other hand, through the information channel, as γ increases, the LC's information about the realized fundamental state becomes more precise and thus she assigns a higher probability to the event that the fundamental state might actually be bad. As a consequence, she is less willing to roll over early. For small γ, the herd behavior channel dominates the information channel so that the increase in expected payoff caused by the potential increase in SCs willing to roll over outweighs the decrease in expected payoff caused by more precise information about the bad fundamental state. The probability of early rollover increases in γ. For larger values of γ, the information channel starts to outweigh the herd behavior channel and the LC is more convinced that the project cannot succeed. The probability of an early rollover decreases in γ.

Results 4.1c) and 4.1d) emerge when comparing the plots across Figures 4.3, C.1 and C.2 (the latter two can be found in Appendix C.1). The two findings also confirm the results from the previous chapter and can be explained by the same arguments. An increase in c_2 increases the opportunity costs of rolling over early, so that the LC is less willing to do so. Increasing the size λ decreases the mass of SCs who could roll over, so that the outcome of the project is less dependent on their action and more on the LC's action, regardless of the stage of the game. Hence, due to the information advantage of waiting in stage 1, the LC is more willing to wait than if she is smaller.

Result 4.1e) restates the finding for game Γ, adjusted for imperfect learning. Again, the mass of SCs who are willing to roll over if and only if the LC rolls over early increases monotonically in the LC's information precision because for larger γ, the LC is better informed about the realized fundamental state and therefore less likely to roll over if θ is bad. Hence, her action is a good predictor for the outcome of the project and SCs are more willing to ignore their private signal and follow the LC. Panels (c) and (d) in Figures 4.3, C.1 and C.2 illustrate this positive relation for different parameter settings.

4.4.3 The conditional probability of coordination failure

The effect of LC's decisions. As previously explained, I interpret the fundamental thresholds as indicators for the conditional probability of coordination failure, because they determine for which range of fundamental states the project fails. Comparing the three thresholds allows us to assess the impact of the LC's decisions on the probability of coordination failure. Analogous to the previous chapter, I find that the probability is lowest if the LC rolls over early, while it is highest if she withdraws. If she rolls over late, the probability of coordination failure is lower than if she withdraws if the LC is not small, i.e., if $\lambda > 0$.[61]

Proposition 4.5. *The conditional probability of coordination failure is smaller if the LC rolls over early than if she rolls over late or withdraws. More specifically,* $\underline{\theta}^{**}_{IL} \geq \overline{\theta}^{**}_{IL} \geq \theta^{*}_{IL}$.

The intuition behind $\underline{\theta}^{**}_{IL} \geq \overline{\theta}^{**}_{IL}$ is trivial: If the LC is large ($\lambda > 0$), a rollover increases the mass of all creditors who roll over ℓ and thereby decreases the threshold, given by the solution to $\theta = 1 - \ell(\theta)$. Hence, the probability of coordination failure is lower than if she withdraws. If the LC is small ($\lambda \to 0$), the two thresholds converge. However, an early rollover increases ℓ even

[60] See Result 4.1e) and panels (c) and (d) in Figure 4.3.
[61] For proof see Appendix C.3.

beyond the LC's own mass λ because SCs interpret the LC's action as an optimistic signal about θ and are more willing to roll over than if the LC waits. It follows that, $\overline{\theta}^{**}_{IL} \geq \theta^{*}_{IL}$.

Numerical Simulation Results. Further insights can be gained by comparing the simulation results in Figure 4.4 for $c_2 = 0.9999$.[62] In the four panels, I plot the fundamental thresholds derived from the three games against the log of γ/η, in order to illustrate how the conditional probability of coordination failure changes. Panels (a) and (b) depict the threshold conditional on the LC rolling over in stage 1 and panels (c) and (d) show the thresholds for the case that she waits/withdraws in stage 1.

Simulation Result 4.2. *Conditional probability of coordination failure:*

a) $\theta^{*}_{NW} \geq \theta^{*}_{IL} \geq \theta^{*}$ *and* $\theta^{**}_{NW} \geq \underline{\theta}^{**}_{IL} \geq \overline{\theta}^{**}_{IL} \geq \overline{\theta}^{**}$ *for all* γ.

b) If γ *increases,* $\overline{\theta}^{**}_{IL}$ *and* $\underline{\theta}^{**}_{IL}$ *increase monotonically. The relation between* θ^{*}_{IL} *and* γ *may be non-monotone.*

c) If c_2 *increases,* θ^{*}_{IL}, $\overline{\theta}^{**}_{IL}$ *and* $\underline{\theta}^{**}_{IL}$ *decrease.*

d) If λ *increases,* θ^{*}_{IL} *and* $\overline{\theta}^{**}_{IL}$ *decrease, while* $\underline{\theta}^{**}_{IL}$ *increases.*

Similar to the findings for the probability of early rollover, the fundamental thresholds in game Γ_{IL} lie between the thresholds from games Γ_{NW} and Γ (Result 4.2a)). Apart from that, the simulations confirm the findings from the previous chapter. The two thresholds $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ increase in γ because SCs consider the LC waiting a pessimistic signal about the fundamental state, hence fewer roll over than if she rolled over early (Result 4.2b)). This effect is even stronger if the LC is better informed (γ large) because SCs rely more on the LC's action as an indicator of the outcome of the project than on their private signal. Hence, for larger γ even fewer SCs roll over if the LC waits. Accordingly, the thresholds $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ and thus the probability of coordination failure increases in γ.

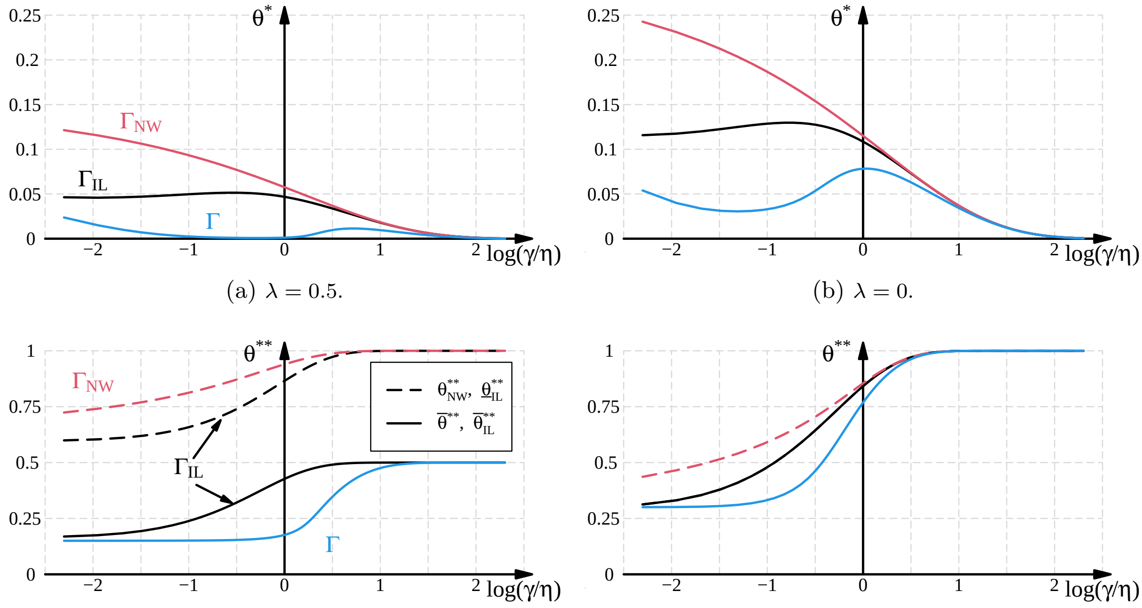
For θ^{*}_{IL} the relation may be non-monotone because for an intermediate range of γ the LC's incentive to roll over only in order to convince SCs to follow her outweighs her conviction that the fundamental state is actually that good. SCs internalize that the LC's decision to roll over early is less driven by the LC's expectation about the fundamental state and more about her influence on other SCs and, as a consequence, some of them are less convinced that she actually observed a good signal when she rolled over early. Therefore, they do not follow her and the probability of coordination failure increases (θ^{*}_{IL} increases) in γ.

Finally, the simulations suggest that the findings with regard to the effect of changes in c_2 and λ hold as well in the setting with imperfect learning (Results 4.2c) and 4.2d)). This is little surprising because payoff c_2 and size λ do not directly affect the formation of expectations and the direction of their effect on equilibrium thresholds is therefore not affected by the additional uncertainty.

Overall, the robustness of the qualitative results can be explained by the fact that the additional noise does not fundamentally change the LC's optimization problem and how she trades off learning and signaling incentives. Rather, if we interpret the game without the option to wait as a game without learning, the two extreme cases with perfect learning and without learning create a boundary for the players' behavior in the game with imperfect learning. Hence, it is logical that the results in this chapter confirm previous findings for the two extreme cases.

[62]Figures C.3 and C.4 in Appendix C.1 show results for $c_2 = 0.8$ and 0.6.

Figure 4.4: Effect of change in $\log(\gamma/\eta)$ on fundamental thresholds for $c_2 = 0.9999$.



(a) $\lambda = 0.5$. (b) $\lambda = 0$.

(c) $\lambda = 0.5$. (d) $\lambda = 0$.

Note: Panels (a) and (b): Black = θ^*_{IL}, blue = θ^* and red = θ^*_{NW}. Panels (c) and (d): Black = $\overline{\theta}^{**}_{IL}$, blue = θ^{**} and red = θ^{**}_{NW}.

4.5 Conclusion

In this chapter I extend the results from the previous chapter by lifting the assumption that the large creditor observes the small creditors' aggregate action perfectly, resulting in a game with imperfect learning. Thereby, I not only approach a more realistic interpretation of the problem described in the previous chapter, but I am also able to show that the two games with and without the option to wait are special cases of the game with imperfect learning. Accordingly, if we reinterpret the game without waiting as a game without learning, the three games can be arranged along the scale of the large creditor's ability to learn from small ones. Moreover, I show that the qualitative results from the previous chapter also hold if the large creditor is imperfectly informed and thereby confirm their robustness with regard to this assumption.

Even though the general results from my previous analysis still hold, several new insights into the interaction between the large creditor and small creditors can be gained: For example, the large creditor is more willing to roll over early if he observes small creditors with noise than if he observes them perfectly. The reason is that due to the increased uncertainty in her information in stage 3, she is more likely to choose "wrong" and e.g. roll over even though the project is likely to fail. This risk is absent in the game with perfect learning and makes the option to roll over early in order to convince small creditors to follow her relatively more attractive. Since the conditional probability of coordination failure is lowest if the large creditor rolls over early (Proposition 4.5), there are also implications for the probability of coordination failure and thus for a hypothetical policy-maker who might want to minimize this probability. E.g., by configuring transparency and disclosure obligations in such a way that a large creditor does not want to wait and learn might help to make a project success more likely.

The interaction between a policy-maker and creditors is not considered explicitly in this chapter as it would add an additional layer of strategic interaction and increase the complexity of the analysis. In particular because a policy-maker's choice could be considered as an additional public signal about the fundamental state. In this sense, the model presented in this chapter leaves space for extensions or re-interpretations, e.g. if one considers the large player as some type of policy-maker.

Moreover, as is well known, global games have been applied to other types of coordination problems such as speculative attacks. Therefore, it might also be interesting to look at the model from the perspective of currency speculators who have to decide to attack a currency peg or not. Giving a large speculator, the choice of moving early or late might help explain why the empirical results on the allegedly aggressive trading practices of large currency speculators during the 1997 Asian financial crisis have been mixed (Corsetti et al., 2002; Financial Stability Forum, 2000), because in this case it would depend on the quality of their information.

To conclude, by extending the results on the impact of a large player and her learning abilities on the coordination of small players, my analysis contributes to the literature on global coordination games and the role of large players and signaling in financial markets. More specifically, I show how signaling may encourage coordination between small players and thereby I complement the literature on catalytic finance and the impact of large or influential players such central banks, governments, international financial institutions, as well as credit rating agencies.

C.1 Appendix: Figures and tables

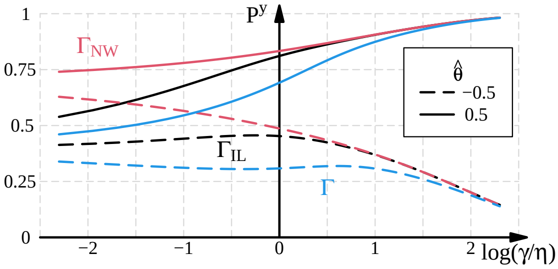
Table C.1: Equilibrium thresholds of Γ_{IL} and their interpretation.

Eqm. Threshold	**Interpretation**
y^*_{IL}	In stage 1, the LC rolls over early if $y > y^*_{IL}$ and waits if $y \leq y^*_{IL}$.
s^*_{IL}	In stage 3, the LC rolls over late if $s > s^*_{IL}$ and withdraws if $s \leq s^*_{IL}$.
x^*_{IL}	In stage 2, conditional on the LC rolling over early in stage 1, an SC i rolls over if $x_i > x^*_{IL}$ and withdraws if $x \leq x^*_{IL}$.
x^{**}_{IL}	In stage 2, conditional on the LC waiting in stage 1, an SC i rolls over if $x_i > x^{**}_{IL}$ and withdraws if $x \leq x^{**}_{IL}$.
θ^*_{IL}	Conditional on the LC rolling over early in stage 1, the investment project succeeds if $\theta > \theta^*_{IL}$ and fails if $\theta \leq \theta^*_{IL}$.
$\overline{\theta}^{**}_{IL}$	Conditional on the LC waiting in stage 1 and rolling over late in stage 3, the investment project succeeds if $\theta > \overline{\theta}^{**}_{IL}$ and fails if $\theta \leq \overline{\theta}^{**}_{IL}$.
$\underline{\theta}^{**}_{IL}$	Conditional on the LC waiting in stage 1 and withdrawing in stage 3, the investment project succeeds if $\theta > \underline{\theta}^{**}_{IL}$ and fails if $\theta \leq \underline{\theta}^{**}_{IL}$.

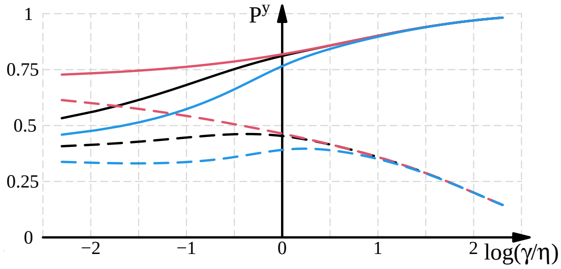
Table C.2: Parameter specification for numerical simulations.

Parameter	**Defined on**	**Values used in Simulation**	**Interpretation**
β	$(0,\infty)$	1	Precision of SC i's private signal x_i.
γ	$(0,\infty)$	$[0.1, 10]$	Precision of LC's private signal y.
η	$(0,\infty)$	1	Precision of LC's private signal z.
λ	$(0,1)$	$\{0, 0.5\}$	LC's size.
c_1	$(0,\infty)$	1	LC's and SCs' payoff for (early) roll over if project is successful.
c_2	$(0,\infty)$	$\{0.6, 0.8, 0.9999\}$	LC's payoff for late roll over if project is successful.
t	$(0,\infty)$	0.3	Costs of rolling over.
α	$(0,\infty)$	-	$\alpha \equiv \frac{\beta\gamma}{\beta+\gamma} = \frac{1}{Var(y\|x_i)}$, i.e., precision of y conditional on x_i.
ν	$(0,\infty)$	-	$\nu \equiv \frac{\gamma(\gamma+\beta\eta)}{\beta\eta} = \frac{1}{Var(s\|y)}$, i.e., precision of s conditional on y.
τ	$(0,\infty)$	-	$\tau \equiv \gamma + \beta\eta = \frac{1}{Var(s\|\theta)}$, i.e., precision of s conditional on θ.

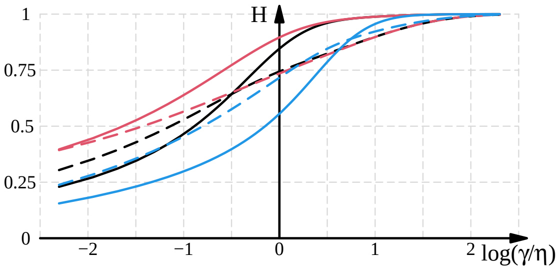
Figure C.1: Effect of change in $\log(\gamma/\eta)$ on probability of early rollover and herd-index for $c_2 = 0.8$.



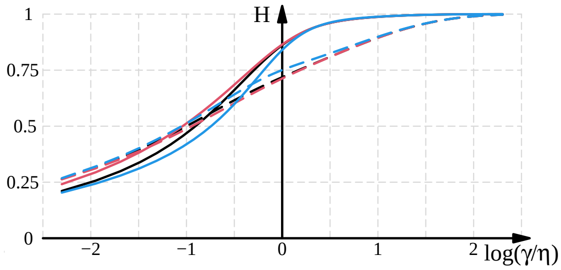
(a) $\lambda = 0.5$.



(b) $\lambda = 0$.



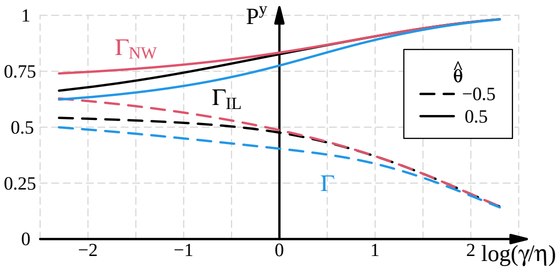
(c) $\lambda = 0.5$.



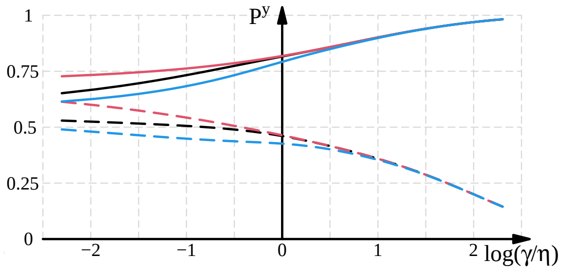
(d) $\lambda = 0$.

Note: Panels (a) and (b): Black = P^y_{IL}, blue = P^y and red = P^y_{NW}. Panels (c) and (d): Black = H_{IL}, blue = H and red = H_{NW}.

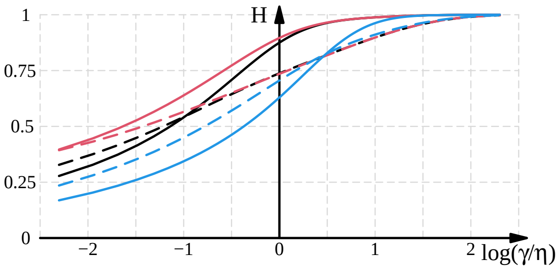
Figure C.2: Effect of change in $\log(\gamma/\eta)$ on probability of early rollover and herd-index for $c_2 = 0.6$.



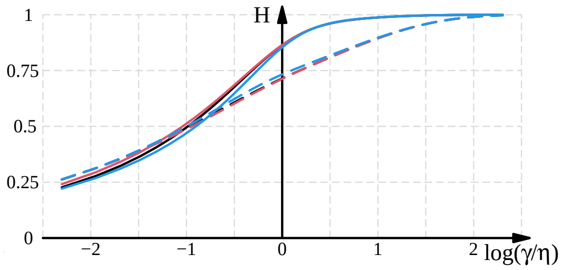
(a) $\lambda = 0.5$.



(b) $\lambda = 0$.



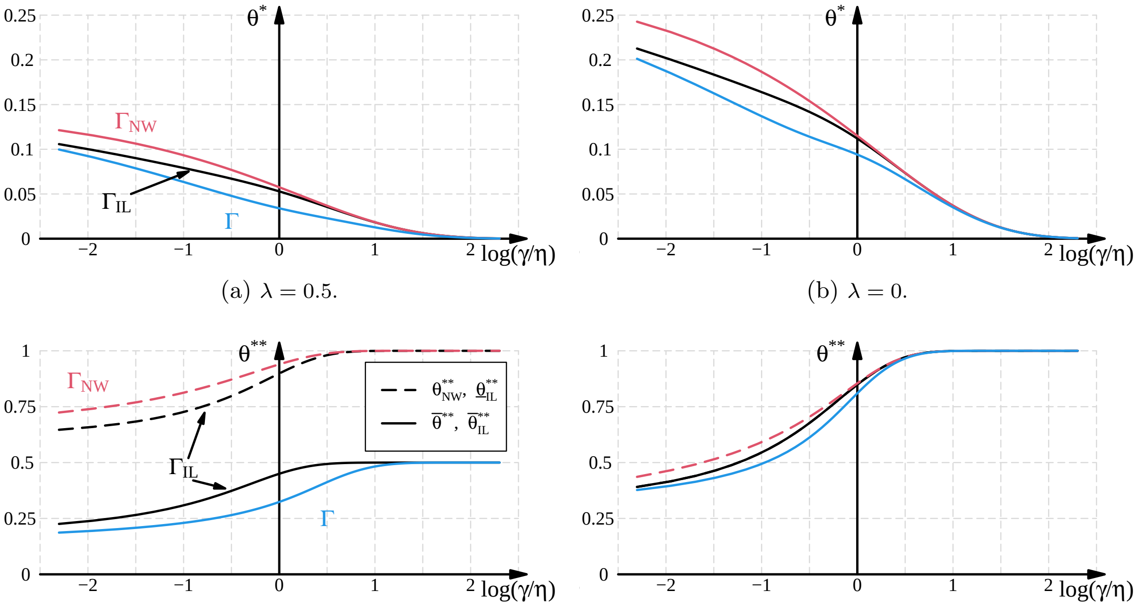
(c) $\lambda = 0.5$.



(d) $\lambda = 0$.

Note: Panels (a) and (b): Black = P^y_{IL}, blue = P^y and red = P^y_{NW}. Panels (c) and (d): Black = H_{IL}, blue = H and red = H_{NW}.

Figure C.3: Effect of change in $\log(\gamma/\eta)$ on fundamental thresholds for $c_2 = 0.8$.

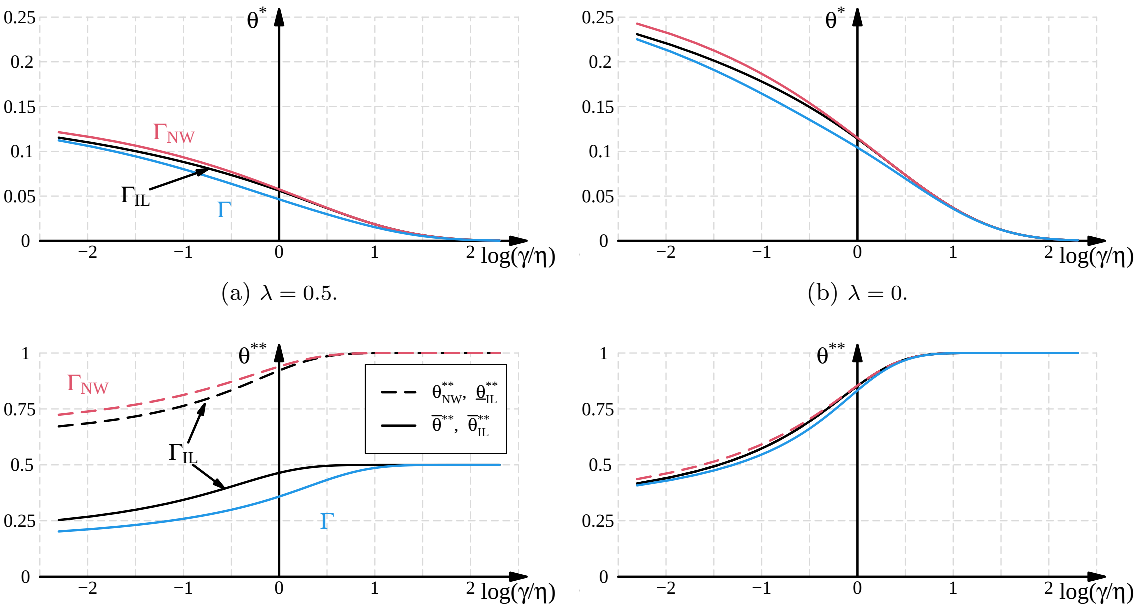


(a) $\lambda = 0.5$. (b) $\lambda = 0$.

(c) $\lambda = 0.5$. (d) $\lambda = 0$.

Note: Panels (a) and (b): Black = θ^*_{IL}, blue = θ^* and red = θ^*_{NW}. Panels (c) and (d): Black = $\overline{\theta}^{**}_{IL}$, blue = θ^{**} and red = θ^{**}_{NW}.

Figure C.4: Effect of change in $\log(\gamma/\eta)$ on fundamental thresholds for $c_2 = 0.6$.

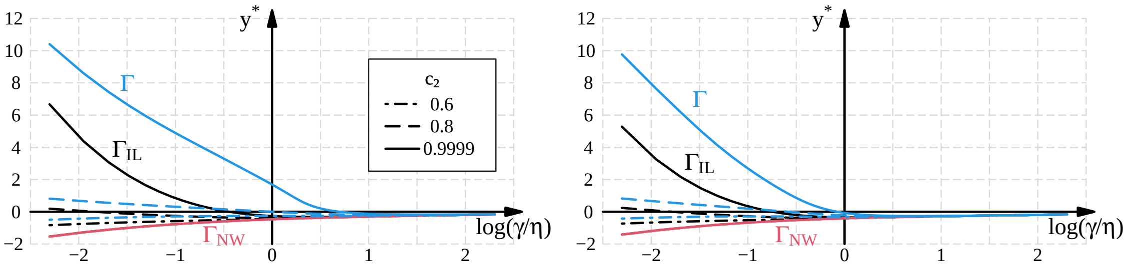


(a) $\lambda = 0.5$. (b) $\lambda = 0$.

(c) $\lambda = 0.5$. (d) $\lambda = 0$.

Note: Panels (a) and (b): Black = θ^*_{IL}, blue = θ^* and red = θ^*_{NW}. Panels (c) and (d): Black = $\overline{\theta}^{**}_{IL}$, blue = θ^{**} and red = θ^{**}_{NW}.

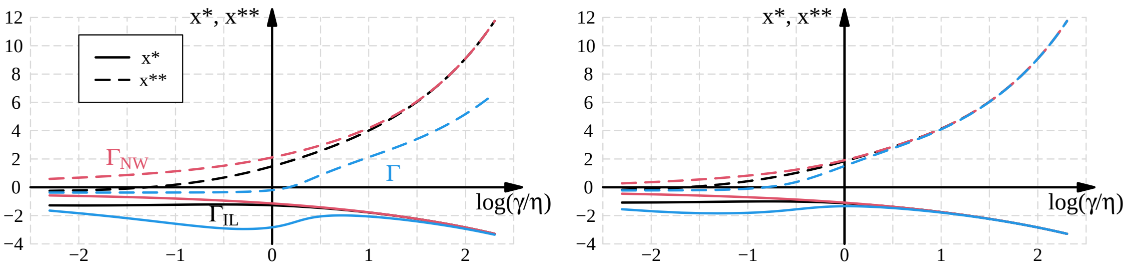
Figure C.5: Effect of change in $\log(\gamma/\eta)$ on signal the LC's private signal threshold.



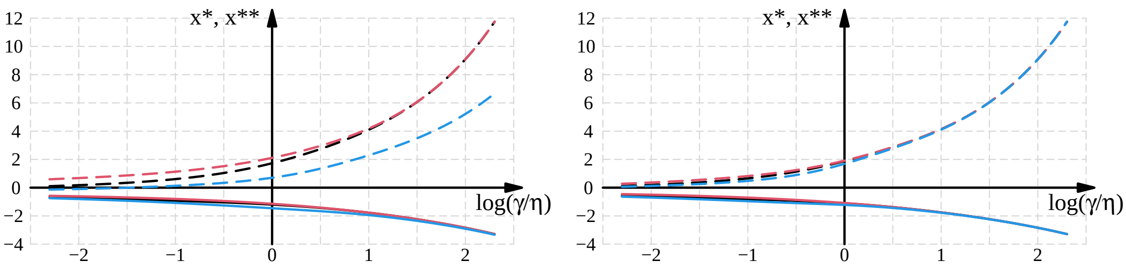
(a) $\lambda = 0.5$. (b) $\lambda = 0$.

Note: Black $= y^*_{IL}$, blue $= y^*$ and red $= y^*_{NW}$.

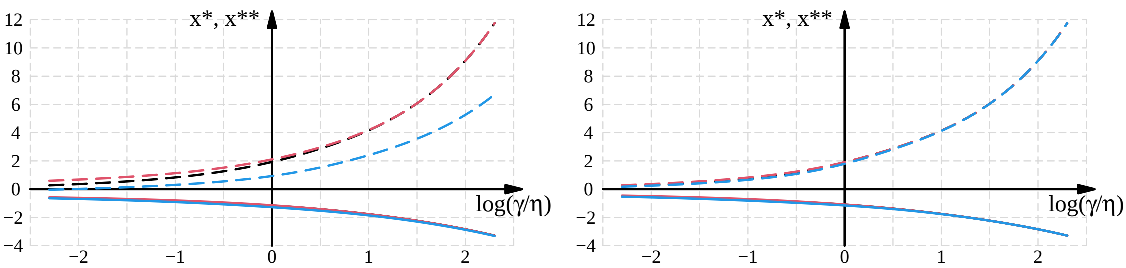
Figure C.6: Effect of change in $\log(\gamma/\eta)$ on the SCs' private signal thresholds.



(a) $\lambda = 0.5$, $c_2 = 0.9999$. (b) $\lambda = 0$, $c_2 = 0.9999$.



(c) $\lambda = 0.5$, $c_2 = 0.8$. (d) $\lambda = 0$, $c_2 = 0.8$.



(e) $\lambda = 0.5$, $c_2 = 0.6$. (f) $\lambda = 0$, $c_2 = 0.6$.

Note: Black $= x^*_{IL}$ or x^{**}_{IL}, blue $= x^*$ or x^{**} and red $= x^*_{NW}$ or x^{**}_{NW}.

C.2 Appendix: Derivation of equilibrium beliefs

For the derivation of equilibrium beliefs I use the same notation as used in the previous chapter. I.e., let $F(a \mid \tilde{b} \leq b, c) = Pr(\tilde{a} \leq a \mid \tilde{b} \leq b, \tilde{c} = c)$ for some random variables $\tilde{a}, \tilde{b}, \tilde{c}$. Let f denote the corresponding probability density/mass function. From the previous chapter (see section B.2 in the appendix of Chapter 3) we know the following conditional distributions, which are also needed in game Γ_{IL} to parameterize the probabilities in the indifference equations:

$$F(\theta \mid y) = \Phi\left(\sqrt{\gamma}\,(\theta - y)\right) \tag{4.34}$$

$$F(\theta \mid \tilde{y} > y, x) = \frac{\Phi\left(\sqrt{\beta}\,(\theta - x)\right) - \Phi_2\left(\sqrt{\beta}\,(\theta - x)\,, \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y - x)\,; \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{1 - \Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y - x)\right)} \tag{4.35}$$

$$= \frac{\Phi_2\left(\sqrt{\beta}\,(\theta - x)\,, \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x - y)\,; -\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x - y)\right)} \tag{4.36}$$

$$F(\theta \mid \tilde{y} \leq y, x) = \frac{\Phi_2\left(\sqrt{\beta}\,(\theta - x)\,, \sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y - x)\,; \sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(y - x)\right)} \tag{4.37}$$

$$F(\theta \mid x) = \Phi\left(\sqrt{\beta}\,(\theta - x)\right) \tag{4.38}$$

$$F(\theta, y \mid x) = \Phi_2\left(\sqrt{\beta}\,(\theta - x)\,, \sqrt{\frac{\beta\gamma}{\beta + \gamma}}\,(y - x)\,; \sqrt{\frac{\gamma}{\beta + \gamma}}\right) \tag{4.39}$$

$$F(y \mid x) = \Phi\left(\sqrt{\frac{\beta\gamma}{\beta + \gamma}}\,(y - x)\right) \tag{4.40}$$

However, distributions that include s must be derived anew. In particular, the equilibrium conditions require that we need the following:

$$F(s \mid y) \tag{4.41}$$

$$F(\theta, s \mid y) \tag{4.42}$$

$$F(\theta, s, y \mid x)\,. \tag{4.43}$$

Regarding $F(s \mid y)$, recall that

$$s = \frac{\gamma y + \eta\sqrt{\beta} z + \eta\beta x_{IL}^{**}}{\gamma + \eta\beta}, \tag{4.44}$$

where $z = \sqrt{\beta}(\theta - x_{IL}^{**}) + \varepsilon_z$ with $\varepsilon_z \sim N(0, \eta^{-1})$. Hence,

$$s = \frac{\gamma}{\gamma + \eta\beta} y + \frac{\eta\beta}{\gamma + \eta\beta}\theta + \frac{\eta\sqrt{\beta}}{\gamma + \eta\beta}\varepsilon_z. \tag{4.45}$$

Conditional on y, s is a linear combination of the two independent normal random variables θ and ε_z and is therefore also normal:

$$s \mid y \sim N\left(y, \frac{\eta\beta}{\gamma\,(\gamma + \eta\beta)}\right) \tag{4.46}$$

In order to derive $F(\theta, s \mid y)$, recognize that

$$f(\theta, s \mid y) = f(s \mid \theta, y)\, f(\theta \mid y)\,. \tag{4.47}$$

Conditional on θ and y, s is a linear transformation of the normal random variable $z \mid \theta \sim N(\theta, \eta^{-1})$, which is independent from y. Hence, $s \mid \theta, y$ is also normal with expectation

$$E\left[s \mid \theta, y\right] = \frac{\gamma y + \eta\beta\theta}{\gamma + \eta\beta} \tag{4.48}$$

and variance

$$Var\left(s \mid \theta, y\right) = E\left[s^2 \mid \theta, y\right] - E\left[s \mid \theta, y\right]^2 = \frac{\eta\beta}{\left(\gamma + \eta\beta\right)^2}. \tag{4.49}$$

For $f(\theta \mid y)$ we already know that $\theta \mid y \sim N(y, \gamma^{-1})$. Hence,

$$f\left(\theta, s \mid y\right) = \frac{1}{2\pi}\frac{\left(\gamma+\beta\eta\right)}{\sqrt{\eta\beta}}\sqrt{\gamma}\exp\left(-\frac{1}{2}\left(\frac{\left(\gamma+\beta\eta\right)^2}{\eta\beta}\left(s - \frac{\gamma y + \eta\beta\theta}{\gamma+\eta\beta}\right)^2 + \gamma\left(\theta - y\right)^2\right)\right) \tag{4.50}$$

$$= \frac{\sqrt{\gamma}\left(\gamma+\beta\eta\right)}{2\pi\sqrt{\eta\beta}}\exp\left(-\frac{1}{2}\frac{\gamma+\beta\eta}{\gamma}\left(\gamma\left(\theta-y\right)^2 + \frac{\gamma\left(\gamma+\eta\beta\right)}{\beta\eta}\left(s-y\right)^2 - 2\gamma\left(\theta-y\right)\left(s-y\right)\right)\right) \tag{4.51}$$

which corresponds to a bivariate normal distribution of with means y and y, variances γ^{-1} and $\eta\beta/(\gamma(\gamma+\eta\beta))$ and correlation $\sqrt{\eta\beta/(\eta\beta+\gamma)}$.

Finally, in order to find $F(\theta, s, y \mid x)$, notice that

$$f\left(\theta, s, y \mid x\right) = f\left(\theta \mid s, x\right) f\left(s \mid y, x\right) f\left(y \mid x\right) \tag{4.52}$$

because s is a sufficient statistic for θ given y. We know that

$$y \mid x \sim N\left(x, \frac{\beta+\gamma}{\beta\gamma}\right). \tag{4.53}$$

Conditional on y and x, s is a linear transformation of the normal random variable z, hence it is also normal and we simply have to find its expected value and variance:

$$s \mid x, y \sim N\left(\frac{\gamma\left(\beta+\gamma+\eta\beta\right)y + \eta\beta^2 x}{\left(\gamma\eta+\gamma\right)\left(\beta+\gamma\right)}, \frac{\eta\beta\left(\beta+\gamma+\eta\beta\right)}{\left(\gamma+\beta\eta\right)^2\left(\beta+\gamma\right)}\right). \tag{4.54}$$

Moreover, conditional on s and x, θ is normal because x and y are (transformations of) independent normal signals about θ. It follows that

$$\theta \mid s, x \sim N\left(\frac{\beta x + \left(\gamma+\beta\eta\right)s}{\beta+\gamma+\beta\eta}, \frac{1}{\beta+\gamma+\beta\eta}\right). \tag{4.55}$$

Hence, we can write

$$f\left(\theta, s, y \mid x\right) = \frac{\sqrt{\gamma}\left(\gamma+\beta\eta\right)}{2\pi\sqrt{2\pi\eta}}\exp\left(-\frac{1}{2}\left[\frac{\beta\gamma}{\beta+\gamma}\left(y-x\right)^2\right.\right.$$
$$\left.\left.+\frac{\left(\gamma+\beta\eta\right)^2\left(\beta+\gamma\right)}{\eta\beta\left(\beta+\gamma+\eta\beta\right)}\left(s - \frac{\gamma\left(\beta+\gamma+\eta\beta\right)y+\eta\beta^2 x}{\left(\gamma\eta+\gamma\right)\left(\beta+\gamma\right)}\right)^2 + \left(\beta+\gamma+\beta\eta\right)\left(\theta - \frac{\beta x + \left(\gamma+\beta\eta\right)s}{\beta+\gamma+\beta\eta}\right)^2\right]\right) \tag{4.56}$$

$$= \frac{1}{\sqrt{\left(2\pi\right)^3\left|\Sigma\right|}}\exp\left(-\frac{1}{2}\left(\mathbf{m}-\mathbf{x}\right)^\top \Sigma^{-1}\left(\mathbf{m}-\mathbf{x}\right)\right), \tag{4.57}$$

where

$$\mathbf{m} = \begin{bmatrix} \theta \\ s \\ y \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x \\ x \\ x \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \beta^{-1} & \beta^{-1} & \beta^{-1} \\ \beta^{-1} & \frac{\beta+\gamma+\beta\eta}{\beta(\gamma+\beta\eta)} & \frac{\beta+\gamma+\beta\eta}{\beta(\gamma+\beta\eta)} \\ \beta^{-1} & \frac{\beta+\gamma+\beta\eta}{\beta(\gamma+\beta\eta)} & \frac{\beta+\gamma}{\beta\gamma} \end{bmatrix}. \tag{4.58}$$

That is, $(\theta, s, y \mid x)$ is trivariate normal with mean vector $\mathbf{x}$ and variance-covariance matrix Σ.

As before, in order to derive probabilities, one simply needs to integrate over the relevant intervals.

C.3 Appendix: Proofs

For the sake of proofs, I restate the indifference and critical mass conditions:

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \theta^*_{IL}\right)\right) = c_2\Phi_2\left(\sqrt{\gamma}\left(y^*_{IL} - \overline{\theta}^{**}_{IL}\right), \sqrt{\nu}\left(y^*_{IL} - s^*_{IL}\right); \sqrt{\frac{\gamma}{\nu}}\right) + t\Phi\left(\sqrt{\nu}\left(s^*_{IL} - y^*_{IL}\right)\right), \tag{4.59}$$

$$c_2\Phi\left(\sqrt{\tau}\left(s^*_{IL} - \overline{\theta}^{**}_{IL}\right)\right) - t = 0, \tag{4.60}$$

$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x^*_{IL} - \theta^*_{IL}\right), \sqrt{\alpha}\left(x^*_{IL} - y^*_{IL}\right); \sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(x^*_{IL} - y^*_{IL}\right)\right)} - t = 0, \tag{4.61}$$

$$c_1\left[\frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL} - \underline{\theta}^{**}_{IL}\right), \sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right); -\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right)\right)} + \frac{\Psi_3\left(\underline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right) - \Psi_3\left(\overline{\theta}^{**}_{IL}, -s^*_{IL}, y^*_{IL}; \mathbf{x}^{**}_{IL}, \Sigma\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL} - x^{**}_{IL}\right)\right)}\right] - t = 0, \tag{4.62}$$

$$\theta^*_{IL} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\theta^*_{IL} - x^*_{IL}\right)\right), \tag{4.63}$$

$$\overline{\theta}^{**}_{IL} = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\overline{\theta}^{**}_{IL} - x^{**}_{IL}\right)\right), \tag{4.64}$$

$$\underline{\theta}^{**}_{IL} = 1 - (1-\lambda)\Phi\left(\sqrt{\beta}\left(\underline{\theta}^{**}_{IL} - x^{**}_{IL}\right)\right), \tag{4.65}$$

Proof of Proposition 4.1:

Proof. If $\eta \to 0$ while $\gamma > 0$, the private signal z becomes completely uninformative. Hence, $\nu \to \infty$ and $\tau \to \gamma$. Solving (4.60) for s^*_{IL} gives

$$s^*_{IL} = \overline{\theta}^{**}_{IL} + \frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{t}{c_2}\right). \tag{4.66}$$

Moreover, notice that because $\nu \to \infty$, we can write (4.59) as

$$\begin{aligned} & c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \theta^*_{IL}\right)\right) \\ & = c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \overline{\theta}^{**}_{IL}\right)\right)\Phi\left(\sqrt{\nu}\left(y^*_{IL} - s^*_{IL}\right)\right) + t\left(1 - \Phi\left(\sqrt{\nu}\left(y^*_{IL} - s^*_{IL}\right)\right)\right) && (4.67) \\ \Leftrightarrow \quad & c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \theta^*_{IL}\right)\right) - t \\ & = \Phi\left(\sqrt{\nu}\left(y^*_{IL} - \overline{\theta}^{**}_{IL} - \frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{t}{c_2}\right)\right)\right)\left[c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \overline{\theta}^{**}_{IL}\right)\right) - t\right] && (4.68) \end{aligned}$$

where the RHS is equal to 0 if $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t\leq 0$ and equal to $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t$ if $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t>0$. Suppose that $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t>0$ so that the indifference condition is

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\theta^*_{IL}\right)\right)=c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right). \tag{4.69}$$

Proposition 4.5 states that $\overline{\theta}^{**}_{IL}\geq\theta^*_{IL}$ so that with $c_2<c_1$ there is no solution y^*_{IL} to the indifference equation because the LHS is always greater than the RHS, i.e. $y^*_{IL}\to-\infty$ (the LC always rolls over early). This in turn contradicts the initial assumption $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t>0$, so that only $c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)-t\leq 0$ can hold and the indifference equation is

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\theta^*_{IL}\right)\right)-t=0 \quad\Leftrightarrow\quad y^*_{IL}=\theta^*_{IL}+\frac{1}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{t}{c_1}\right). \tag{4.70}$$

Equation (4.62) also adjusts because the probability of a late rollover by the LC converges to zero due to the LC not learning from private signal z. Therefore, the indifference condition is

$$c_1Pr\left(\theta>\underline{\theta}^{**}_{IL}\mid x^{**}_{IL},y>y^{**}_{IL}\right)-t=0 \tag{4.71}$$

$$\Leftrightarrow\quad c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL}-\underline{\theta}^{**}_{IL}\right),\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right);-\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right)\right)}-t=0. \tag{4.72}$$

The remaining indifference conditions do not change, and hence we are left the system of indifference equations which corresponds to game Γ_{NW}. □

Proof of Proposition 4.2:

Proof. As $\eta\to\infty$ and $\gamma<\infty$, $\tau\to\infty$ and $\nu\to\gamma$. It follows that $s\to\theta$ and from equation (4.60) that $s^*_{IL}\to\overline{\theta}^{**}_{IL}$. Hence, equation (4.59) collapses to

$$c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\theta^*_{IL}\right)\right)=c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL}-\overline{\theta}^{**}_{IL}\right)\right)+t\Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**}_{IL}-y^*_{IL}\right)\right). \tag{4.73}$$

Moreover, equation (4.62) collapses to

$$c_1\frac{\Phi_2\left(\sqrt{\beta}\left(x^{**}_{IL}-\overline{\theta}^{**}_{IL}\right),\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right);-\sqrt{\frac{\alpha}{\beta}}\right)}{\Phi\left(\sqrt{\alpha}\left(y^*_{IL}-x^{**}_{IL}\right)\right)}-t=0 \tag{4.74}$$

because one can show that as $s\to\theta$ and $s^*_{IL}\to\overline{\theta}^{**}_{IL}$. We can rewrite the indifference equation as

$$c_1\left[Pr\left(\theta>\underline{\theta}^{**}_{IL}\mid y\leq y^*_{IL},x^{**}_{IL}\right)+Pr\left(\underline{\theta}^{**}_{IL}\geq\theta>\overline{\theta}^{**}_{IL}\mid y\leq y^*_{IL},x^{**}_{IL}\right)\right]-t=0 \tag{4.75}$$

$$\Leftrightarrow\quad c_1Pr\left(\theta\geq\overline{\theta}^{**}_{IL}\mid y\leq y^*_{IL},x^{**}_{IL}\right)-t=0 \tag{4.76}$$

which is parameterized by equation (4.74). Hence, Γ_{IL} collapses to game Γ. □

Proof of Proposition 4.3:

Proof. For $\gamma\to 0$ and $\eta>0$ it is straightforward to show that equation (4.60) simplifies to

$$s^*_{IL}=\overline{\theta}^{**}_{IL}+\frac{1}{\sqrt{\beta\eta}}\Phi^{-1}\left(\frac{t}{c_2}\right). \tag{4.77}$$

Moreover, since the LC's action in stage 1 becomes uninformative with regards to θ, SCs do not rely on it as a signal and the indifference equation (4.61) simplifies to

$$x^*_{IL} = \theta^*_{IL} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}}. \tag{4.78}$$

Plugging this expression into equation (4.63) gives

$$\theta^*_{IL} = (1-\lambda)\,\frac{t}{c_1} \tag{4.79}$$

which results in

$$x^*_{IL} = (1-\lambda)\,\frac{t}{c_1} + \frac{\Phi^{-1}\left(\frac{t}{c_1}\right)}{\sqrt{\beta}}. \tag{4.80}$$

Moreover, since $\gamma \to 0 \quad \Rightarrow \quad \alpha \to 0$, the LC's action is also not informative if she waits. Hence, y drops out of the SCs' indifference equation (4.62), so that we are left with

$$c_1\left[Pr\left(\theta > \underline{\theta}^{**}_{IL} \mid x^{**}_{IL}\right) + Pr\left(\underline{\theta}^{**}_{IL} \geq \theta > \overline{\theta}^{**}_{IL}, s > s^*_{IL} \mid x^{**}_{IL}\right)\right] - t = 0 \tag{4.81}$$

$$\begin{aligned}\Leftrightarrow \quad c_1\Bigg[&\Phi\left(\sqrt{\beta}\left(x^{**}_{IL} - \underline{\theta}^{**}_{IL}\right)\right) + \Phi_2\left(\sqrt{\beta}\left(\underline{\theta}^{**}_{IL} - x^{**}_{IL}\right), -\sqrt{\frac{\eta}{1+\eta}}s^*_{IL}; -\sqrt{\frac{\eta}{1+\eta}}\right) - \\ &\Phi_2\left(\sqrt{\beta}\left(\overline{\theta}^{**}_{IL} - x^{**}_{IL}\right), -\sqrt{\frac{\eta}{1+\eta}}s^*_{IL}; -\sqrt{\frac{\eta}{1+\eta}}\right)\Bigg] - t = 0.\end{aligned} \tag{4.82}$$

As for equations (4.64) and (4.65), they remain the same. Plugging the expression for θ^*_{IL} into equation (4.59) simplifies to

$$\begin{aligned}&c_1\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - (1-\lambda)\,\frac{t}{c_1}\right)\right) \\ &= c_2\Phi\left(\sqrt{\gamma}\left(y^*_{IL} - \overline{\theta}^{**}_{IL}\right)\right) + t\Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**}_{IL} + \frac{1}{\sqrt{\eta\beta}}\Phi^{-1}\left(\frac{t}{c_2}\right) - y^*_{IL}\right)\right)\end{aligned} \tag{4.83}$$

Now, as $\gamma \to 0$, the LC's choice becomes a coin toss, hence, she assigns equal probability to success or failure of the project, so that the three probability expressions in the above equation converge to 0.5. It follows that she always wants to roll over ($y^*_{IL} \to -\infty$) if $c_1 > c_2 + t$ and she always wants to wait ($y^*_{IL} \to \infty$) if $c_1 < c_2 + t$. Only if $c_1 = c_2 + t$, her decision is less straightforward as she wants to roll over whenever $y > \hat{y}^*$, where $\hat{y}^*$ solves

$$\begin{aligned}&c_1\Phi\left(\sqrt{\gamma}\left(\hat{y}^* - (1-\lambda)\,\frac{t}{c_1}\right)\right) \\ &= c_2\Phi\left(\sqrt{\gamma}\left(\hat{y}^* - \overline{\theta}^{**}_{IL}\right)\right) + t\Phi\left(\sqrt{\gamma}\left(\overline{\theta}^{**}_{IL} + \frac{1}{\sqrt{\eta\beta}}\Phi^{-1}\left(\frac{t}{c_2}\right) - \hat{y}^*\right)\right).\end{aligned} \tag{4.84}$$

□

Proof of Proposition 4.4:

Proof. As $\gamma \to \infty$, the LC's initial signal becomes infinitely precise and in effect she perfectly knows the realization of θ. Hence, the LC always chooses to roll over early if $y > y^*_{IL} = \theta^*_{IL}$ and wait otherwise. However, since the second signal z does not add any information, she will never roll over late so that $s^*_{IL} \to \infty$. Moreover, SCs know that the LC perfectly observes θ and therefore always follow her action: All SCs roll over if the LC rolls over and they withdraw if she waits.

They completely ignore their private signals so that $x^*_{IL} \to -\infty$ and $x^{**}_{IL} \to \infty$. In turn, since every creditor rolls over if the LC rolls over and the LC rolls over early if $\theta > \theta^*_{IL}$, the threshold θ^*_{IL} and therefore y^*_{IL} converge to 0 because the LC rolls over for any fundamental state that could support a project success $\theta > 0$. On the contrary, if the LC waits, no creditor rolls over and the project will never be successful. Hence, $\underline{\theta}^{**}_{IL} \to 1$. Finally, for the theoretical case that the LC decides to roll over late, $\overline{\theta}^{**}_{IL} \to 1 - \lambda$ because all SCs will have withdrawn. □

Proof of Proposition 4.5:

Proof. The proof essentially follows the analog proof of Proposition 3.3 in the appendix B.3 in the previous chapter 3. First, define

$$h(\theta, x) = 1 - (1 - \lambda)\,\Phi\left(\sqrt{\beta}\,(\theta - x)\right). \tag{4.85}$$

Then the critical mass conditions (4.63), (4.64) and (4.65) can be written as

$$\theta^*_{IL} = h(\theta^*_{IL}, x^*_{IL}) - \lambda \tag{4.86}$$
$$\overline{\theta}^{**}_{IL} = h\left(\overline{\theta}^{**}_{IL}, x^{**}_{IL}\right) - \lambda \tag{4.87}$$
$$\underline{\theta}^{**}_{IL} = h(\underline{\theta}^{**}_{IL}, x^{**}_{IL}). \tag{4.88}$$

For any x, h is strictly monotonously decreasing in θ. In a diagram with θ on the x-axis, the thresholds θ^*_{IL}, $\overline{\theta}^{**}_{IL}$ and $\underline{\theta}^{**}_{IL}$ are given by the intersection of the (shifted) function h with the 45-degree line. Hence, it must hold that $\overline{\theta}^{**} \leq \underline{\theta}^{**}$ for any $\lambda \geq 0$. More specifically, for any $\lambda \geq 0$, the RHS of (4.87) is equal to the RHS of (4.88) shifted downward by λ. Accordingly, the intesection with the 45-degree line must lie to the left (or below) the intersection of (non-shifted) h with the line. Hence, $\overline{\theta}^{**} \leq \underline{\theta}^{**}$.

Moreover, notice that (4.86) and (4.87) define θ^*_{IL} and $\overline{\theta}^{**}_{IL}$ as implicit functions of x. In fact, the function is the same in both cases, it is just evaluated at different points of x, namely x^*_{IL} and x^{**}_{IL}. Hence, I write $\theta^*_{IL} = \tilde{\theta}(x^*_{IL})$ and $\overline{\theta}^{**}_{IL} = \tilde{\theta}(x^{**}_{IL})$, where $\tilde{\theta}$ denotes the implicit function. From the Implicit Function Theorem we know that

$$\frac{\partial \tilde{\theta}}{\partial x}(x) = \frac{\sqrt{\beta}\phi\left(\sqrt{\beta}\left(\tilde{\theta} - x\right)\right)}{\sqrt{\beta}\phi\left(\sqrt{\beta}\left(\tilde{\theta} - x\right)\right) + \frac{1}{1+\lambda}} > 0, \tag{4.89}$$

so that $\theta^*_{IL} \leq \overline{\theta}^{**}_{IL}$ iff $x^*_{IL} \leq x^{**}_{IL}$.

The proof that $x^*_{IL} \leq x^{**}_{IL}$ follows the same idea as the proof for Proposition 3.3 in appendix B.3 in the previous chapter. That is, $x^*_{IL} \leq x^{**}_{IL}$ holds if

a) x^*_{IL} is the unique solution to $Pr\left(\theta > \tilde{\theta}(x^*_{IL}) \mid y > y^*_{IL}, x^*_{IL}\right) = t/c_1$,

b) $Pr\left(\theta > \tilde{\theta}(x) \mid y > y^*_{IL}, x\right)$ as a function of x crosses t/c_1 from below and

c)

$$\begin{aligned} Pr\left(\theta > \overline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) &\geq Pr\left(\theta > \underline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) \\ &+ Pr\left(\underline{\theta}^{**}_{IL}(x) \geq \theta > \overline{\theta}^{**}_{IL}(x), s > s^*_{IL} \mid y \leq y^*_{IL}, x\right). \end{aligned} \tag{4.90}$$

The idea is that if the three statements are true, then the unique intersection of $Pr\left(\theta > \tilde{\theta}(x) \mid y > y^*_{IL}, x\right)$ with t/c_1 always lies to the left of any intersection of $Pr\left(\theta > \tilde{\theta}(x) \mid y \leq y^*_{IL}, x\right)$ with t/c_1, so that $x^*_{IL} \leq x^{**}_{IL}$.

As in the previous chapter, I could not show analytically that a) holds, however, numerical simulations suggest that it is true. Hence, for this proof I assume that it is true. Condition b) is shown to hold in the proof of Proposition 3.3 in the previous chapter, which is why I omit it here. For condition c), we can write

$$\begin{aligned} & Pr\left(\theta > \overline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) \geq Pr\left(\theta > \underline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) \\ & + Pr\left(\underline{\theta}^{**}_{IL}(x) \geq \theta > \overline{\theta}^{**}_{IL}(x), s > s^*_{IL} \mid y \leq y^*_{IL}, x\right) \qquad (4.91) \\ \Leftrightarrow \quad & Pr\left(\theta > \overline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) - Pr\left(\theta > \underline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) \\ & \geq Pr\left(\underline{\theta}^{**}_{IL}(x) \geq \theta > \overline{\theta}^{**}_{IL}(x), s > s^*_{IL} \mid y \leq y^*_{IL}, x\right) \qquad (4.92) \\ \Leftrightarrow \quad & Pr\left(\underline{\theta}^{**}_{IL}(x) \geq \theta > \overline{\theta}^{**}_{IL}(x) \mid y \leq y^*_{IL}, x\right) \geq Pr\left(\underline{\theta}^{**}_{IL}(x) \geq \theta > \overline{\theta}^{**}_{IL}(x), s > s^*_{IL} \mid y \leq y^*_{IL}, x\right) \\ & \qquad (4.93) \end{aligned}$$

where the last inequality is obviously true. Hence, since b) and c) hold and a) is assumed to hold, $x^*_{IL} \leq x^{**}_{IL}$ holds as well. □

Proof of Corollary 4.1:

Proof. Recall that from Proposition 3.3 that for $\gamma \to 0$, x^*_{IL} converges to

$$(1-\lambda)\frac{t}{c_1} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{t}{c_1}\right) \qquad (4.94)$$

and x^{**}_{IL} converges to the solution of

$$\begin{aligned} c_1 \Bigg[& \Phi\left(\sqrt{\beta}\left(x^{**}_{IL} - \underline{\theta}^{**}_{IL}\right)\right) + \Phi_2\left(\sqrt{\beta}\left(\underline{\theta}^{**}_{IL} - x^{**}_{IL}\right), -\sqrt{\frac{\eta}{1+\eta}} s^*_{IL}; -\sqrt{\frac{\eta}{1+\eta}}\right) - \\ & \Phi_2\left(\sqrt{\beta}\left(\overline{\theta}^{**}_{IL} - x^{**}_{IL}\right), -\sqrt{\frac{\eta}{1+\eta}} s^*_{IL}; -\sqrt{\frac{\eta}{1+\eta}}\right)\Bigg] - t = 0. \qquad (4.95) \end{aligned}$$

The herd index is defined by

$$H_{IL} \equiv Pr(x > x^*_{IL} \mid \hat{\theta}) - Pr(x > x^{**}_{IL} \mid \hat{\theta}) = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^*_{IL}\right)\right) - \Phi\left(\sqrt{\beta}\left(\hat{\theta} - x^{**}_{IL}\right)\right), \qquad (4.96)$$

hence, for $\gamma \to 0$ it converges to

$$\hat{H}_{IL} = \Phi\left(\sqrt{\beta}\left(\hat{\theta} - (1-\lambda)\frac{t}{c_1} + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{t}{c_1}\right)\right)\right) - \Phi\left(\sqrt{\beta}\left(\hat{\theta} - \hat{x}^{**}_{IL}\right)\right), \qquad (4.97)$$

where $\hat{x}^{**}_{IL}$ is the solution to (4.95). Moreover, in the proof for Proposition 4.5 I show that $x^*_{IL} \leq x^{**}_{IL}$ under the assumption that x^*_{IL} is unique, hence it must hold that $\hat{H}_{IL} \geq 0$ as well.

For b), recall that if $\gamma \to \infty$, $x^*_{IL} \to -\infty$ and $x^{**}_{IL} \to \infty$ so that $H_{IL} \to 1$. □

C.4 Appendix: Numerical stability and equilibrium uniqueness

Numerical solution: For the comparative statics results, the R-command `nleqslv` from the package of the same name is used (Version 3.3.4) (Hasselman, 2023). In general, I used the following set of arguments:

- `method="Newton"`
- `global="dbldog"`
- `xscalm="fixed"`
- `allowSingular=FALSE`
- `xtol=1e-14`

CDF of multivariate normal distribution: To solve the CDF of the bi-variate normal distribution, I use the Genz-Bretz-algorithm as implemented in the R-command `pmvnorm` of the R-package `mvtnorm` (Version 1.2-4) (Genz and Bretz, 2009).

Algorithm to determine equilibrium uniqueness: If there are multiple equilibria, the Newton-algorithm should converge to any of them if the initial conditions are sufficiently close. Therefore, I compute the solution to the equation system for a set of different initial conditions in the set of possible solutions, for a subset of the parameter space. In R, the package `BB` (Version 2019.10-1) (Varadhan and Gilbert, 2009) offers the command `multiStart` to check for multiple solutions in this manner. Essentially, I employ the same algorithm as in chapter 3 adjusted for the additional parameter and equilibrium variables:

Algorithm:

A. Define a parameter-vector Ω with values $(\beta, \gamma, \eta, \lambda, c_1, c_2, t)$ and a matrix M with 6 columns and N rows.

B. Get a set of different initial values and save them in M: For each n in N repeat:

   1. Draw randomly y^*_{IL} from $[-3, 9]$ and save in column 1, row n of matrix M.
   2. Draw randomly x^*_{IL} from $[-5, 1]$ and save in column 2, row n of matrix M.
   3. Draw randomly x^{**}_{IL} from $[-4.5, 16.5]$ and save in column 3, row n of matrix M.
   4. Draw randomly θ^*_{IL} from $(0, 1)$ and save in column 4, row n of matrix M.
   5. Draw randomly $\overline{\theta}^{**}_{IL}$ from $(0, 1)$ and save in column 5, row n of matrix M.
   6. Draw randomly $\underline{\theta}^{**}_{IL}$ from $(0, 1)$ and save in column 6, row n of matrix M.

C. Use a numerical algorithm (see documentation of `multiStart` (Varadhan and Gilbert, 2009)) to solve the model N times given Ω and using a new row of M as initial values in each iteration. For each iteration, document whether the algorithm converged or not and store the solution, if one is found.

The idea of determining the set of initial values in steps B.1. to B.5. is as follows: From the results in this chapter (see Figures C.5 and C.6) one can see, that for the given sets of parameters the solutions y^*_{IL}, x^*_{IL} and x^{**}_{IL} usually lie inside the intervals $[-1, 7]$, $[-4, 0]$ and $[-1, 13]$ respectively. In order to cover cases where additional equilibria may lie outside of these intervals, I extend both of their borders by 25% of the range of the interval and draw initial values for y^*_{IL}, x^*_{IL} and x^{**}_{IL} from these extended intervals. Of course, I cannot exclude that solutions lie outside of these intervals as well. However, in order to have the numerical algorithm converge in a sufficient number of cases, extending the interval would require to increase the number of initial values drawn, too, which would in turn overreach the limits of computational capacity available. Hence, to keep a balance, I limit my analysis to these intervals. With regards to the fundamental thresholds, they are naturally bounded between 0 and 1. Therefore, I draw initial values from these intervals. Given Ω, the numerical algorithm then tries to find a solution for the system of equations for each set of initial values. The outcomes of these attempts are stored and evaluated with regard to multiplicity of solutions. Using the parameter values described in the text for the vector Ω, I could not find evidence for multiple solutions.

Uniqueness of y^*_{IL} (Assumption 4.1): Given different values of θ^*_{IL}, $\overline{\theta}^*_{IL}$ and s^*_{IL}, I solve equation (4.59) for y^*_{IL} numerically using the same parameter sets and algorithm as before. Results indicate that y^*_{IL} is always unique, which supports Assumption 4.1.

Uniqueness of x^*_{IL} and x^{**}_{IL} (Assumption 4.2): The simulations to determine the uniqueness of the solution x^*_{IL} and x^{**}_{IL} is essentially the same as for the whole system of equilibrium equations. I.e., I use the same algorithm as before, however, I only solve equations (3.19) and (3.22) separately. Again I find no evidence of multiple solutions for a large set of parameters and initial values. The results therefore indicate that Assumption 4.2 is true.

Uniqueness of x^* given θ^* as an implicit function of x^* (Proposition 4.5): Using simulations, I check for requirement a) in the proof of Proposition 4.5. For that purpose, I numerically solve the following system of equations

$$c_1 \cdot \frac{\Phi_2\left(\sqrt{\beta}\,(x^* - \theta^*)\,,\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*)\,;\sqrt{\frac{\gamma}{\beta+\gamma}}\right)}{\Phi\left(\sqrt{\frac{\beta\gamma}{\beta+\gamma}}\,(x^* - y^*)\right)} = t, \tag{4.98}$$

$$\theta^* = 1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}\,(\theta^* - x^*)\right) \tag{4.99}$$

for x^* and θ^*, and y^* being given. Using the same algorithm and parameter sets as in the other simulations in this chapter, I cannot find evidence for multiple solutions. Hence, the results support the assumption that requirement a) in the proof of Proposition 4.5 is satisfied.

5

German Summary - Zusammenfassung

I know you think you understand what you thought I said, but I'm not sure you realize that what you heard is not what I meant.

- Unbekannten Ursprungs, fälschlich
Alan Greenspan zugeschrieben

Obwohl Alan Greenspan das obige Zitat[63] in der Form wahrscheinlich nie geäußert hat, so ist es nicht verwunderlich, dass es ihm oft fälschlicherweise zugeschrieben wird. Als Vorsitzender der US-Notenbank war er berüchtigt für seine meisterhafte Anwendung des sogenannten "Fedspeak", einer Kommunikationsstrategie, bei der Aussagen zu Geldpolitik und wirtschaftlichen Entwicklung gegenüber der Öffentlichkeit bewusst vage und zweideutig formuliert werden um eine Überreaktion an Finanzmärkten zu verhindern. Das obige Zitat ist ein treffendes Beispiel für diese Kommunikationspraxis und unabhängig davon, ob es nun von Alan Greenspan stammt oder nicht, so verdeutlicht es das Gewicht, das kleinere Marktteilnehmer seinen Einschätzungen und denen seiner Nachfolgerinnen und Nachfolger beimessen.

Als große Akteurin auf den Finanzmärkten nutzt eine Zentralbank in erster Linie ihre gesetzlichen Befugnisse, um die Märkte durch direkte Interventionen oder durch Offenmarktgeschäfte zu beeinflussen. In Zeiten hoher Unsicherheit und Volatilität (wie bspw. in Krisen), orientieren sich kleine Marktteilnehmer jedoch vermehrt an ihren Prognosen und Entscheidungen, sodass ihre Handlungen einen überproportionalen Einfluss auf Märkte und Volkswirtschaften gewinnen können. Entsprechend sind ausgeklügelte Kommunikations- und Verschleierungsstrategien zur Steuerung von Erwartungen (wie Forward Guidance und Fedspeak) inzwischen etablierte Werkzeuge im Werkzeugkasten von Zentralbanken. Aber nicht nur Zentralbanken können als große Akteure einen solchen Einfluss ausüben. Regierungen, internationale Finanzinstitutionen, private Investitionsfirmen und sogar Individuen können als "Gravitationszentren" oder Orientierungspunkte für kleine Akteure wirken. Ein Beispiel dafür ist George Soros, dessen Short-Position gegenüber dem britischen Pfund Sterling 1992 andere Devisenhändler dazu ermutigte, gegen das Pfund zu wetten, und damit angeblich wesentlich zum Austritt des Vereinigten Königreichs aus dem Europäischen Wechselkursmechanismus beitrug.

[63] "Ich weiß, Sie glauben zu verstehen, was Sie dachten, das ich gesagt hätte, aber ich bin mir nicht sicher, ob Sie verstehen, das, was Sie gehört haben, ist nicht das, was ich gemeint habe."

In der vorliegenden Dissertation untersuche ich in drei Kapiteln, wie große Akteure Einfluss auf kleinere Akteure ausüben und wie sich dies auf die Entscheidungen der großen Akteure auswirkt. Ein besonderer Fokus liegt dabei darauf, wie die großen Akteure in einem unsicheren Umfeld Informationen verarbeiten, Erwartungen bilden und diese durch ihr Handeln an kleinere Akteure kommunizieren. Ich untersuche diese Zusammenhänge zum einen empirisch beispielhaft im Devisenmarkt und zum anderen innerhalb eines spieltheoretischen Modells eines Investitionsprojekts.

In Kapitel 2 dieser Arbeit untersuche ich die Beziehung zwischen der Devisenhandelsaktivität großer US-amerikanischer Marktteilnehmer und der Volatilität des nominalen Devisenkassakurses. Die Mixture-of-Distributions-Hypothese (MDH) sagt einen positiven Zusammenhang zwischen der Handelsaktivität von Marktteilnehmern und der Volatilität von Anlagenrenditen voraus. Frühere empirische Studien bestätigen diese Vorhersage für Aktien- und Futures-Märkte. Vergleichbare Studien für den Devisenmarkt sind jedoch aufgrund seiner dezentralen und undurchsichtigen Struktur und des damit verbundenen Mangels an Daten zur Handelsaktivität seltener. Unter Verwendung von Daten aus den sogenannten "Foreign Currency Reports", die im "Treasury Bulletin" veröffentlicht werden (U.S. Department of the Treasury, nd), verwende ich die wöchentliche Wachstumsrate der aggregierten Fremdwährungspositionen großer Marktteilnehmer, um die Handelsaktivität auf dem Devisenmarkt zu ermitteln. Durch die Schätzung des Heterogenen Autoregressiven Modells der Realisierten Volatilität (HAR-RV by Corsi, 2009), das um exogene Regressoren erweitert wurde, finde ich Belege dafür, dass die auf anderen Vermögensmärkten festgestellte positive Beziehung zwischen Handelsaktivität und Volatilität auch auf dem Devisenmarkt besteht. Darüber hinaus zeige ich, dass die Beziehung hauptsächlich durch unerwartete Veränderungen in der Handelsaktivität bedingt ist und dass sie für einige der betrachteten Währungen asymmetrisch ist. Die Ergebnisse erweitern die empirischen Befunde für die MDH auf Devisenmärkte und tragen zum Verständnis der Triebkräfte der Wechselkursvolatilität und der Rolle großer Akteure für den Informationsfluss bei.

In den Kapiteln 3 und 4 untersuche ich innerhalb eines spieltheoretischen Modells eines Investitionsprojektes, wie ein großer Kreditgeber mit seiner Kreditentscheidung die Entscheidungen kleiner Kreditgeber beeinflusst. Ein besonderes Augenmerk lege ich dabei auf den Zeitpunkt der Entscheidung des großen Akteurs, d.h. ob er seine Entscheidung über die Verlängerung eines Kredits vor oder nach der Entscheidung der kleinen Akteure fällt. Kapitel 3 behandelt dieses Szenario in einem sequenziellen Globalen Spiel eines Regimewechsel (*global game of regime change*). Der große Kreditgeber wird vor die Wahl gestellt, eine Investition vorzeitig zu verlängern oder seine Entscheidung zu verschieben. Ein frühzeitiger Rollover sendet ein positives Signal an die kleinen Kreditgeber, die dadurch veranlasst werden, ebenfalls einen Rollover vorzunehmen. Gleichzeitig kann der große Kreditgeber durch den Aufschub die aggregierte Entscheidung der kleinen Kreditgeber beobachten und von ihnen lernen, um eine besser informierte Entscheidung zu treffen. Er muss sich entscheiden, ob er in der Masse der Gläubiger ganz vorne oder ganz hinten stehen will. Da seine Entscheidung einen Orientierungspunkt für die kleinen Kreditgeber darstellt, hat sein Handeln einen erheblichen Einfluss auf die Wahrscheinlichkeit eines Scheiterns der Koordinierung und damit auf das Scheitern des Investitionsprojekts. Durch die Hervorhebung der Bedeutung der Signalwirkung eröffnen die Ergebnisse eine neue Perspektive auf die Idee der katalytischen Finanzierung (*catalytic finance*) und den Einfluss eines *Lenders-of-last-resort* in sich selbst erfüllenden Schuldenkrisen.

In Kapitel 4 erweitere ich diese Erkenntnisse, indem ich die Annahme aufhebe, dass der große Kreditgeber den Anteil der kleinen Kreditgeber, die ihren Kredit verlängern, perfekt beobachtet, was zu einem Spiel mit unvollkommenem Lernen führt. Ich untersuche die Empfindlichkeit des

Gleichgewichts in Bezug auf die Informationsgenauigkeit. Es ergeben sich zwei zentrale Ergebnisse: 1) Ich zeige, dass das Spiel mit perfektem Lernen und das Spiel ohne die Option zu warten (beide werden in Kapitel 3 eingeführt) Spezialfälle des Spiels mit unvollkommenem Lernen sind. Dabei bestimmt der neue Parameter für die Signalpräzision, wie nahe das Spiel mit unvollkommenem Lernen den beiden anderen Spielen ist. 2) Ich bestätige, dass die qualitativen Ergebnisse aus Kapitel 3 immer noch gelten, selbst wenn der große Kreditgeber unvollkommen informiert ist. Daher sind diese Ergebnisse robust gegenüber der Annahme, dass der große Kreditgeber die kleinen Kreditgeber perfekt beobachtet.

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