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Fachbereich IV

Structured Eurobonds

Optimal Construction, Impact on the Euro and the Influence of Interest Rates

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Vorgelegt von:

Marc-Patrick Adolph, M. Sc.

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Gutachter: Univ.-Prof. Dr. Christian Bauer (Universität Trier)
Univ.-Prof. Dr. Matthias Neuenkirch (Universität Trier)

Preface

This doctoral thesis was submitted at the department IV (Economics, Social Sciences, Mathematics, and Informatics) for obtaining the degree doctor rerum politicarum (Dr. rer. pol.). It is written cumulative, containing three separate articles. The first and third paper were written as a co-authorship with Christian Bauer in the first and Tobias Kranz in the third. Christian Bauer was responsible for the methodology and several examination ideas, and Tobias Kranz for the theoretical framework and the Monte Carlo simulations. All topics are interlinked in these three papers and form a reasonable structure, which will be explained in detail.

This thesis starts with a motivation of the topic, followed by the three separate articles. Every article represents a chapter of the thesis. A german summary – as prescribed in the promotion regulation – can be found at the end.

The articles were written between 2017 to 2020, simultaneously working as a research and teaching assistant at the Chair of Monetary Economics at the University of Trier.

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Motivation

The European Union (EU) and especially the European Monetary Union (EMU) have undertaken many actions towards a complete political, economic, and fiscal union. Whereas the political and the economic union are on a good way, e.g. with a common parliament and common trade negotiations, the fiscal union is still a theoretical concept yet to be fully implemented. There is a common currency with the Euro, which leads to a strong connection between the countries, and one central bank responsible for this currency, the European Central Bank (ECB). The Euro is the link between the countries in the EMU, but a common budget or fiscal authority and a common bond issuance scheme are missing, which is necessary to complete a fiscal union. These missings have been a point of criticism ever since the EMU started with the introduction of the Euro in 1999, but had been neglected in its first years.

Only a few years after the introduction of the Euro in January 1999 as demand deposit and in January 2002 as cash, a large housing/real estate bubble grew in the subprime mortgage market of the USA and had its peak in the bankruptcy of Lehman Brothers in 2008. The impact on the European market became visible a few years later in several countries such as Greece through bailouts granted to banks which had invested in structured mortgage products in the US. The mortgage products depreciated throughout the crisis and were written off afterwards. The depreciation led to the necessity for the affected countries to rescue the involved banks as higher risk provisions burden the balance sheets. As a consequence, the European sovereign debt crisis evolved because it became unclear whether all countries could serve their debt. A flight to relatively safe bonds, e.g. German bonds, began and as a result, several support programmes started, e.g. for Greece and Portugal, through the European Stability Mechanism. In this context, the previously mentioned criticism got more attention, because a fiscal union might have previously predicted these difficulties or could have easier resolved the issues.

A second crucial issue that can be resolved by issuing Eurobonds is the sovereign-bank nexus. Banks primarily hold sovereign debt of the countries they are incorporated in. If countries must rescue banks in a financial crisis and have difficulties to serve their debt afterwards, the value of their bonds will depreciate. As a consequence, these banks need new equity, and a spiral starts to spin. A fiscal union with several control mechanisms such as fiscal surveillance and a common bond issuance scheme might have been in a better position to handle the crisis.

One of the central requirements to reach a full fiscal union and to complete the EMU is the implementation of a common debt instrument. With the introduction of the Euro, the idea of this instrument came up. The Giovannini Group (2000) performed the groundwork. In their work and with Eurobonds per se, the countries shall no longer be solely responsible for their debt issuance. The discussion of the designated shape of the envisioned common debt instrument is highly political due to the beforementioned sovereign debt crisis and its necessary support packages. A wide range of possibilities were discussed and Claessens et al. (2012) and the Securities Industry and Financial Markets Association (2008) give an overview of these possibilities. Delpla and von Weizsäcker (2010) drew another possibility with two types of bonds. The first type is the "blue bond" with a common liability which are issued up to 60% of the GDP. The Stability and Growth Pact inspires this threshold. The second type is "red bonds" who are issued for every debt above 60% of the GDP for every country. The countries have to issue the bonds on their own liability. Nevertheless, the focus turned towards structured products in the form of an Asset-Backed Security (ABS). By their construction (Pooling & Tranching of assets) and their handling of liability (transfer to the capital market), ABS mitigate the problems of other approaches. These approaches have a common liability which can violate the "no-bailout"-clause and the resulting rating of the bonds is not secured. However, a certain aversion against ABS products still exists. Collateral Default Obligations (CDOs), a subgroup of ABS products and a key factor for the financial crisis in 2008/2009, are the reason for this criticism.

The introduction of a novel, not yet established type of product such as structured Eurobonds comes with uncertainty, hurdles, and various questions to overcome. Many of the questions are concerning their feasibility and are of legal nature, e.g. how to handle budget authorities. Nevertheless, there is a great chance to diminish several negative effects such as too large common liability or "moral hazard" because the product can be conceptualised and constructed from scratch. Several ideas to issue Eurobonds have been developed, but their consequences, e.g. on the capital market, and the sensitivity have not been examined yet. The quantification of these two aspects is the aim of this work.

The effects under investigation can be split up into two phases, a pre- and a post-introduction phase. The first chapter focuses on the pre-introduction phase by analysing the sensitivity of the gains, which can be achieved by issuing structured Eurobonds. In this chapter, the focus lies on the optimal design for issuance. The basis is an ABS approach. The cost of higher joint liability, which is represented by a model-inherent trust fund, and the sensitivity towards several factors such as the risk-free interest rate are analysed.

It turns out that the realised gains are highly connected to the risk-free interest rate and the recovery rate. Whereas the recovery rate effect is strictly positive, the interest rate effect is mixed, dependent on the trust fund size. In this context, the choice of the joint liability – or the capitalisation of the trust fund – can be crucial to maximising the gains. In particular, the right choice depends on the current risk-free interest rate and ranges between 9% and 18%.

These results are important for the political discussion because the two parameters – risk-free interest rate and recovery rate – determine the optimal joint liability. The question regarding the liability is essential in every Eurobond discussion, and the results in this chapter can help to determine an optimal construction. Also, the simulation in this chapter points out that an issuance through a supranational organisation, e.g. the European Stability Mechanism (ESM), can be advantageous. The choice of the common liability has a direct impact on the resulting structure, which consequently has a severe impact on the post-introduction phase. Especially, in an uncertain context where the risk-free interest rate and the recovery rate might change in the future, the variation of the joint liability might be important.

The second chapter evaluates a post-introduction scenario. Legal challenges that arise from the introduction are neglected here and afterwards because the focus lies on the economic perspective. The introduction of structured Eurobonds and its impact on exchange rates is the subject of this chapter. This analysis of the impact on exchange rates has its origin in the assumption that the Euro will be a reserve currency, as highlighted in the Green Paper of the European Commission (2011). This position as a reserve currency is assumed because an equivalent to the US T-Bill market will exist due to Eurobonds, and it strengthens the investor attraction towards the Euro. A complete replacement of single-level national bonds through structured Eurobonds would deliver a new yield curve. Thus, the current connection between exchange rate predictability and changes in the yield curve is analysed to quantify the impact of this shock. As a result, the Euro would depreciate against the US-Dollar and appreciate against British Pound, Swiss Franc, and Chinese Renminbi. Since the depreciation against the US-Dollar is counter-intuitive, a subset, excluding the time around the recent financial crisis, is examined. The effect stays the same four all four currency pairs. When a trade-weighted exchange rate of all four currency pairs is calculated, the Euro would appreciate.

A ride to "normal" monetary policy might have stronger influences because the yield curve heavily depends on the central bank refinancing rates and expectations. A prediction of this can be connected with severe errors which over- or underestimate the real exchange rate impact. This prediction error could have several next-round effects, e.g. on trade and stock prices, which might also be of interest for further examinations. The results of this chapter confirm the assumption of a strengthening of the Euro as an international reserve currency.

Finally, the third chapter leaves the field of structured Eurobonds and sets its focus on an approximation bias as a result of the use of Jensen's Inequality in DSGE models. This error occurs in non-linear, forward-looking models, which are also used by the ECB and the IMF, e.g. ECB-Global and IMF's Global Projection Model, as discussed by Dieppe et al. (2018) and Carabenciov et al. (2013). Two different solutions to measure this bias are presented – an analytical and a numerical. The analytical solution can be used when the dataset is sufficient in its quality and size. The bias can be calculated with deterministic factors derived from the dataset. If the dataset is not sufficient, a numerical solution – especially a Monte Carlo simulation – is useful. For this purpose, the Consensus Forecasts survey is used, which consists of forecasts given by several capital market institutions regarding macroeconomic factors. A calibration against the dataset is run to receive the distribution parameters for two distributions. They are used in the following Monte Carlo simulation. This simulation shows that the bias, which arises from Jensen's Inequality, can reach values around 25 basis points and should not be neglected – especially for large-scale models, such as the above mentioned ECB-Global. A correction of the results in these models would have a direct impact on the inflation forecast of the ECB. Therefore, their choice of the refinancing rate might change. A resulting adjustment, e.g. of the refinancing rate of the ECB, will change the risk-free interest rate and therefore the yield curve. Both changes do have a significant impact on the results of the two previous chapters. The change of the risk-free interest rate influences the first chapter, where the optimal choice of joint liability is connected to this rate. Since the yield curve is also changing, an impact on the exchange rates will also be observable. The two factors – yield curve and risk-free interest rate – are of central importance to determine the impact on exchange rates and the optimal choice of common liability, and they are affected by the choice of the refinancing rate of the central bank.

The central goal of this work is to quantify the impact of several macroeconomic factors on the issuance of structured Eurobonds and its following effect. It shall shed some light on the considerations needed before Eurobonds are issued as a structured product. Moreover, this dissertation is meant to sensitise policymakers regarding the effect of Eurobonds on exchange rates and several second-round effects, e.g. the impact on trade. Another purpose is to highlight the circumstances when a financial crisis with a high level of uncertainty is present. An independent institution such as the ECB should account for this uncertainty. An adjustment of the interest rate would be the consequence. This uncertainty needs to be accounted for in the effects of structured Eurobonds.

After all, it is on policymakers to decide whether or not they are in favour to introduce Eurobonds. There are many effects – some of them might be negative – that need to be accounted for. Nevertheless, to complete the fiscal union of the EMU, it is inevitable to issue Eurobonds, as a structured product or not.

1 Sensitivity of Structured Eurobonds

Limited Joint Liability in Structured Eurobonds: Pricing the political costs

Abstract

We introduce an analytical tool to study the effects of ABS-based Eurobonds. Our approach allows to optimise the degree of jointness and therefore could overcome the huge and emotionally influenced political obstacle of joint liability. The approach is stable over time. Interest savings reach 0.5 percentage points depending on the degree of joint liability even in the current economic environment. Based on an optimal degree of joint liability between 9% and 18% of each countries individual share, we can price the political cost of joint liability. In the sensitivity analysis, we examine different scenarios as, e.g. a PIIGS bond and an EU-6 bond which are all beneficial for each country as well as the community. Due to risk diversification, countries with high-interest load are most profitable for the community. Summarising, structured Eurobonds could be a stable, very beneficial and political feasible tool for a European fiscal system.

Keywords: Structured Eurobonds, Joint Liability, Fiscal Union, EMU, Sovereign Debt, ABS

1.1 Introduction

Since the beginning of the European Monetary Union (EMU), there is an ongoing discussion about a fiscal union to complete and stabilise the economic union. This discussion was fueled by the persistent crisis, which makes joint or coordinated fiscal policies a necessary stabilising pillar of the economic union. One of the most debated options is the common issuance of sovereign bonds of all countries in the EMU, the so-called Eurobonds.

There is as much literature on the political, macroeconomic, fiscal and stabilising advantages of the different proposals of Eurobonds as there is on the political and economic dangers of joint liabilities, moral hazard, and redistributions. To our knowledge, none of these approaches is tested on its quantitative sensitivity to the most typical political and economic stress situations. Nor are there quantitatively supported studies trying to optimise and price the degree of joint liabilities which seems to be a political tabu. Our paper aims to fill this gap.

In this paper, we want to shape a discussion about the optimal degree of joint liability in structured Eurobond approaches. While preserving the general macroeconomic and fiscal advantages of Eurobonds, structured approaches as in Hild et al. (2014) and Bauer and Herz (2019) (Brunnermeier et al. (2016) can be easily extended as well) allow for varying the degree of joint liability. We analyse how the degree of joint liability affects the efficiency and stability of Eurobonds. Thus, we can price the different degrees of jointness to facilitate political debates on the optimal design of a Eurobond system. Also, we analyse how structured approaches react to (1) political stress, e.g. the drop out of one member, (2) two speed scenarios, i.e. "Euro-Sub-Group Bonds", such as PIIGS or EU6, (3) the global financial situation, and (4) the issuance of short term bonds. Structured Eurobonds yield explicitly more interest than single country sovereign bonds. These interest gains serve in our paper to quantify the advantages of Eurobonds. Furthermore, these interest gains can be distributed in different ways, e.g. allowing for compensation of AAA countries for the political and economic costs of joint liability. These costs lead to a discussion on the optimal degree of joint liability and compensation. Besides, suitable distributions increase the internal stability and reduce moral hazard.

The rest of the paper is structured as follows. Section 1.2 gives the theoretical background on the advantages and disadvantages of Eurobonds, in general, and structured Eurobonds, in particular. Section 1.3 has a look at the data and methodology. In Section 1.4, we discuss the optimal degree of joint liabilities, while Section 1.5 presents different settings and robustness checks. Section 1.6 concludes our findings.

1.2 Theoretical Background

The Giovannini Group (2000) has published first ideas of joint issuance of sovereign bonds. The variety of approaches proposed after that is wide. The Securities Industry and Financial Markets Association (2008) and Claessens et al. (2012) give overviews of the proposals. The suggestions range from partial to full joint refinancing and vary in several other aspects, including the degree of joint liability. Boonstra (2005) introduced the possibility to use a fund for issuance and a much-noticed approach has been given by Delpla and von Weizsäcker (2010) who differentiate between the issuance of blue (Eurobonds) and red (national) bonds.

The discussion of joint issuance shows that besides strengthening the connection between member countries, joint bonds reduce interest expenses and deepen the market for sovereign bonds. Another purpose is to create an equivalent to the US-American T-Bill market and supply the financial markets with an Eurozone wide yield curve.

More advantages are higher liquidity in the sovereign bond market and a higher volume of EMU-wide AAA-rated bonds. Eurobonds might even be able to stop the sovereign-bank nexus. Also, some authors claim that the role of the Euro as an international reserve currency will be strengthened after the introduction of Eurobonds.[1]

Besides the many advantages, the introduction of Eurobonds is confronted with several difficulties. One of the most significant political barriers to the introduction of Eurobonds is the "joint liability - moral hazard" problem. Usually, in the political debate issuing bonds with joint liabilities comes down to the argument "We [the 'strong' countries] will have to pay for your [the 'weak' countries'] debt." E.g., in September 2011 German Chancellor Angela Merkel said in the German Parliament that Eurobonds are only a "communitisation of debt" and a "way into a debt union". One year later she said that Eurobonds would not be established "as long as I live". Another part of the debate is the "moral hazard" argument. This states that 'weak' countries, meaning high-interest countries, could easily increase debt as (a) they have the opportunity to do it at lower interest rates and (b) the individual repayment or default decision is influenced by the potential of externalisation of negative default effects. The individual repayment or default decision transforms from a merely internal economic to an intergovernmental political bargaining process.

Of course, joint liability means an additional risk and thus hard financial costs, and there is a serious academic and political discussion about the moral hazard involved in Eurobonds. The debate, whether common issuance will set negative incentives for countries with refinancing problems, e.g. through a financial crisis, is controversial. More stable countries, e.g. Germany, fear a situation in which they de facto have to pay for countries which use Eurobonds as a cross-financing instrument. This means that countries with refinancing problems can borrow money from the capital market in a higher volume than in a single issuance scheme. This advantage is due to a good rating and a high nominal amount which allows borrowing money at a low-interest rate and the effect of one country needing more money is not crucial for the whole Eurobond scheme. If they cannot repay the debt, they default and the other countries in the Eurobond system, e.g. the more stable countries and especially their taxpayers, have to repay the debt of the defaulted country. The joint liability of Eurobonds is widely seen as the demise of a common budget authority. Also, the design of the Eurobond system has to account for defaults of single countries to prevent negative impacts on the credibility of the system itself with a subsequent probability of super-contagion-like effects.

[1] See for example the Green paper on stability bonds of the European Commission (2011).

On the national level, after the introduction of Eurobonds, the decreasing liquidity in the remaining sovereign country-specific bonds will be challenging for the fiscal authorities. Also, the uncertainty in rating single country bonds increases when liquidity is low in the remaining individual markets. Due to lower liquidity, existing bondholders will face some liquidity premia which lowers their yield. Also, new issuances will be more costly for emitting countries - if they don't want to be part of the Eurobond programme - since the demand is lower with existing Eurobonds.

Nearly all of these problems, however, can be addressed by a proper design of the Eurobonds. Previously mentioned literature and overview papers have highlighted the arising problems and proposed adequate solutions. Although theory unanimously argues in favour of Eurobonds, the issuance is accompanied by uncertainty in a crisis and discomfort against the unknown. Nonetheless the European Commission (2017) highlights in their current Reflection Paper of the on the Deepening of the Economic and Monetary Union, the necessity of common issuance to deepen the bond market in the EMU. Mcevoy (2016) links public support of the EU to the level of economic expectations. Thus, the positive economic effects of Eurobonds can raise expectations on economic conditions and therefore support the public opinion of both Eurobonds themselves and the European Union in general.

1.2.1 Structured Eurobonds

The European Commission suggests Eurobonds as one element to solve the current debt crisis and to prevent new adverse situations in the European Monetary Union.

The above mentioned Reflection Paper of the European Commission indicates a theoretically supported preference for structured products to introduce Eurobonds. Hild et al. (2014) and Brunnermeier et al. (2016) propose Asset-Backed Security (ABS) approaches to construct Eurobonds. To construct an ABS, a Special Purpose Vehicle (SPV) has to buy a portfolio of bonds of all participating countries. To avoid additional uncertainty and issuer risks, these portfolios must be filled with physical bonds and not through synthetic contracts such as credit default swaps (CDS). Collateral Default Obligations (CDOs) are a special type of ABS which uses sovereign bonds as collateral. Longstaff and Rajan (2008), Coval et al. (2009), and Coval et al. (2009b) give a deeper explanation of CDOs, their pricing and the correlation effects. After this pooling of assets, tranches with different risk and interest payments are emitted. The diversification and tranching effects are attributed to a correlation less than one between the countries of the EMU. Essentially, both approaches use similar techniques with slight but nevertheless fundamental differences. While Brunnermeier et al. (2016) propose only two tranches, "European Safe Bonds" (ESBies) and "European Junior Bonds" (EJ-Bies), Hild et al. (2014) allow for a more optimised tranching. The advantages of both ABS-approaches are a reduction of the negative aspects mentioned above. Market liquidity will be improved due to the different tranches, and the volume of AAA sovereign bonds in Europe will strongly increase. Additionally, Hild et al. (2014) allow varying the degree of joint liability. This is achieved by using a reserve fund to absorb first losses in case of default. Joint liability is limited to this fund which has an optimal size of 10% to 15% of the nominal volume of emitted debt. Losses that exceed this size will cause depreciation of the junior tranches.[2] Besides the above named political and macroeconomic advantages of Eurobonds in general, the structure generates a significant interest gain, which (in the approach of Hild et al. (2014)) can be distributed among all participating countries. The new emitted tranches have a lower average interest rate than the weighted average of the single issuance interest rates of participating countries since sovereign yield curves clearly show the market's risk aversion. The probably largest hindrance to this approach is the limited credibility of the structure and the associated interest spread, which is to be expected. An implementation via a supranational organisation, such as the ESM, would solve this problem, ensure low issuer risk and high-interest savings for the participants. Also, when single level country bonds vanish, it will be challenging to determine the interest rates to be paid in by participating countries.[3]

This design, in combination with an ABS product, prevents the above mentioned negative aspects of Eurobonds concerning "moral hazard" and joint liability. Properly designed structured Eurobonds have two features that would allow overcoming these problems. Firstly, the degree of joint liability is not a binary black and white choice but a continuous variable which can be optimised to balance out efficiency and stability. Secondly, the interest advantage over single issuance is an explicit income stream which can be distributed among all participants. By proposing two exemplified distributions of this interest gain, we show that high rated low-interest countries like Germany also profit financially from participation. In the empirical part, we will analyse the trade-offs between varying degrees of joint liability, financial profitability, and political costs. A discussion on the topic of structured Eurobonds can also be found in the feasibility study of the ESRB (2018). Different possibilities of Eurobonds in various configurations are portrayed as well as advantages and disadvantages. These possibilities are also thematised by van Riet (2017).

[2]Formally, the joint liability can be lowered until even zero although this extreme would dramatically reduce the efficiency of the structure as we will show in the next chapter. Hild et al. (2014) clarify the model of structured Eurobonds in a SPV as well as the cascading effects.

[3]Few technical problems to be expected during the conversion can certainly be handled.

1.3 Methodology and Data

1.3.1 Methodology

Our analysis follows the model introduced by Hild et al. (2014). A SPV is established, which buys a portfolio of bonds of the participating countries and sells different tranches with different interest rates to the capital market. A trust fund (in the benchmark model 10% of the issued nominal volume) absorbs first losses and invests the capital at the risk-free interest rate. If a country defaults and the cash flows cannot be served, the trust fund steps in while the recovered nominal value is transferred to the trust fund and the country drops out of the scheme.

The probability of default of every country is calculated using the method of Sturzenegger and Zettelmeyer (2010). They use CDS-spreads and recovery rates to calculate the default probability. As a benchmark, we use a fixed recovery rate of 50%. Other possible methods to calculate a default probability have been shown by Polito and Wickens (2015) and Bi (2012). In the first article, the authors use a rolling-window VAR model while in the second risk premia are matched to default probabilities in a closed economy model. Polito and Wickens (2015) mainly focus their analysis on the possibility of countries to repay their debt by tax revenues and Bi (2012) uses a model with several macroeconomic factors. Since none of them presents a connection to the recovery rate, we stick to the method of Sturzenegger and Zettelmeyer (2010) and assume that the market appropriately displays the default probability. When structured Eurobonds substitute the whole single issuance bonds, a derivation of the default probability by CDS-spreads is impossible. In this case, the other two methods presented above might be more appropriate. In addition, the contingent claims approach by Merton (1974) can be used.[4] As structured Eurobonds generate gains and improve the fiscal situation of all countries, their default probability will get better.

We apply the strongest possible parameter values for contagion effects to present the most conservative model. For this, we assume the highest possible correlation without violating the default probabilities of every participating country. This means that if one country defaults all countries with a higher default probability also default. Also, we do not allow a member to come back into the SPV after defaulting once. If a country defaults, it drops out of the SPV, and the recovered nominal is transferred to the trust fund.

In this paper, we present results for a period of ten years to model long-term refinancing. After this time, the remaining capital in the trust fund is redistributed to

[4] An overview of credit risk models for sovereigns can be found by Gray and Malone (2008).

the non-defaulted countries. We allow for two different distribution methods of the net surplus, even or relative, although other distribution schemes are possible to set political incentives. In this context, an even distribution means that the net surplus is allocated according to the individual debt level. In the relative case, the funds are distributed in relation to the interest payments. The first method is better for countries with high nominal volume and low-interest rates, i.e. AAA countries, whereas the second method is advantageous for countries with a high-interest burden, e.g. PIIGS countries. Therefore, it is a political topic to decide between the methods of distribution. The direct economic advantage, i.e. the average interest reduction for the debtors, is independent of this distribution. But the indirect benefits, e.g. higher economic stability through repayment schemes based on prudential economic policies or low budget deficits, do indeed depend on the outcome of the political bargaining process defining the distribution schemes.

We run Monte Carlo simulations to find the structure, calculate the extra costs, and gains/losses. The simulation is done with m=100,000 loops.

1.3.2 Data

For the calculation, we need the yields of the government bonds of every single member country of the EMU to calculate the individual interest rate. We additionally require the CDS-spreads for different rating classes to calculate the interest outflows from the SPV to the capital market. We thus include the issuer risk assigned to non-sovereign bonds into our model. Since these spreads are not available for every single possible rating, we use a spline interpolation to approximate the few that are missing. The 10-year CDS-spreads for every member country are taken to derive their probability of default. These datasets and the rating of every member country are taken from Reuters Datastream for the end of August 2018, the end of December 2012 and the end of December 2008. The three dates are chosen to represent current circumstances, the European sovereign debt turmoil and the financial crisis starting in 2008, respectively. The individual interest rates of every participating country can be found in Table A1 in the Appendix. Sovereign debt and GDP to calculate the emitted volume of debt are also taken for the end of June 2018 or the latest possible date from Eurostat, the European Statistical Office.

1.4 Optimal degrees of joint liability

The initial trust fund volume is a crucial determinant of both the political bargaining process and the economic advantages of structured Eurobonds. The trust fund is equivalent to the degree of joint liability every country is willing to choose. The main implications of a larger trust fund are (1) an increased security buffer for the investors resulting in a more efficient tranching and lower average interest rates, (2) an increase of the void interest payments of the participating countries as the trust fund share is also credit financed and (3) an increase of the risk of the participating countries to lose money in case of a default of a member and thus a potential rise in moral hazard. The net financial effect of the trust fund for the participants is nonlinear in the size of the trust fund and also depends on other parameters such as the global economic situation and the recovery rate. In the efficiency optimum, the positive marginal interest rate reduction is offset by the marginal financing cost of the trust fund. The political and economic risk is an additional argument in the political bargaining process that determines the realisation of the trust fund size. The deviation from the optimum is the political price for the joint liability risk.

First, we examine the effects of the trust fund on nominal gain and interest rate advantages generated by Eurobonds. In Figure 1.1 we have a look at three benchmark trust fund shares - 5%, 10%, and 20% - and their impact on the nominal gains as a function of the recovery rate and the risk-free interest rate. In all scenarios, the recovery rates have a qualitatively similar positive effect. The sensitivity to the recovery rate, however, decreases with the trust fund size, since a larger trust fund stabilises the repayment cash flow and thus enhances the structuring. The effect of the risk-free interest rate is dependent on several factors. It is neither strictly positive nor strictly negative. For low recovery rates, the effect is negative with decreasing gains for growing interest rates. For higher recovery rates, we can observe a change and the effect is strictly positive for increasing interest rates. The respective reversal point in the recovery rate is dependent on the initial trust fund share. For a trust fund of 5% the reversal takes place at a recovery rate of 40% whereas for a 20% trust fund share this point is at a 20% recovery rate. These different reversal points can be explained by the interest generated within the trust fund, which depends on the risk-free interest rate. Also, a larger initial trust fund can even bear losses when the recovery rate is low. In this case, the sensitivity to the risk-free interest rate decreases with the trust fund size. For an assumed total issuance of 10% of GDP, the gains reach from 3.63 billion Euro to 69.46 billion Euro. These extremes both happen in a 5% trust fund and 6% risk-free interest rate scenario with the recovery rate ranging from 0% to 80%.

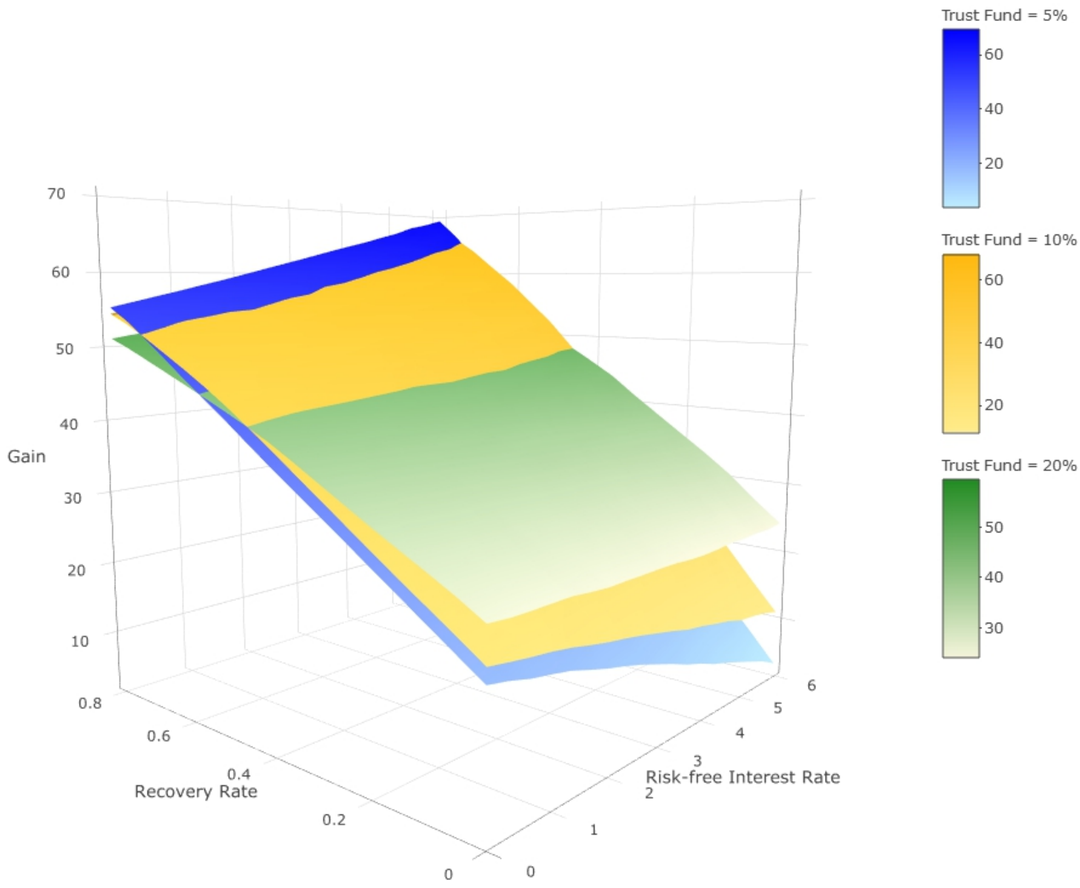

Figure 1.1: Nominal gains in a structured Eurobonds scenario with different trust fund shares over a grid of several recovery values as well as risk-free interest rates.

For recovery rates below 60%, gains rise with the initial trust fund share but this effect is reversed for higher recovery rates. The marginal effect of the trust fund share on the average rating of the structure declines while the individual countries' marginal interest rate burden to finance it remains constant.

We also see that excess returns of the Eurobonds are closer to each other for low risk-free interest rate and diverge stronger for higher values. For low-interest rates the differences range only from 4.19 to 6.31 billion Euro, i.e. the structure is not sensitive w.r.t. the trust fund share in a low-interest rate environment. The model does also support the possibility of negative risk-free interest rates. Even in this environment, the structure delivers gains, e.g. 42.87 billion Euro for a trust fund of 50% and a risk-free interest rate of -0.5%.[5]

[5]Exact results for negative risk-free interest rates are available upon request.

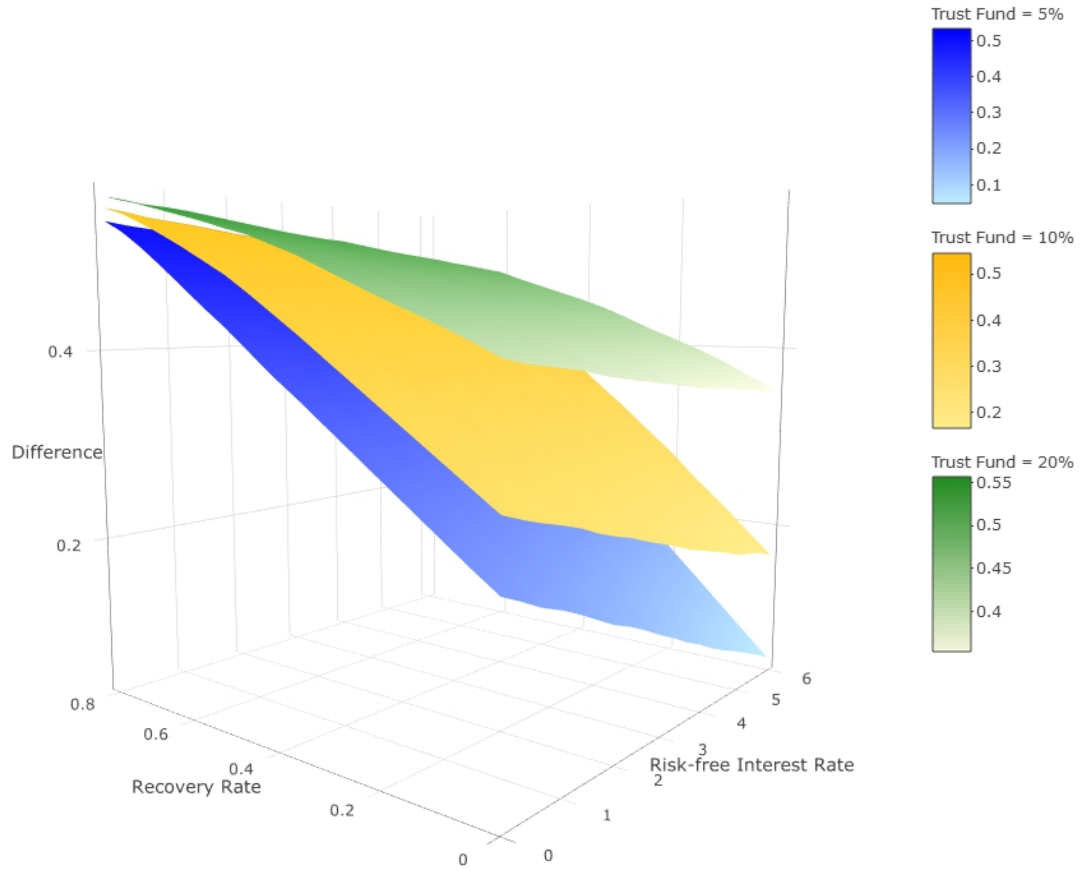

Figure 1.2: Interest rate difference in a structured Eurobonds scenario with different trust fund shares over a grid of several recovery values as well as risk-free interest rates.

The results highlighted in Figure 1.2 present the interest savings of structured Eurobonds, i.e. the net gains displayed in Figure 1.1 less the financing cost of the trust fund. We see that the topmost layer, which is representing the 20% initial trust fund rate, has the flattest slope and the lowest layer (5%) the steepest slope. Low trust fund structures are more sensitive to changes in the recovery rate and risk-free interest rates. This interest advantage (of a smaller initial trust fund) is decreasing with higher recovery rates because the risk structures and thus the tranching converges with growing recovery rates.

We turn to optimise the trust fund size w.r.t. the countries' net gain. We use a benchmark recovery rate of 50%[6] and vary the trust fund shares between 5% and 30%. Figure 1.3 shows the results.[7] Since a total issuance of 10% of the GDP is assumed, this

[6]Meyer et al. (2019) and Cruces and Trebesch (2013) estimate haircut sizes of 44% and 40%, respectively. This haircut size is equivalent to a recovery rate of 56% and 60%. We use a more conservative assumption and fix the recovery rate at 50%.

[7]Robustness checks with 40% and 60% recovery rate can be found in the Appendix.

implicates a liability between 0.5% and 3% of the GDP. As a consequence, the liability ranges from 5.5 million Euro for Malta in the 5% trust fund scenario to 9.79 billion Euro for Germany in the 30% trust fund scenario.

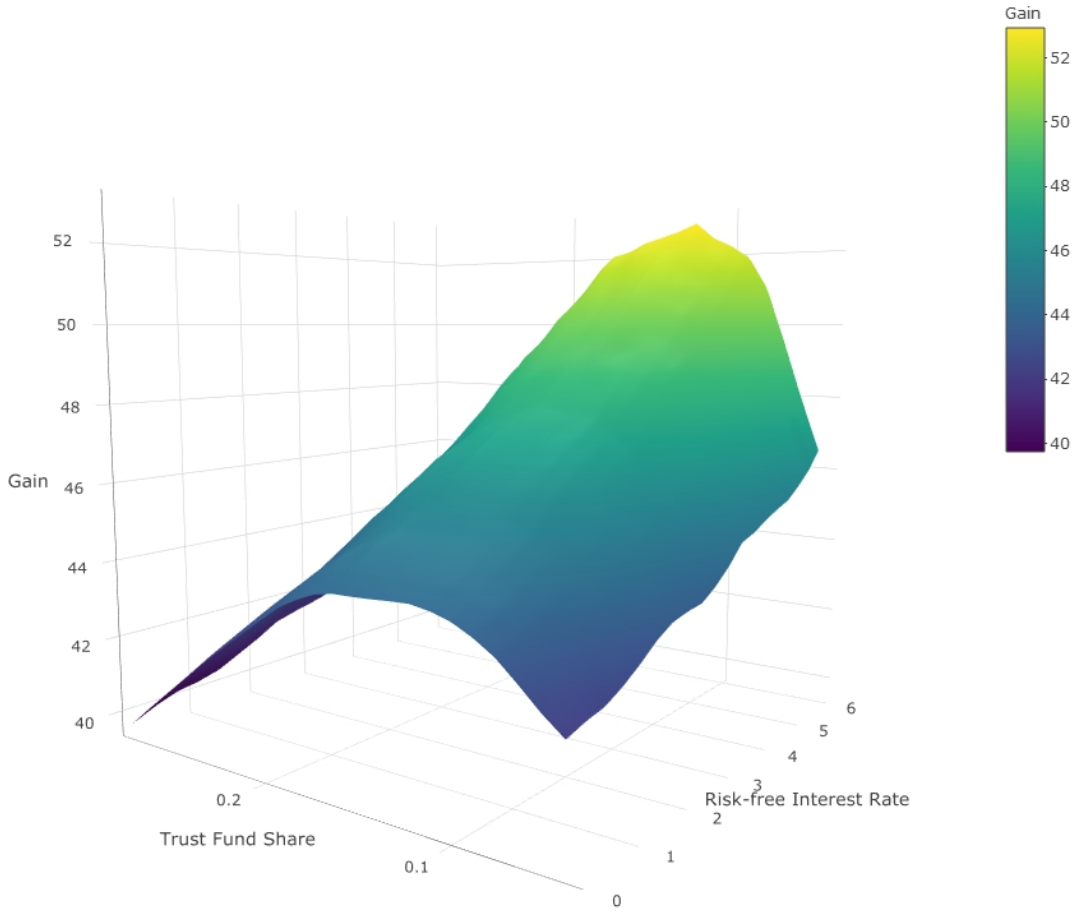

Figure 1.3: Sensitivity of gains for different trust fund shares in a fixed recovery rate scenario where it is fixed to 50%.

We see that the net gain as a function of the trust fund share is nonlinear, asymmetrically inverse U-shaped and has a unique maximum. In Figure 1.4, we see that, dependent on the risk-free interest rate, the optimal trust fund size is between 12% and 16%. It starts at 12% in a 0% interest rate scenario and jumps to 16% at 1% interest rate. The jump can be explained by gains that are close to each other, and they are building a flat plain in this area. Afterwards, it is steadily declining to 12% again, which is reached at a risk-free interest rate of 5.6%.

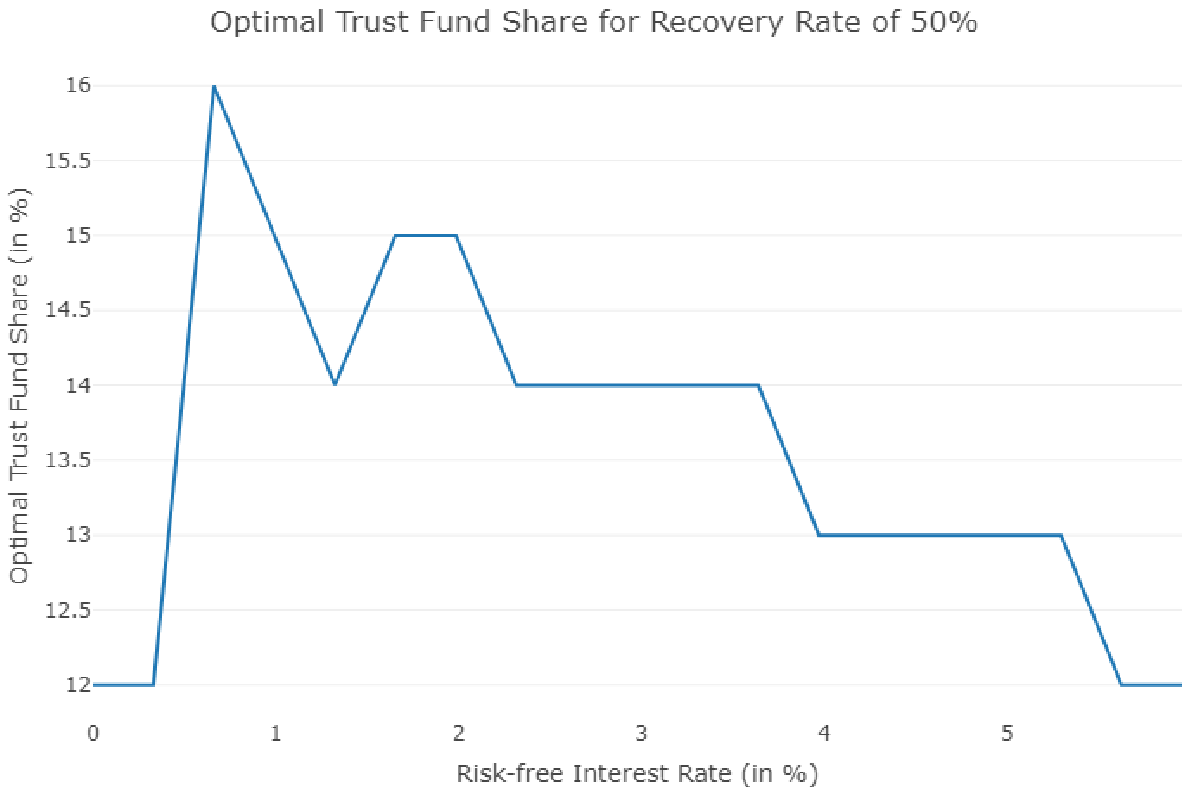

Figure 1.4: Sensitivity of the optimal initial trust fund share in respect to the risk-free interest rate with a fixed recovery rate of 50%.

In the boom scenario (6% risk-free interest rate), gains reach a maximum of 53 billion Euro with an initial trust fund of 12% and decline to 42.5 billion Euro for an initial trust fund of 30%. In the global recession or post-crisis scenario with a risk-free interest rate of 0% an optimal gain is also reached for a trust fund of 12%. However, the sensitivity is lower. The optimal gain is 45 billion Euro and declines to 40 billion Euro for a 30% trust fund choice. The results are also absolutely supported for other recovery rates (see the Appendix). The main difference lies in the optimal choice of the trust fund size. For 40% recovery, it varies between 14% and 18% and for 60% recovery between 9% and 13%.

Summarising, the recovery rate increases the monetary advantages of structured Eurobonds, whereas the risk-free interest rate has a mixed effect. The higher the trust fund share, the higher is the interest advantage and the lower is the sensitivity of gains to the recovery rate and the global economic situation. But in the current low-interest rate environment, the structure is not sensitive w.r.t. the trust fund share, i.e., in and after a global recession, it appears not advantageous to have a high degree of joint liability. On the other hand, when interest rates return to normal levels again, interest advantages of structured Eurobonds would decrease, and the decline is faster, the lower the degree of joint liability. In the same instant, interest increases deteriorate the economic situation of struggling members potentially reducing their expected recovery rate and thus inducing a very steep reduction of the structured Eurobonds' advantages if the level of joint liability is low. It might currently appear reasonable to establish structured Eurobonds with a very low degree of joint liability, e.g. 5% or less since there are only little losses in efficiency compared to higher joint liabilities. This parametrisation certainly is politically much easier feasible and eventually the only way to overcome current reservations against such structures in the northern countries. However, only sufficiently high degrees of joint liabilities, 10% to 15%, ensure that the system remains significantly advantageous (in financial terms) if the global economic situation returns to normal. The efficiency of structured Eurobonds, i.e. the net gain, which equals the interest advantage less the financing cost for the trust fund, depends on the size of the trust fund. The optimal degree of joint liability lies between 10% and 15%, dependent on the recovery rate and the risk-free interest rate. However, the efficiency losses between the optimum and more extreme cases, such as 5% or 30%, are quite moderate and remain below 20% of the maximal gains. The results indicate that optimising common liability has positive effects on the gains of structured Eurobonds and the stability of these gains to economic shocks despite the political costs that might arise.

1.5 Alternative scenarios and Robustness checks

In this section, we present some alternative scenarios and robustness checks for structured Eurobonds and give a brief overview on the effects. This includes country subsets and variations in macro- and country-specific factors. An extensive explanation and more robustness checks can be found in the Appendix.

1.5.1 EMU-wide with different introduction dates

Firstly, we vary the general economic environment by altering the time for the introduction of structured Eurobonds. As in the benchmark case, we look at an EMU-wide introduction where every country is participating, and 10% of GDP is issued. The three different scenarios where the dates for introduction are August 2018, December 2012 and December 2008, respectively, yield heterogeneous results for the structures and the gains. Changes in three factors mainly drive the differences in the results. The first factor is the risk-free interest rate, the second is the CDS-spreads, which changes the default probability of the countries, and the third is the interest rate spread every country and the SPV have to pay. Whereas the results are positive for 2012 and 2018, they can get negative in 2008. The net gains reach from 0.72% of issued nominal for Germany to 20.97% for Greece dependent on the time of issuance and distribution method (relative and even). The negative results in 2008 are due to a growing issuer risk which pushes the interest paid by the SPV. The negative results can be overcome when a supranational institution like the ESM issues the structured Eurobonds. When this can be achieved, the interest rate is decreasing, and the losses turn into gains again. These are well distributed through all countries and above 1.5% for every one of them.

1.5.2 Country subsets

Next, we focus on several country subsets with an introduction in 2018. The subsets consist of an EMU-wide introduction without (1) Italy or (2) Germany, an introduction in only (3) the so-called PIIGS-countries and (4) the EU-6 countries, the founders of the European Economic Community. In case (1) the proceeds drop significantly. The maximum gain that can be reached is for Greece, with 5.01% of the issued nominal, and the lowest is for Germany with 0.16%. This shows that the inclusion of a relatively high share of low rated countries such as Italy has substantial advantages for other participating countries due to the average cash inflow. Case (2) draws a contrary picture. Here the gains rise. The average rating of the structure decreases only very little since the trust fund can easily bear first losses. Therefore the structure's capital outflows are significantly reduced due to a lower nominal volume (Germany has a share of approximately 29%) while the inflows stay nearly the same since the German interest payments would have been very low. Summarising, the diversification and tranching effects of structured Eurobonds imply a high relevance of the participation of low rated countries for the advantageousness of this financing instrument. Top-rated countries such as Germany, on the other hand, are less important for the economic performance of structured Eurobonds, but undoubtedly inevitable for their political credibility.

In case (3), gains grow significantly compared to our base scenario. The average rating of the structure declines but the interest rate differential stemming from the diversification effect is stronger than the rating effect. The proceeds reach from 4.27% for Ireland to 22.62% for Greece. Savings per year can reach 1.2% of issued nominal for all of the five participating countries in the even distribution scheme. Case (4) shows the best average rating with 97% rated AA or better. However, the gains are, on average, seven basis points lower than in the baseline scenario due to the already high average rating of the participants. This result supports the notion that the countries which founded the predecessor of the European Union can again play a pioneering role in establishing new European cointegration measures like common debt financing.

1.5.3 Shorter duration

A possibility to introduce structured Eurobonds and to make them politically more feasible is to choose a shorter duration. An advantage is that countries with aversions against structured Eurobonds, e.g. Germany, might be more interested in a shorter commitment period. Therefore, we vary the duration and calculate the gains for a one- and two-year duration. As a consequence of negative risk-free interest rates, only the even distribution is evaluated. It produces gains and losses for some countries for a one-year duration. The total profits amount to 0.3 billion Euro. In this case, a new distribution method might be appropriate to avoid losses for a single country.

For a two-year duration, the total gains rise to 3.98 billion Euro, which is a significant rise compared to the one-year duration. When the even distribution method is used, all countries gain from the issuance of structured Eurobonds. Therefore, this distribution method is appropriate for short term bonds with more than one-year duration.

1.5.4 Comparison

All of these alternative scenarios show the robustness of the model. We can also see that even with small country-subsets gains can be achieved and that they are sometimes larger than for an all Euro-area scenario. Italy, as a large low rated country, has a crucial impact on the gains as can be seen in the calculation where Italy is missing. Politically it might be more feasible to start with a subset of the Euro-area and extend afterwards. As we see, there will be no economic disadvantages with a smaller initial group, and the process is open for other countries to join after realising the advantages. It might also be more feasible to start with a shorter duration. In this case, it is crucial to choose the even distribution method or to implement a completely new method since some countries face losses otherwise.

Also, capital market trust and fiscal discipline indirectly determine the economic efficiency of structured Eurobonds, as is shown in the results regarding the sensitivity to the risk-free interest rate and recovery rate. A high fiscal discipline can boost both capital market trust and the expected recovery rate.

1.6 Conclusion

The European Commission pushes forward the joint issuance on sovereign bonds as a vehicle to deepen integration and foster financial stabilisation in the European Monetary Union. Although the refinancing costs in the EMU are low even now, we show that an ABS-approach yields fiscal advantages for all participating countries.

We optimise the size of the trust fund share, i.e. the degree of joint liability, with respect to the efficiency of the structure. Higher joint liability not necessarily results in higher gains. But higher degrees of joint liability are more resistant against changes in the recovery rate as well as the risk-free interest rate. We also show that for typical recovery rates, the optimal range of joint liability varies between 10% and 15%.

Also, under the global economic circumstances in 2018, the issuance of structured Eurobonds creates gains for participating countries between 1% to nearly 14%, depending on the distribution scheme. Even if structured Eurobonds would have been established during the government bond crisis or a global financial crisis, they would still have had positive effects.

Finally, using scenarios without Italy and Germany, we show the importance of low rated participants for creating a large share of the interest inflow. In the case of the dropout of Italy, the overall surplus would shrink by over 80%. Dropouts of high rated countries, such as Germany, lead to significant improvements of the surplus because their average rating is better than the average rating of the structure, i.e. they pay less interest than refinancing costs are, and the improvement of the structure has a smaller effect.

Appendix A

A.1 Collateral Default Obligations

Collateral Default Obligations (CDOs) are a special type of Asset-Backed Securities (ABS). They are built of a portfolio of loans which is bought by a Special Purpose Vehicle (SPV). The SPV buys the portfolio and structures it into different tranches. The ratings of these tranches depend on the default risk and correlation of the underlying credit facilities. In our case, the SPV buys sovereign bonds. Since the ratings of the tranches are different, their cash flow from interest payments are also different. The highest-rated tranche typically receives a rating of AAA - the best possible rating - and faces the lowest interest payments. The lowest rated tranche has to pay the highest interest rate. Investors in the lowest-rated part of the CDO are first to suffer losses, whereas investors in the AAA part are last to do so. The lowest rated tranche, called “equity tranche”, often remains in the holdings of the SPV.

A deeper explanation of CDOs, their pricing and the correlation effects can be found by Longstaff and Rajan (2008), Coval et al. (2009), and Coval et al. (2009b). An exact clarification of structured Eurobonds in a SPV, as well as the cascading effects, can be found by Hild et al. (2014).

A.2 Financial Data

Country	Interest Rate 2008	Interest Rate 2012	Interest Rate 2018
Belgium	4.63%	2.05%	0.70%
Germany	4.01%	1.31%	0.33%
Estonia	4.32%	2.41%	0.78%
Ireland	4.69%	4.56%	0.86%
Greece	4.94%	11.92%	4.40%
Spain	4.59%	5.27%	1.49%
France	4.34%	2.00%	0.69%
Italy	4.88%	4.51%	3.24%
Cyprus	4.60%	7.00%	2.30%
Latvia	6.60%	3.24%	0.80%
Lithuania	6.11%	4.00%	1.20%
Luxembourg	4.84%	1.43%	0.47%
Malta	4.82%	3.79%	1.50%
Netherlands	4.34%	1.49%	0.45%
Austria	4.44%	1.75%	0.55%
Portugal	4.72%	7.05%	1.92%
Slovenia	5.21%	5.03%	0.95%
Finland	4.37%	1.51%	0.52%
Slovakia	4.77%	3.92%	0.79%

Table A1: Interest Rates per country

A.3 Optimal degrees of joint liability

A crucial and very much political topic is the choice of the initial trust fund volume or, to put it precisely, the trust fund's percentage share of the whole nominal volume. The choice displays the joint liability every country is willing to enter. We examine the effects of different trust fund shares on the nominal gain as well as the interest rate differential of the complete EMU. At first, we have a look at three different possible trust fund shares - 5%, 10% and 20% - and their impact on the nominal gains. The results are presented in Figure A1. The gain can be understood as a function of the degree of joint liability since a higher joint liability is connected with a higher trust fund share and a higher trust fund share is equivalent to a better average rating of the structure. The attributable profit resulting from this connection is also highly dependent on the chosen initial trust fund share. On the other hand, political costs are also directly related to joint liability. Higher liability delivers higher moral hazard. In this case, we assume an issuing of 10% of the GDP with structured Eurobonds. The effects of variations in the risk-free interest rate and recovery rates are the same for all three possibilities of trust fund shares. The gain reaches from 3.63 billion Euro in a

5% trust fund scenario with 0% recovery rate and 6% risk-free interest rate to 69.46 billion Euro with a 5% trust fund, 80% recovery rate and 6% risk-free interest rate. It is remarkable that the lowest, as well as the highest gains, are achieved with a trust fund share of 5%. We can see this in Figure A1 where the other two layers are not as sensitive to the variables “risk-free interest rate” and “recovery rate” as the 5% layer.

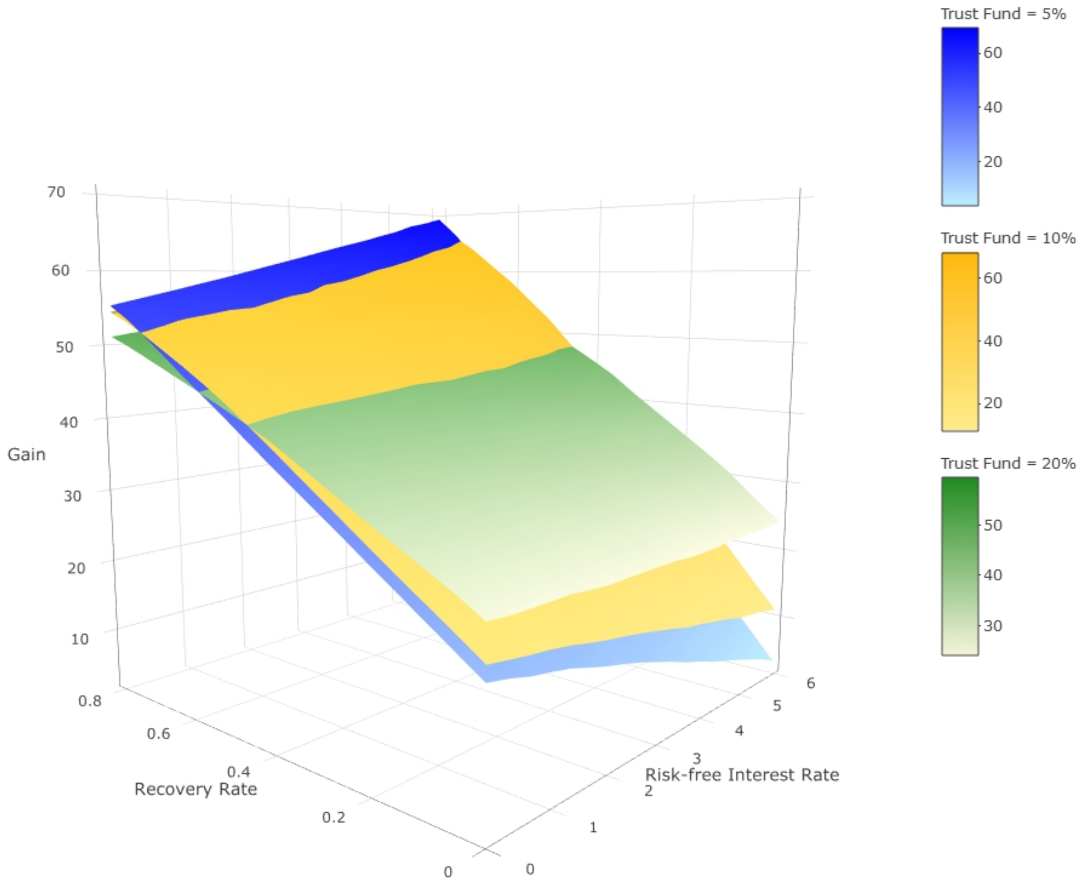

Figure A1: Nominal gains in a structured Eurobonds scenario with different trust fund shares over a grid of several recovery values as well as risk-free interest rates.

We have to split our observations into different areas of interest. When we face recovery rates below 60%, the gains depend strongly on the initial trust fund share. Here we can conclude that the higher the trust fund share is, the higher the profits are. As long as the recovery value is between 60% and 75%, structured Eurobonds with an initial trust fund of 10% will deliver the highest gains. At last, when recovery values are above the 75% threshold the 5% trust fund will generate the highest profits. This can be explained by a declining effect of the trust fund rate on the average rating of the structure. Also, the interest rate burden is growing for every country due to their payment on the total debt. Notably, the results for a low risk-free interest rate are close to each other whereas they diverge stronger for higher values in the interest rate. The differences for low-interest rates range from 4.19 to 6.31 billion Euro. These differences are an indication that the system is not sensitive to the trust fund share in a low-interest rate environment. The 5% trust fund share is relatively sensitive to a change in the recovery value or the risk-free interest rate. The results mentioned above can be combined with Figure A2.

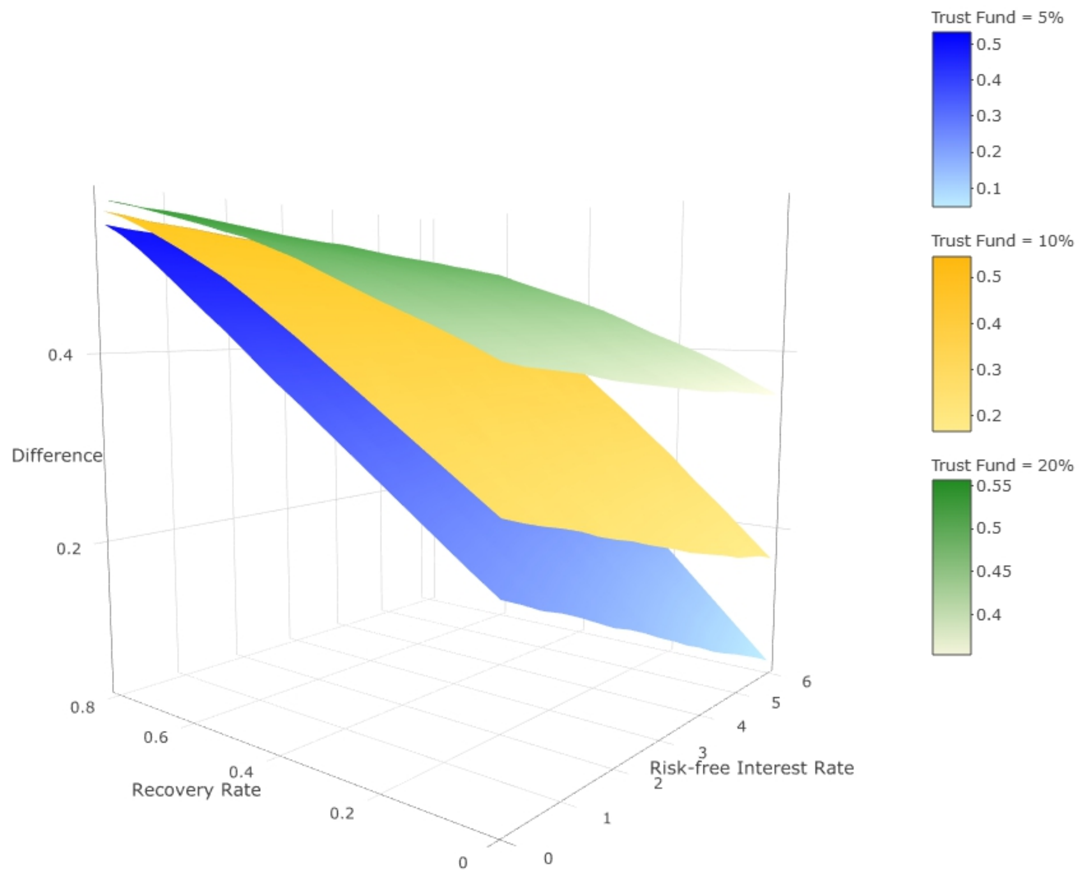

Figure A2: Interest rate difference in a structured Eurobonds scenario with different trust fund shares over a grid of several recovery values as well as risk-free interest rates.

We can see that the topmost layer, which is representing the 20% initial trust fund rate, has the flattest growth and the lowest layer (5%) the steepest growth. It is noticeable that the difference is decreasing for higher recovery rates with a fixed risk-free interest rate. This decreasing difference is due to converging structures for different trust fund rates with growing recovery rates. The 20% trust fund layer is the topmost because the reached structure is better than for the 10% case. As a result, the interest rate from the SPV to the investors is lower and the capital inflows from the EMU countries to the SPV stay the same. Besides, it is relevant to remark that for lower recovery rates, especially below a rate of 40%, the jump in the interest difference from a 5% to 10% trust fund share is lower than from a 10% to a 20% share. This is in line with the results from Figure A1. For higher recovery rates, e.g. 65%, it is the reverse and the jump in the interest difference from 5% to 10% is higher than from 10% to 20%. The first increase in the trust fund share from 5% to 10% is enough to offset the cost of joint liability, which is reflected in the higher interest payments from every country, against the higher interest payments of the SPV. The second jump in the trust fund share from 10% to 20% is not enough to offset the additional payments of every country against the advantages of a larger joint liability. Due to this, the gains for a 10% trust fund share are above the ones in a 20% scenario. We also have to mention that the interest differences are converging for higher recovery values. This can be explained by a declining effect of structuring in the ABS-approach and a declining effect of the trust fund.

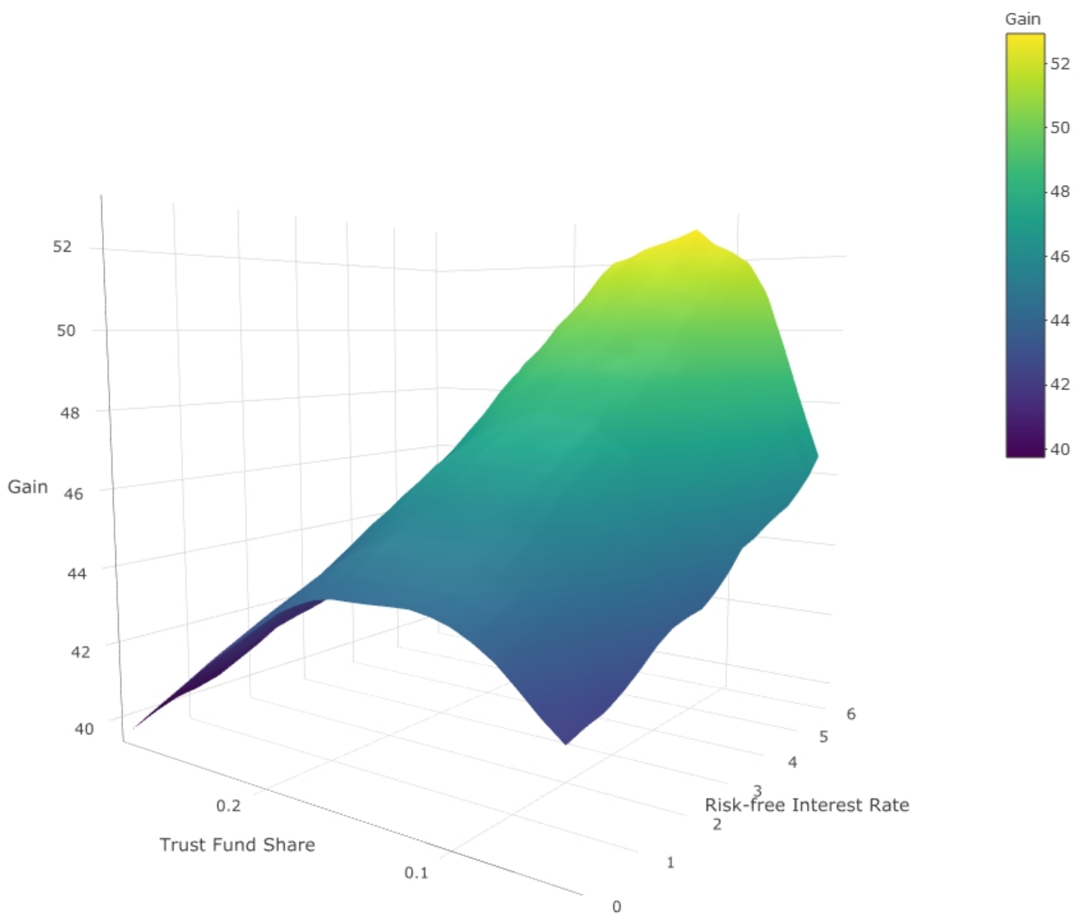

Figure A3: Sensitivity of gains for different trust fund shares in a fixed recovery rate scenario where it is fixed to 50%.

Now we want to analyse at which trust fund rate we can reach the maximum gain when the recovery rate is fixed. We start with a fixed recovery rate of 50% and initial trust fund shares between 5% and 30%. The result can be seen in Figure A3.

We can conclude that the gain is a parabola function of the trust fund share. The gains are rising from a trust fund share of 5% until we reach a maximum of 52.94 billion Euro with an initial trust fund of 12% and a risk-free interest rate of 6%. Afterwards, the gains are declining for higher trust fund shares. It is noticeable that the decline is getting stronger, the larger the initial trust fund is. For comparison, we take a look at a trust fund of 30% and a risk-free interest rate of 6%. Here, the gain is 42.48 billion Euro which is about 10 billion Euro or 20% smaller than in the optimal scenario. We also want to examine the profits arising from higher political costs which is equivalent to higher trust fund shares and common liability every country wants to enter. In a 5% trust fund scenario with a risk-free interest rate of 6%, the gain is 46.56 billion Euro. In our optimal scenario, that has a higher common liability and therefore higher political costs, the gains are about 6 billion Euro or 13% higher. Also, we want to evaluate the results for a lower risk-free interest rate with a value of 0%. The results are staying the same with an optimal gain for a trust fund of 12%, but the sensitivity is lower. The optimal gain is 45.31 billion Euro in this case and declines to 40.30 billion Euro or about 11% for the maximal observed trust fund choice of 30%. In a low trust fund case of 5% the gains are 42.66 billion Euro which is 2.65 billion Euro or 6% lower than in the optimal case. The results indicate that a higher common liability can have positive effects on the gains of structured Eurobonds despite the political costs that might arise.

The optimal choice can be seen in Figure A7. It starts at 12% in a 0% interest rate scenario and jumps to 16% at 1% interest rate. Afterwards, it is again steadily declining down to 12% which is reached at a risk-free interest rate of 5.6%.

The results are also supported for other recovery rates. The structure of the gains can be seen in Figures A4 and A5, and the optimal choice of the initial trust fund size in Figures A6 and A8.

Again, the connection between the trust fund share and the gains is parabola-like. It is noticeable that the optimal trust fund share is changing for different recovery values. When the recovery rate is fixed at 40% the optimal trust fund is between 14% and 18% of the issued nominal volume. When the recovery rate grows to 60%, the picture is changing with an optimal share between 9% and 13%. From these findings, we can conclude a negative connection between the fixed recovery rate and the optimal trust fund share, representing the common liability.[8] We can explain this connection

[8]The link is also supported for other fixed recovery rates. They can be delivered upon request.

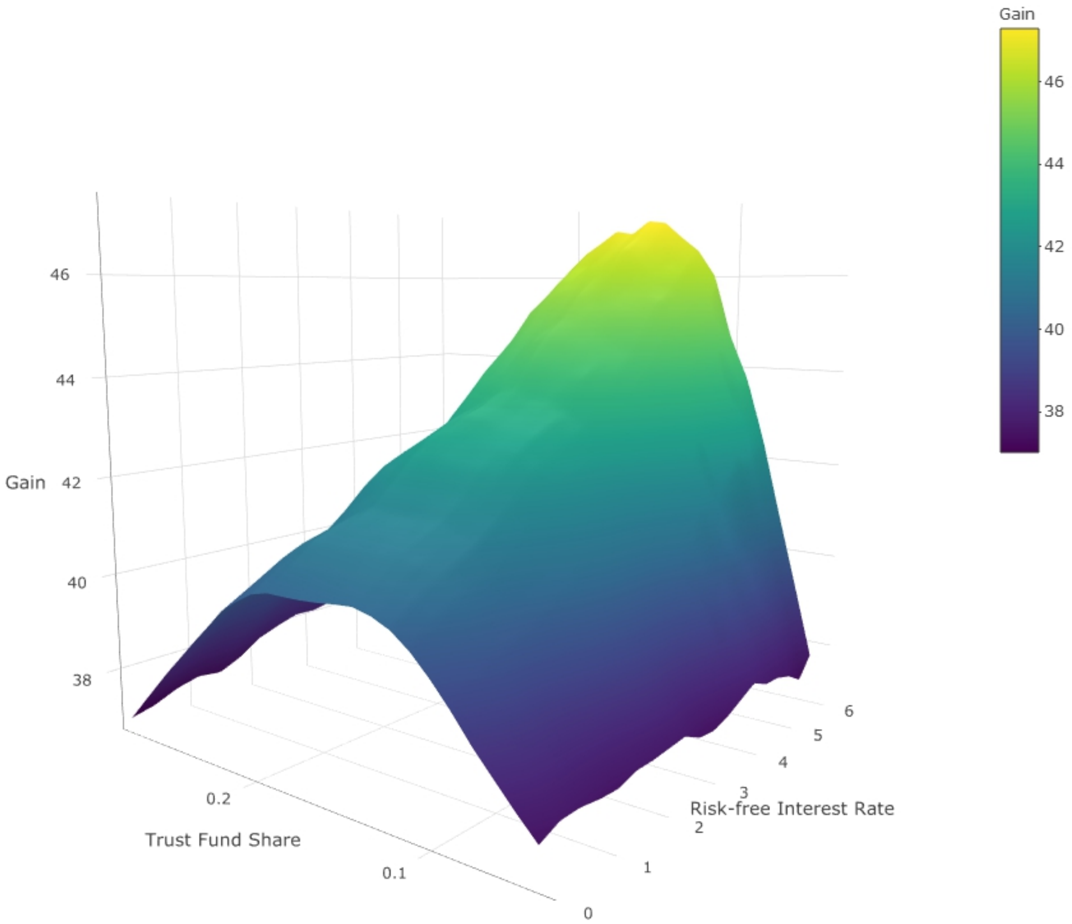

Figure A4: Sensitivity of gains for different trust fund shares with a fixed recovery rate scenario at 40%.

with a higher average rating resulting from structuring. Due to this, the impact of the trust fund share on the average rating is highly dependent on the recovery rate. The higher the recovery rate is, the lower the trust fund share is allowed to be to reach the optimal outcome. A further increase of the trust fund share brings with it an additional interest burden that is higher than the advantage from a better rating.

We start with a more in-depth examination of the results for a fixed recovery rate of 40%. As mentioned before, the optimal share is around 15% and we can see that the impact of the right choice of the initial trust fund share can be crucial. A lower trust fund with an initial volume of 5% delivers gains which are about 10 billion Euro or 21% lower than in the optimal scenario with gains of 47.28 billion Euro in the highest interest rate case. When the initial trust fund rises to 30%, we face a drop in the profits to 40.2 billion Euro, which is equivalent to a decline of 7.08 billion Euro or nearly 15%. The effects in a lower interest rate scenario are the same but not as strong

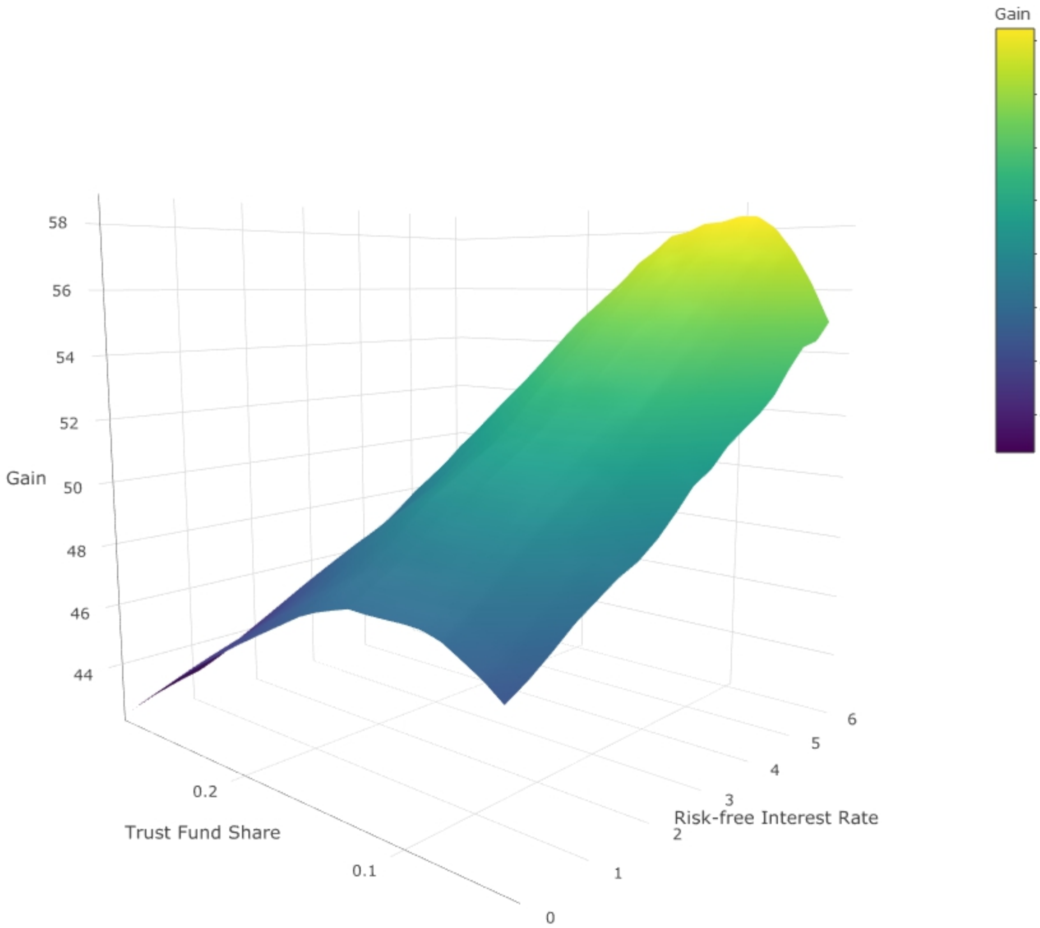

Figure A5: Sensitivity of gains for different trust fund shares with a fixed recovery rate scenario at 60%.

as in the recently discussed case. The optimal gain is now 40.45 billion Euro and the decline on both ends is about 3 billion Euro which is equivalent to 7.5%.

In a 60% recovery rate scenario, the gains for higher common liability do generate an advantage of 3.43 billion Euro or 6.2% when the trust fund grows from 5% to 10%. On the other hand, when the common liability grows above 10% to 30%, the gains decrease from 58.45 billion Euro to 44.81 billion Euro, which is equivalent to a decline of 23.4%. This occurs at a risk-free interest rate of 6%. Again, the differences in the results for a low-interest rate scenario are not as large when compared to the higher interest rates. The gains resulting from a higher joint liability, which is equivalent to higher political costs, rise from 46.75 to 47.99 billion Euro which is a rise of 2.7%. Also, when the common liability grows above this optimal share of 10% to 30%, the gains decline to 42.58, which is equivalent to a decrease of 11.3%.

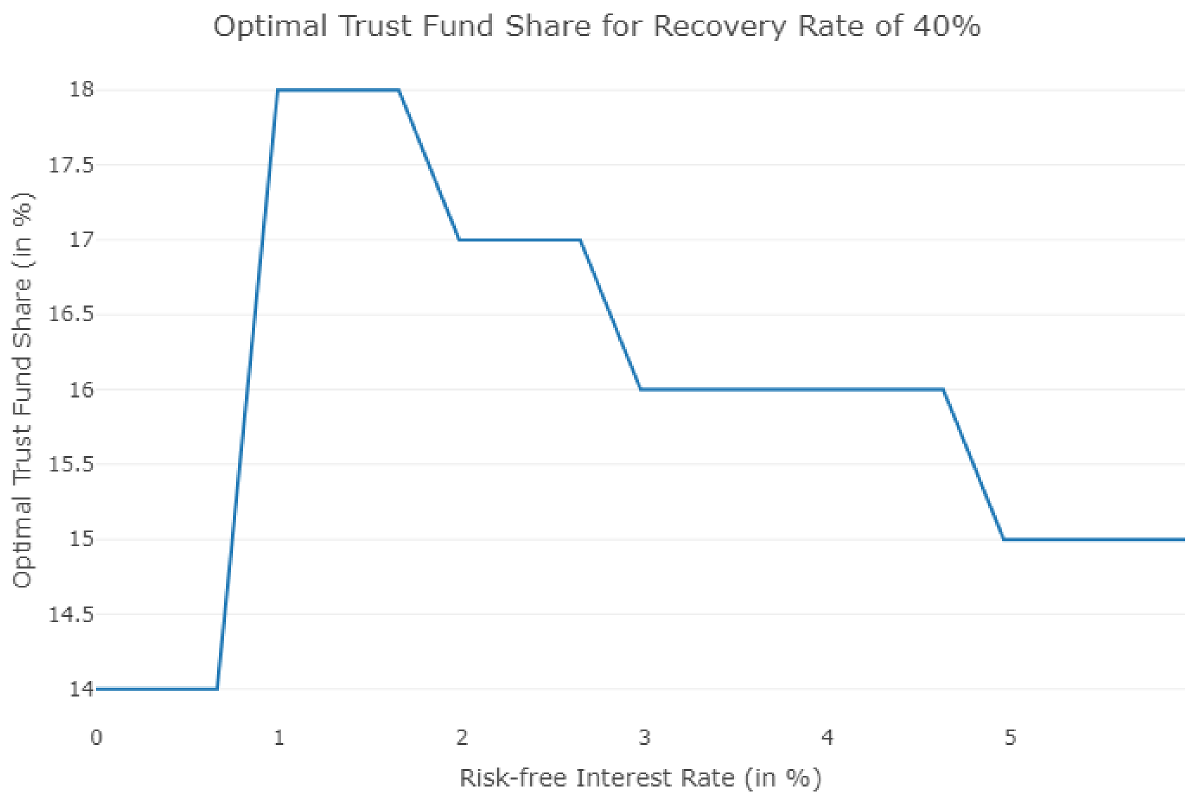

Figure A6: Sensitivity of the optimal initial trust fund share in respect to the risk-free interest rate with a fixed recovery rate of 40%.

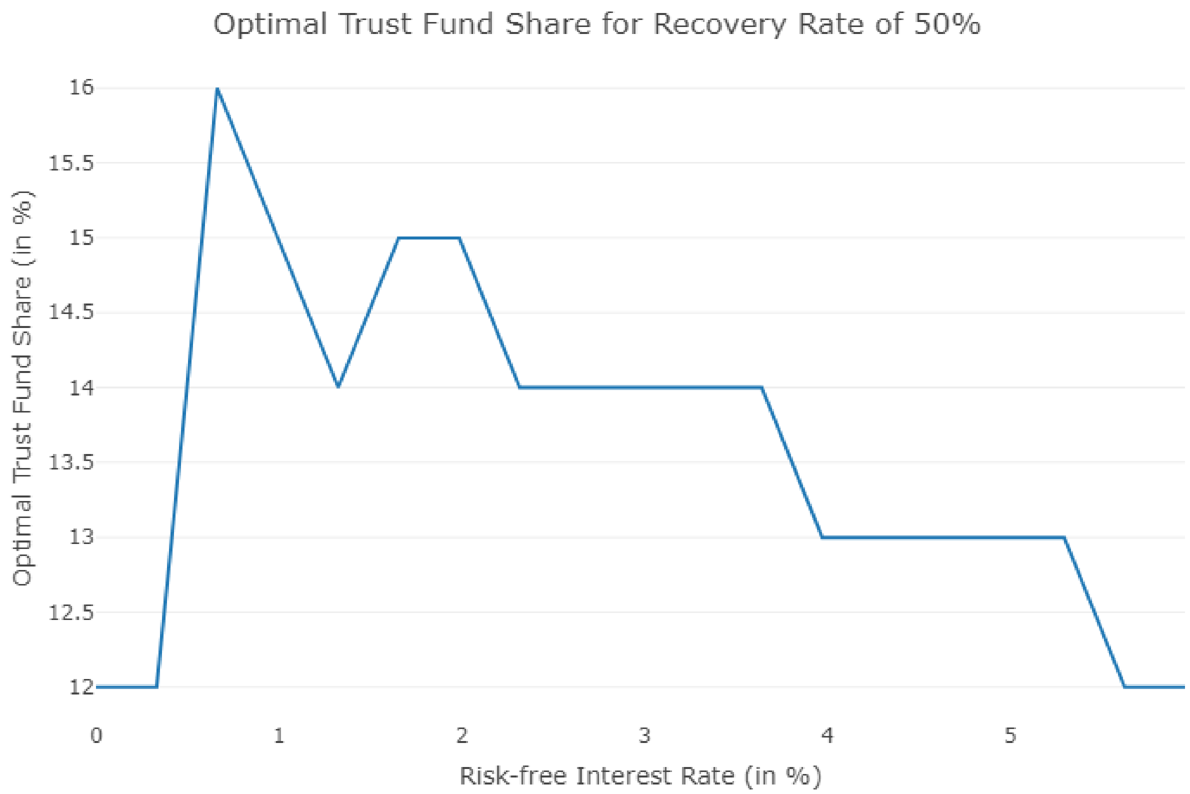

Figure A7: Sensitivity of the optimal initial trust fund share in respect to the risk-free interest rate with a fixed recovery rate of 50%.

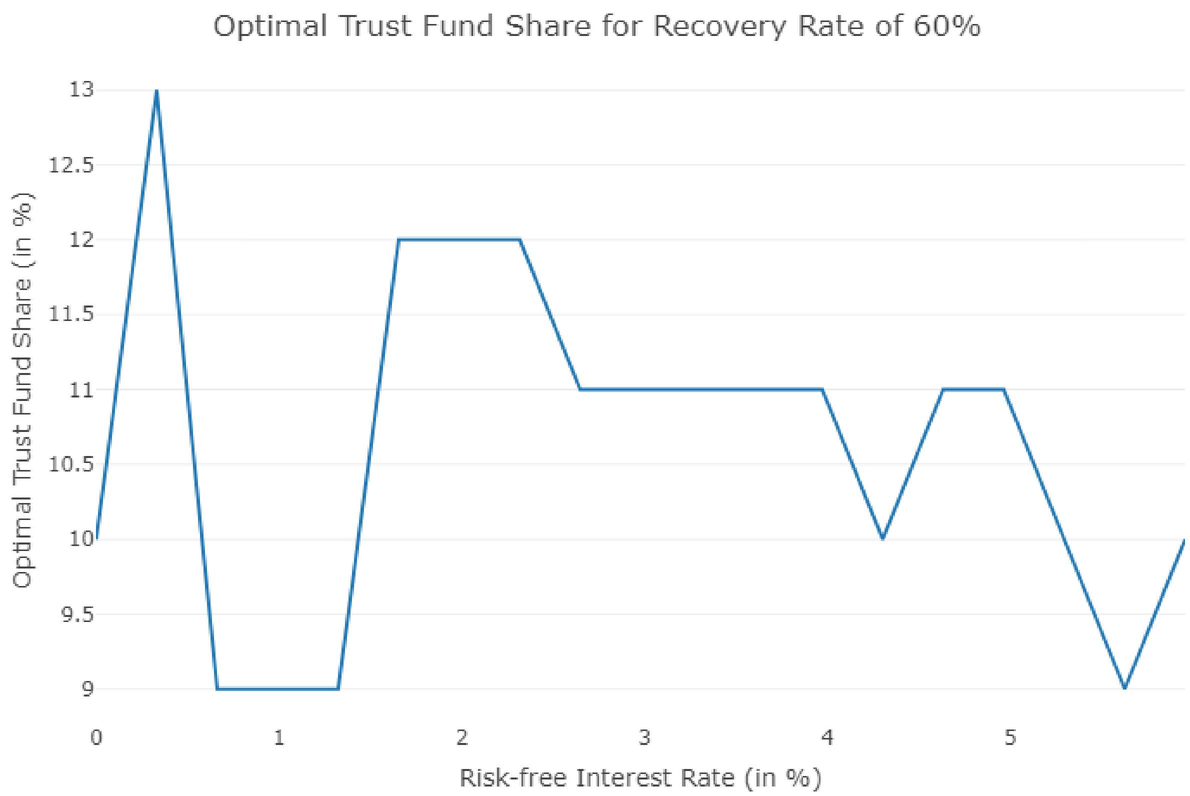

Figure A8: Sensitivity of the optimal initial trust fund share in respect to the risk-free interest rate with a fixed recovery rate of 60%.

Summarised, this points at a crucial political question of how large the initial trust fund has to be chosen. The larger the initial trust fund, the larger is the common liability, but due to a decreasing effect of improvement in the average rating of the structure, the effect of a rising trust fund share declines. The gains do not rise infinitely when the common liability increases. It can be seen that the profits are more sensitive to changes in the parameters the lower the common liability is. Also, the gains depend on recovery values. When we focus on the results for an optimal trust fund share with a fixed recovery rate, the gains are higher for higher recovery values whereas the optimal trust fund share is lower. This is a direct consequence of higher nominal volumes that flow into the trust fund after default. When we combine these insights, an optimal trust fund share would be between 10% and 15% with lower sensitivity in a low risk-free interest rate scenario.

A.4 EMU-wide introduction

Now we assume an introduction of structured Eurobonds where every member country is participating in August 2018, a time of low-interest rates and yields at the sovereign bond markets. Table A2 displays the resulting structure from the Monte Carlo simulation. This structure stays the same independent from the distribution scheme. It only changes when the data or considered countries change. The AAA tranche represents the main part of the structure. Here, the advantages of the trust fund and correlation effects can be seen. With the end of July ratings, we have three countries with an AAA rating which are representing 36.27% of the GDP of the whole EMU. In this structure, the AAA-part is nearly 74%, more than double the volume. Also, the lowest-rated tranche receives an A rating whereas the lowest rating in the EMU is at B.

Tranche	Thickness	Rating	Interest Rate
Tranche I	73.99%	AAA	0.56%
Tranche II	4.15%	AA+	0.65%
Tranche III	13.81%	AA	0.85%
Tranche IV	2.24%	AA-	0.99%
Tranche V	3.63%	A+	1.17%
Tranche VI	2.14%	A	1.32%

Table A2: The structure for an introduction of structured Eurobonds for the whole EMU. The first column displays the thickness of the tranche, the second the rating, and the third shows the interest rate that is paid by every tranche to the capital market.

We want to have a closer look at the two distribution schemes where the extra costs and the initial funding are subtracted from the trust fund before distribution and the remaining capital is distributed with an even or a relative distribution. We again assume an issuing of 10% of the GDP with structured Eurobonds. This delivers a total volume of 1,240.81 billion Euro and an AAA-rated nominal of 918.08 billion Euro. For comparison, the total nominal volume of AAA-rated debt in the EMU is 2,540.9 billion Euro for the end of June 2018. The results of the Monte Carlo simulation and the distribution methods can be seen in Table A3. In column 2, the portion of every country of the whole nominal can be seen, and column 3 displays the nominal net gain after subtraction of the extra costs in billion Euro. The next columns are showing the gain in relation to the nominal in total and per year. In parenthesis are the results for the even distribution.

In a relative distribution scheme, there are some advantages for lower-rated countries. On the other hand, this changes when the distribution scheme changes to an even distribution. For better comparison, we use as a reference Germany, France, and Italy. With a change of the distribution method, the nominal net gain rises from 2.62 to 12.59 (4.83 to 8.92) for Germany (France) which is equivalent to an increase of 2.75 (1.61) percentage points relative to nominal. The gain for Italy falls from 21.62 to 7.18 billion Euro. This decline is equivalent to a decrease of 7.57 percentage points. On the other hand, the yearly interest savings for Italy are 1.08% in the relative distribution.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	3.95%	0.94 (1.70)	1.93% (3.50%)	0.19% (0.34%)
Germany	29.26%	2.62 (12.59)	0.72% (3.47%)	0.07% (0.34%)
Estonia	0.24%	0.06 (0.09)	2.27% (3.60%)	0.22% (0.35%)
Ireland	2.66%	0.81 (1.16)	2.46% (3.51%)	0.24% (0.35%)
Greece	1.61%	3.12 (0.73)	15.82% (3.67%)	1.46% (0.36%)
Spain	10.40%	5.98 (4.63)	4.62% (3.58%)	0.45% (0.35%)
France	20.47%	4.83 (8.92)	1.90% (3.51%)	0.19% (0.35%)
Italy	15.39%	21.62 (7.18)	11.33% (3.76%)	1.08% (0.37%)
Cyprus	0.16%	0.16 (0.08)	7.68% (3.70%)	0.79% (0.37%)
Latvia	0.24%	0.07 (0.11)	2.35% (3.58%)	0.23% (0.35%)
Lithuania	0.40%	0.17 (0.17)	3.67% (3.61%)	0.34% (0.36%)
Luxembourg	0.48%	0.07 (0.21)	1.15% (3.49%)	0.12% (0.34%)
Malta	0.08%	0.06 (0.05)	4.78% (3.73%)	0.46% (0.37%)
Netherlands	6.53%	0.91 (2.84)	1.12% (3.49%)	0.11% (0.35%)
Austria	3.30%	0.59 (1.43)	1.44% (3.49%)	0.14% (0.34%)
Portugal	1.69%	1.33 (0.78)	6.21% (3.65%)	0.62% (0.36%)
Slovenia	0.40%	0.14 (0.18)	2.89% (3.64%)	0.28% (0.36%)
Finland	2.01%	0.33 (0.87)	1.33% (3.48%)	0.13% (0.34%)
Slovakia	0.73%	0.22 (0.34)	2.30% (3.59%)	0.24% (0.35%)

Table A3: The results for an introduction of structured Eurobonds in the whole EMU. The first column displays the stake every country has at the nominal volume, the second the nominal net gain, the third shows the nominal net gain in relation to the debt, and the last column the yearly savings from the third column. In parenthesis are the results for the even distribution.

This is equivalent to a reduction of the effective interest rate from 3.24% to 2.16%. In the even distribution scheme, the yearly savings are only 0.37%, but they are growing for a high rated Germany from 0.07% to 0.34% p.a. This would turn the effective interest rate for Germany from 0.33% into -0.01%, a negative interest rate. This effect is a result of the different interest rates paid by the SPV and the countries. The SPV has to pay 0.65% to the investors, whereas the countries are on average paying 1.13% to the SPV. A part of the difference is used to pay the additional costs resulting from the higher nominal volume. The remaining cash is transferred into the trust fund where it is interest-paying with the risk-free interest rate. All countries can have gains in an environment of low-interest rates.

In the next step, we want to show that the ABS-structure is stable for other dates of introduction. We start by focussing on the end of 2012, a peak of the European sovereign debt crisis. Naturally, the default probabilities of nearly all countries are higher and the risk-free interest rate is higher at 1.31% compared to 0.33% in 2018. We use the same assumptions as before with 10% of GDP issuing and 10% initial trust fund volume. Due to different values of the default probability, the structure changes. This can be seen in Table A4.

Tranche	Thickness	Rating	Interest Rate
Tranche I	47.89%	AAA	2.06%
Tranche II	4.88%	AA+	2.11%
Tranche III	13.62%	AA	2.21%
Tranche IV	8.46%	AA-	2.29%
Tranche V	7.73%	A+	2.39%
Tranche VI	1.84%	A	2.48%
Tranche VII	3.35%	A-	2.66%
Tranche VIII	6.66%	BBB+	2.95%
Tranche IX	5.55%	BBB	3.26%

Table A4: The structure for an introduction of structured Eurobonds for the whole EMU in 2012. The first column displays the thickness of the tranche, the second the rating, and the third shows the interest rate that is paid by every tranche to the capital market.

In comparison to the results for 2018, we can see that the number of tranches grows from six to nine with a sharp drop in the thickness of the AAA tranche. This drop is a result of higher CDS-spreads and finally higher default probabilities. The interest rate of every tranche grows due to a higher risk-free and individual interest rate.

Noticeable is that the thickness of smaller tranches is growing compared to the evaluation for 2018. In 2018 a considerable part of the nominal volume is concentrated in the AAA and AA tranches with approximately 88%. Now the AAA and AA tranche only represent nearly 62%, a decline of 26 percentage points. The gains for every country with the two distribution schemes can be seen in Table A5.

At first, we ascertain that the gains remain positive for every country. The stakes change but the differences are negligible. The profits are more diversified in the relative distribution than in the first case. This is a result of a higher level of the risk-free interest rate and more uniform interest spreads that have to be paid by the countries. As a result, higher-rated countries such as Germany have received higher net gains. Nonetheless countries with problems in the sovereign debt crisis, e.g. Greece and Portugal, have more significant gains than in the first case. This can be seen in the relative net gain which is about 20.97% for Greece and 10.32% for Portugal compared to 15.82% (6.21%) for Greece (Portugal) in 2018. The effective interest rate for Portugal is reduced by 0.97% per year. As before the gains are shifting when we use the even distribution. Higher rated countries now face higher net gains and the interest rate reduction per year is 0.33%. This will reduce the interest burden for Germany from 1.31% to 0.98%.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	3.93%	0.84 (1.43)	1.94% (3.31%)	0.19% (0.33%)
Germany	28.00%	3.10 (10.13)	1.01% (3.31%)	0.10% (0.33%)
Estonia	0.18%	0.05 (0.07)	2.46% (3.36%)	0.24% (0.33%)
Ireland	1.83%	1.14 (0.70)	5.83% (3.58%)	0.55% (0.35%)
Greece	1.92%	4.46 (0.76)	20.97% (3.57%)	1.94% (0.35%)
Spain	10.61%	8.28 (4.32)	7.16% (3.74%)	0.69% (0.37%)
France	21.23%	4.44 (7.76)	1.91% (3.35%)	0.19% (0.33%)
Italy	16.38%	10.59 (6.71)	5.91% (3.74%)	0.58% (0.37%)
Cyprus	0.18%	0.21 (0.08)	9.70% (3.56%)	0.95% (0.35%)
Latvia	0.18%	0.09 (0.09)	3.71% (3.46%)	0.37% (0.34%)
Lithuania	0.37%	0.18 (0.13)	4.78% (3.40%)	0.44% (0.34%)
Luxembourg	0.46%	0.06 (0.16)	1.16% (3.31%)	0.11% (0.33%)
Malta	0.09%	0.04 (0.03)	4.77% (3.77%)	0.37% (0.37%)
Netherlands	6.59%	0.88 (2.36)	1.23% (3.30%)	0.12% (0.32%)
Austria	3.20%	0.55 (1.16)	1.55% (3.29%)	0.16% (0.32%)
Portugal	1.74%	1.93 (0.70)	10.32% (3.73%)	0.97% (0.37%)
Slovenia	0.37%	0.27 (0.15)	6.74% (3.71%)	0.66% (0.36%)
Finland	2.01%	0.28 (0.73)	1.24% (3.28%)	0.13% (0.32%)
Slovakia	0.73%	0.38 (0.28)	4.66% (3.40%)	0.46% (0.34%)

Table A5: The results for an introduction of structured Eurobonds in the whole EMU in 2012. The first column displays the stake every country has at the nominal volume, the second the nominal net gain, the third shows the nominal net gain in relation to the debt, and the last column the yearly savings from the third column. In parenthesis are the results for the even distribution.

Now we want to evaluate the third scenario with an introduction at the end of 2008 when the financial crisis was at its peak. As before the default probabilities of the countries are changing and the ABS have a new structure. This can be seen in Table A6.

We now have eight tranches and all are rated BBB or better, which is equivalent to an investment-grade rating for every tranche. The nominal debt is 1,070.67 billion Euro. We also have a sharp rise in the risk-free interest rate up to 1.58%[9] and the different tranches also have higher interest rate burdens. The risk-free interest rate is now chosen as the yield of the 10 year US-bond because it is lower than the German yield and displays a lower risk. Besides, the default probability of the participating countries cannot be calculated with the CDS-spreads for every country due to the market turmoils in the financial crisis. In these cases, we again use an approach of Sturzenegger and Zettelmeyer (2010) who calculate the default probability with the yield spread

[9] The interest rate is adjusted with the CDS-spread for the US due to the financial crisis and to wipe out the risk premium.

Tranche	Thickness	Rating	Interest Rate
Tranche I	39.50%	AAA	2.57%
Tranche II	6.44%	AA+	2.70%
Tranche III	12.26%	AA	3.01%
Tranche IV	13.81%	AA-	3.23%
Tranche V	6.78%	A+	3.52%
Tranche VI	6.37%	A	4.35%
Tranche VII	9.91%	A-	5.24%
Tranche VIII	4.93%	BBB	6.10%

Table A6: The structure for an introduction of structured Eurobonds for the whole EMU in 2008. The first column displays the thickness of the tranche, the second the rating, and the third shows the interest rate that is paid by every tranche to the capital market.

of every country over the risk-less yield. The results of the Monte Carlo simulation can be seen in Table A7.

In comparison to the first two scenarios (introduction in 2012 and 2018), the average effect is lower. It is noticeable that the minimum gain is 1.57% for Germany in a relative distribution scheme which is higher than in the two cases before. On the other hand, the highest profit is 4.73% for Latvia compared to 15.82% and 20.97% for Greece in 2018 and 2012, respectively. The gains are better distributed, independent of the chosen distribution method. It can be seen that only Lithuania and Latvia have gained above 4% in a relative distribution. These small average gains are a result of well-distributed credit spreads and similar default probabilities of the participating countries. The capital inflows from the countries to the SPV, which are only determined by government bond yields, reach 3.66%. On the other hand, the capital outflows from the SPV, which are determined by CDS-spreads, to the capital market are 3.20%, which is only 0.46% below the inflows.

We draw two crucial conclusions. Firstly, a long-term refinancing through an ABS-approach in this way might not be optimal in a crisis. Short-term refinancing could be more efficient. Secondly, the trust in structured Eurobonds or the assigned issuer risk determines the effectivity and stability of the approach. Introducing Eurobonds by a supranational organisation such as the ESM, ECB or a newly formed institution would eliminate this market risk. Thus, interest rates paid by the SPV to the capital market would be determined by sovereigns that have the same rating and significantly reduce the structures interest outflow.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	3.64%	0.89 (0.86)	2.27% (2.20%)	0.23% (0.22%)
Germany	26.62%	4.48 (6.03)	1.57% (2.12%)	0.16% (0.21%)
Estonia	0.19%	0.06 (0.04)	3.32% (2.34%)	0.30% (0.21%)
Ireland	1.96%	0.64 (0.51)	3.04% (2.43%)	0.30% (0.24%)
Greece	2.52%	0.99 (0.65)	3.69% (2.43%)	0.36% (0.24%)
Spain	11.58%	2.93 (2.79)	2.36% (2.25%)	0.23% (0.22%)
France	20.74%	4.27 (4.72)	1.93% (2.13%)	0.19% (0.21%)
Italy	16.91%	5.43 (4.39)	2.99% (2.42%)	0.30% (0.24%)
Cyprus	0.18%	0.07 (0.05)	3.13% (2.37%)	0.33% (0.25%)
Latvia	0.28%	0.13 (0.06)	4.73% (2.37%)	0.42% (0.21%)
Lithuania	0.37%	0.15 (0.09)	4.02% (2.40%)	0.37% (0.22%)
Luxembourg	0.37%	0.13 (0.09)	2.95% (2.17%)	0.30% (0.22%)
Malta	0.09%	0.02 (0.02)	3.38% (2.35%)	0.23% (0.16%)
Netherlands	6.63%	1.59 (1.66)	2.24% (2.33%)	0.22% (0.23%)
Austria	3.08%	0.83 (0.78)	2.53% (2.40%)	0.25% (0.24%)
Portugal	1.87%	0.49 (0.44)	2.46% (2.23%)	0.24% (0.22%)
Slovenia	0.37%	0.15 (0.09)	3.44% (2.21%)	0.36% (0.23%)
Finland	2.06%	0.46 (0.46)	2.13% (2.11%)	0.21% (0.20%)
Slovakia	0.65%	0.23 (0.17)	3.12% (2.33%)	0.32% (0.24%)

Table A7: The results for an introduction of structured Eurobonds in the whole EMU in 2008. The first column displays the stake every country has at the nominal volume, the second the nominal net gain, the third shows the nominal net gain in relation to the debt, and the last column the yearly savings from the third column. In parenthesis are the results for the even distribution.

A.5 Country subsets

In the last chapter, we have required that every country of the EMU participates in the structured Eurobonds programme. Since some countries might disagree to participate, due to country-specific jurisdiction or other reservations, we have a look at a country subset to work out their possible advantages of this programme. We focus on four different scenarios. We start by evaluating the advantages or disadvantages if a single country drops out. In the first scenario, we establish Eurobonds without Italy. This shall contribute to the discussion about the current uncertainty about Italian debt and government deficit. Also, the growth in Italy has been down in the last years. Thus, the government deficit can be a crucial factor for non-participating. In the second scenario, we analyse the EMU without Germany. The primary purpose of this construction is to see whether Germany can draw direct financial advantages from participation. It is inspired by the resistance of German authorities against common Eurobonds. For better comparison, we only evaluate the scenario of 2018.

We start with a scenario where all countries are participating, besides Italy. This is close to the case that was discussed in the previous chapter. Since a country with a lower rating and higher default probability is not in the issuance scheme, we can expect a better overall rating in the structure than in the case with all EMU countries. The results of the Monte Carlo simulation support this. The resulting structure can be seen in Table A8. When we compare this with the structure in Table A2, we can see that the thickness of the tranches with a rating of AA or better is increasing and the thickness of the lower-rated tranches is decreasing. In addition, the A tranche drops. As a result, we receive a better average rating. On the other hand, the complete volume drops from 1,240.8 billion Euro to 1,050 billion Euro with an absolute decline from 916.5 billion Euro to 790.3 billion Euro in the AAA-tranche from the whole EMU to this scenario.

Tranche	Thickness	Rating	Interest Rate
Tranche I	75.27%	AAA	0.56%
Tranche II	5.83%	AA+	0.65%
Tranche III	17.44%	AA	0.85%
Tranche IV	1.10%	AA-	0.99%
Tranche V	0.36%	A+	1.17%

Table A8: The structure for an introduction of structured Eurobonds for EMU countries ex Italy.

We can see in the results, which are displayed in Table A9, that the gains for all countries are strongly declining although the overall rating is better. Nevertheless, every country has a nominal net gain. In this case, we focus on the impact on Germany and Greece, as they are representing a high and low rated country, and we compare it with the results from the whole EMU scenario from Table A3. The nominal net gain for Germany drops from 2.62 (12.59) billion Euro to 0.57 (2.13) billion Euro in a relative (even) distribution. This is a reduction of 10 billion Euro or 80% of the nominal gain in the even distribution. The interest rate reduction decreases from 0.07% (0.34%) in the whole EMU scenario to 0.02% (0.06%) for a relative (even) distribution in this scenario. We can conclude that the gains for Germany are even lower for their advantageous distribution where the nominal net gain is 2.13 billion Euro than for the relative distribution in the whole EMU scenario where they would receive 2.62 billion Euro.

The same effect can be seen for the gains of Greece. They are also reduced from 3.12 (0.73) billion Euro to 0.99 (0.15) billion Euro in the relative (even) distribution scheme, which is a drop of 70% (80%). The interest rate savings per year are reduced from 1.46% (0.36%) to 0.49% (0.07%). Contrary to the German case, Greece could have more gains in this subset and the relative distribution in comparison to the whole EMU scenario with an even distribution.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	4.67%	0.26 (0.29)	0.54% (0.60%)	0.05% (0.06%)
Germany	34.57%	0.57 (2.13)	0.16% (0.59%)	0.02% (0.06%)
Estonia	0.29%	0.02 (0.02)	0.70% (0.61%)	0.06% (0.06%)
Ireland	3.14%	0.24 (0.20)	0.72% (0.61%)	0.07% (0.06%)
Greece	1.90%	0.99 (0.15)	5.01% (0.74%)	0.49% (0.07%)
Spain	12.29%	1.83 (0.85)	1.42% (0.66%)	0.14% (0.07%)
France	24.19%	1.37 (1.56)	0.54% (0.61%)	0.05% (0.06%)
Cyprus	0.19%	0.05 (0.01)	2.34% (0.66%)	0.25% (0.07%)
Latvia	0.29%	0.02 (0.02)	0.74% (0.70%)	0.07% (0.07%)
Lithuania	0.48%	0.05 (0.03)	1.14% (0.69%)	0.11% (0.07%)
Luxembourg	0.57%	0.02 (0.04)	0.29% (0.59%)	0.03% (0.06%)
Malta	0.10%	0.02 (0.01)	1.46% (0.65%)	0.18% (0.07%)
Netherlands	7.71%	0.23 (0.49)	0.29% (0.60%)	0.03% (0.06%)
Austria	3.90%	0.16 (0.25)	0.39% (0.60%)	0.04% (0.06%)
Portugal	2.00%	0.41 (0.14)	1.90% (0.66%)	0.19% (0.07%)
Slovenia	0.48%	0.04 (0.03)	0.89% (0.71%)	0.09% (0.07%)
Finland	2.38%	0.09 (0.15)	0.35% (0.60%)	0.04% (0.06%)
Slovakia	0.86%	0.07 (0.07)	0.72% (0.69%)	0.08% (0.07%)

Table A9: The results for an introduction of structured Eurobonds in the whole EMU ex Italy. In parenthesis are the results for the even distribution.

Different reasons can explain the sharp decline of the gains. The main reason is that the average interest rate paid by the SPV does not significantly decrease due to the absence of Italy. The total interest paid is lower since the total volume is lower, but the capital inflows from Italy are missing which is roughly 40% of total interest inflows. The average interest rate paid by the SPV in the whole EMU scenario is 0.65%, which is nearly AA+. In the now viewed scenario, we have an average interest rate of 0.62%. So, the payments from the SPV to the capital market are only slightly lower. On the other hand, the capital inflows from all countries are significantly lower. If no country defaults, the average interest rate paid by the countries is 1.123% which is equivalent to capital inflows into the SPV of 14.82 billion Euro per year in the whole EMU scenario. In the scenario without Italy the average interest rate of the countries is only 0.745% representing 8.64 billion Euro. This is a decline of 6 billion Euro per year which is not transferred to the trust fund. We can conclude that Italy would face approximately 40% of the interest burden in the whole EMU scenario under the current circumstances. A minor issue is that the trust fund has a lower initial volume and due to this, the compound interest effect is weaker.

The combination of these two factors results in lower gains. In the whole EMU scenario we have a total net profit of approximately 43 billion Euro which drops to 7 billion Euro without Italy. We conclude that a common issuance with all countries is preferable and high rated countries can have higher gains if they work together with lower-rated countries.

Now, we focus on the case without Germany, the country with the lowest default probability. The effect is contrary to the one we have calculated for the issuance without Italy. The structure is getting worse with eight tranches ranging from AAA to BBB+. The main volume is concentrated in the AAA tranche with 60% and the whole amount is 878.21 billion Euro.[10] The average interest rate paid by the structure is 0.73%, a small increase from 0.65% paid in the whole EMU scenario. The main reason for the positive effect on the structure and afterwards on the gains can be found in the average interest rate paid by the countries to the SPV. The average interest is now at 1.46% compared with 1.13% in the whole EMU scenario. Due to the high inflows versus relatively low outflows, the trust fund stores a high volume and defaults can be better compensated.

This leads to the counterintuitive conclusion that a participation of Germany might be disadvantageous in a low-interest rate, relatively stable environment. The main reason is that although almost the entire German share increases the volume of the AAA tranche, the interest paid by Germany is still much less than the structure pays on its AAA tranche because there is an additional issuer risk included in the interest payments of the SPV. The net effect of German participation thus is negative. Also, the marginal effects of diversification and tranching are decreasing with the number of countries. Therefore, the positive effects induced by Germany are diminished by the large group. For political reasons and the market's perception of the structures credibility, an inclusion of Germany is mandatory.

We also evaluated two other subsets. These are the so-called PIIGS countries and EU6. They confirm the results we have seen so far.

In the next step, we have a closer look at the PIIGS countries introducing Eurobonds in 2018 and at the peak of the sovereign debt crisis in 2012. We start with the results for 2018.

The resulting structure can be seen in Table A10. Two interesting issues shall be mentioned. Although no involved country has an AAA rating, a tranche with this rating remains in the structure. Besides this, it is only 19 percentage points smaller than the one containing all EMU-countries. Second, the number of tranches increases

[10]The results can be presented upon request.

to eight with the second greatest thickness besides the AAA tranche concentrated in the lowest-rated tranche. The nominal total volume in this subset is 394.17 billion Euro.

Tranche	Thickness	Rating	Interest Rate
Tranche I	54.55%	AAA	0.56%
Tranche II	2.35%	AA+	0.65%
Tranche III	4.97%	AA	0.85%
Tranche IV	4.56%	AA-	0.99%
Tranche V	6.78%	A+	1.17%
Tranche VI	5.99%	A	1.32%
Tranche VII	7.01%	A-	1.54%
Tranche VIII	13.81%	BBB+	1.72%

Table A10: The structure for an introduction of structured Eurobonds for PIIGS countries in 2018.

The results for an even and relative distribution can be seen in Table A11. The results for all countries are getting better compared to a system involving all countries of the EMU. In this selection, Spain and Italy are representing more than 80% of the nominal volume.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Savings rel. to nom. p.a.
Ireland	8.37%	1.41 (4.17)	4.27% (12.67%)	0.42% (1.20%)
Greece	5.07%	4.47 (2.50)	22.65% (12.63%)	2.04% (1.18%)
Spain	32.73%	9.88 (16.47)	7.64% (12.73%)	0.74% (1.21%)
Italy	48.46%	32.71 (24.74)	17.15% (12.97%)	1.59% (1.23%)
Portugal	5.33%	2.15 (2.75)	10.02% (12.82%)	0.98% (1.24%)

Table A11: The results for an introduction of structured Eurobonds through government debt crisis countries in 2018. In parenthesis are the results for the even distribution.

Now, we focus on 2012 as the introduction year. This leads to the following structure displayed in Table A12.

The nominal volume drops to 354.25 billion Euro and we can see that the thickness of the AAA-tranche declines from 54.58% to 34.72% in this scenario compared to an introduction in 2018. Also, it is noticeable that the lowest-rated tranche is the second thickest, and nearly 40% of the whole nominal is concentrated in the two lowest-rated tranches with a rating of BBB+ and BBB.

The results can be seen in Table A13. For every distribution method, the relative gain is above 10% and the savings per year in relation to the nominal are consistently above 1.6%. The impact of the different distribution schemes is the same as above. In an even scheme, the countries with higher extra costs, here Greece and Italy, are

Tranche	Thickness	Rating	Interest Rate
Tranche I	34.72%	AAA	2.06%
Tranche II	5.35%	AA+	2.11%
Tranche III	5.14%	AA	2.21%
Tranche IV	0.14%	AA-	2.29%
Tranche V	5.21%	A+	2.39%
Tranche VI	4.66%	A	2.48%
Tranche VII	5.56%	A-	2.66%
Tranche VIII	14.84%	BBB+	2.95%
Tranche IX	24.39%	BBB	3.26%

Table A12: The structure for an introduction of structured Eurobonds for PIIGS countries in 2012.

having a higher gain than others. Contrary to this, we see in a relative distribution scheme that countries with lower extra costs have a higher relative gain.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Ireland	5.65%	3.43 (4.27)	17.58% (21.86%)	1.95% (2.08%)
Greece	5.93%	12.65 (4.29)	59.54% (20.19%)	4.82% (1.87%)
Spain	32.75%	24.48 (25.45)	21.19% (22.03%)	1.93% (2.00%)
Italy	50.53%	31.35 (39.49)	17.49% (22.03%)	1.61% (2.00%)
Portugal	5.36%	5.62 (4.03)	30.02% (21.54%)	2.62% (1.94%)

Table A13: The results for an introduction of structured Eurobonds through government debt crisis countries in 2012. In parenthesis are the results for the even distribution.

Now, we observe the effect of an issuance by the founding members of the European Union, the European Economic Community. These are Belgium, France, Italy, Luxembourg, Netherlands, and Germany. As before the structure, represented by the thickness of the tranches, is changing with a higher weighting of the high rated ones and the nominal volume is now 943.74 billion Euro. The structure for an introduction in 2018 can be seen in Table A14. 97% of the structure is rated AA or better. This good average rating will result in lower interest costs than in the cases before.

Table A15 displays the gains for every country. Germany, France, and Italy have the main stake of nominal debt, representing more than 85%. The results are similar to the results when all countries of the Monetary Union are participating in the programme. Again, every country has a positive effect from enrolling at the Eurobond programme and does not fall beyond a 1% gain in relation to nominal volume, except for Germany. This can be a hint that it is possible to introduce structured Eurobonds as a test only in some countries at the beginning and widen the circle afterwards without lowering the gains. On the contrary, the gains will rise if more countries are participating.

Tranche	Thickness	Rating	Interest Rate
Tranche I	78.46%	AAA	0.56%
Tranche II	3.27%	AA+	0.65%
Tranche III	15.37%	AA	0.85%
Tranche IV	0.64%	AA-	0.99%
Tranche V	0.64%	A+	1.17%
Tranche VI	1.28%	A	1.32%
Tranche VII	0.34%	A-	1.54%

Table A14: The structure for an introduction of structured Eurobonds for founding members.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	5.19%	0.86 (1.47)	1.78% (3.02%)	0.18% (0.30%)
Germany	38.46%	2.32 (10.85)	0.64% (2.99%)	0.06% (0.31%)
France	26.91%	4.56 (7.70)	1.75% (3.03%)	0.17% (0.30%)
Italy	20.24%	20.43 (6.32)	10.71% (3.31%)	1.02% (0.33%)
Luxembourg	0.64%	0.07 (0.18)	1.05% (2.99%)	0.10% (0.30%)
Netherlands	8.58%	0.83 (2.45)	1.01% (3.00%)	0.10% (0.30%)

Table A15: The results for an introduction of structured Eurobonds through founding members.

The results are showing that even during the sovereign debt crisis the most problematic countries could have gained and gains would have been higher than in the whole EMU scenario. They would have more advantages when starting without the higher-rated countries. This is supported by the results we have seen in a scenario where all countries besides Germany are participating. Lower-rated countries drive the gains for high rated countries whereas the higher-rated ones preserve the stability.

A.6 Shorter duration

In this case, we go back to the baseline scenario with an issuance in 2018 with all countries of the EMU participating, a trust fund rate of 10% and a recovery rate of 50%. The only difference is that the duration of the structured Eurobonds is chosen as one and two years. As a consequence, the default probability is calculated by using the one- and two-year CDS spread, and the risk-free interest rate is determined by the one- and two-year bond yield. For a one-year time horizon, the risk-free interest rate is -0.64% and is derived from the German bond. In the two year scenario, the yield of the Netherlands bond is lower and has a value of -0.59%. Figure A16 shows the resulting structures for the two durations. As a consequence of the shorter time horizon and the lower default probability, the average rating is increasing compared to the baseline scenario. For a two year duration, the average rating is getting worse relative to the one-year duration, but it is still better than in the baseline scenario. The thickness of the AAA tranche is above 93% in both scenarios. Due to a negative risk-free interest rate and a relatively low credit spread for the tranches, the structure delivers negative yields for the capital market.

Tranche	Thickness 1 Year (2 Years)	Rating	Yield 1 Year (2 Years)
Tranche I	93.86% (93.44%)	AAA	-0.60% (-0.54%)
Tranche II	5.86% (0.61%)	AA+	-0.58% (-0.51%)
Tranche III	0.29% (5.94%)	AA	-0.55% (-0.45%)

Table A16: The structure for an introduction of structured Eurobonds with a shorter duration.

The results of the simulation for both durations can be seen in Tables A17 and A18. Because some countries face negative interest rates, it is better to stick with the even distribution. In the relative distribution, the interest burden is used to calculate the individual repayment scheme. With negative interest rates, some countries do not face interest burdens, and therefore we focus on the even distribution.

For a duration of one year, many countries face losses and the total gain of structured Eurobonds is only 0.3 billion Euro. With this distribution method, the lower (higher) rated countries have positive (negative) results, and in the even distribution, this is reversed.

The results are increasing for a longer duration with a gain of 3.98 billion Euro, more than ten times the result for a one-year issuance. In an even distribution scheme, all countries realise gains. Also, the results are getting better for all countries compared to the one year time horizon.

A reason for the negative results for short durations is that the interest-bearing effect of the trust fund is not present, primarily due to a negative risk-free interest rate. In addition, the spread for every tranche of the structure is low. Nevertheless, some countries with a higher risk, e.g. Italy and Greece, have a relatively higher yield and therefore their additional costs are larger than their gains.

Therefore, an issuance of structured Eurobonds with a shorter duration can also produce gains. But with a short duration, e.g. one year, a new distribution method needs to be implemented to ensure that every country realises gains from structured Eurobonds.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	3.95%	-0.003	-0.01%	-0.01%
Germany	29.26%	-0.057	-0.02%	-0.02%
Estonia	0.24%	0.001	0.04%	0.04%
Ireland	2.66%	-0.001	-0.00%	-0.00%
Greece	1.61%	0.034	0.17%	0.17%
Spain	10.40%	0.032	0.03%	0.03%
France	20.47%	-0.016	-0.01%	-0.01%
Italy	15.39%	0.303	0.16%	0.16%
Cyprus	0.16%	0.003	0.14%	0.14%
Latvia	0.24%	-0.000	-0.00%	-0.00%
Lithuania	0.40%	0.001	0.02%	0.02%
Luxembourg	0.48%	-0.001	-0.02%	-0.02%
Malta	0.08%	0.000	0.00%	0.00%
Netherlands	6.53%	-0.011	-0.01%	-0.01%
Austria	3.30%	-0.005	-0.01%	-0.01%
Portugal	1.69%	0.011	0.05%	0.05%
Slovenia	0.40%	0.002	0.00%	0.00%
Finland	2.01%	-0.003	-0.01%	-0.01%
Slovakia	0.73%	0.000	0.00%	0.00%

Table A17: The results for an introduction of structured Eurobonds in the whole EMU with a duration of one year in 2018. The first column displays the stake every country has at the nominal volume, the second the nominal net gain, the third shows the nominal net gain in relation to the debt, and the last column the yearly savings from the third column. The gains are distributed according to the even distribution.

Country	Stake at nom.	Nom. net gain	Rel. net gain	Gain rel. to nom. p.a.
Belgium	3.95%	0.14	0.29%	0.14%
Germany	29.26%	1.01	0.28%	0.14%
Estonia	0.24%	0.01	0.31%	0.16%
Ireland	2.66%	0.10	0.30%	0.15%
Greece	1.61%	0.09	0.48%	0.24%
Spain	10.40%	0.42	0.32%	0.16%
France	20.47%	0.73	0.29%	0.14%
Italy	15.39%	0.89	0.47%	0.23%
Cyprus	0.16%	0.01	0.42%	0.21%
Latvia	0.24%	0.01	0.30%	0.15%
Lithuania	0.40%	0.02	0.32%	0.16%
Luxembourg	0.48%	0.02	0.28%	0.14%
Malta	0.08%	0.01	0.32%	0.16%
Netherlands	6.53%	0.23	0.28%	0.14%
Austria	3.30%	0.12	0.28%	0.14%
Portugal	1.69%	0.08	0.35%	0.18%
Slovenia	0.40%	0.02	0.31%	0.16%
Finland	2.01%	0.07	0.28%	0.14%
Slovakia	0.73%	0.03	0.31%	0.15%

Table A18: The results for an introduction of structured Eurobonds in the whole EMU with a duration of two years in 2018. The first column displays the stake every country has at the nominal volume, the second the nominal net gain, the third shows the nominal net gain in relation to the debt, and the last column the yearly savings from the third column. The gains are distributed according to the even distribution.

2 Effects of Structured Eurobonds

The Impact of Structured Eurobonds on Exchange Rates

Abstract

This paper discusses the impact of the introduction of structured Eurobonds on exchange rates. We will concentrate on the connection between five currency pairs (Euro vs. US-Dollar, Swiss Franc, British Pound, Japanese Yen, and Chinese Renminbi), but the analysis can as well be used for every other foreign currency compared to the Euro. The impact is analysed in a context where Eurobonds are issued through an Asset-Backed Security (ABS). The issuance results in a new yield curve for the European Monetary Union (EMU). As previous research has shown, there exists a link between the relative shape of yield curves to each other and exchange rate changes. Therefore, I compare the European yield curve with the counterparts mentioned above and evaluate their link to the exchange rates. This comparison is made for both scenarios, with and without structured Eurobonds. It will be shown that after the introduction of structured Eurobonds, the European yield curve would experience a sudden change. This abrupt change in the yield curve results in a depreciation of the Euro against the US-Dollar. The depreciation is in a range of 1.07% to 3.57% in the following 12 months. In contrast, the Euro will appreciate against the other three significant foreign currencies, ranging from 0.53% for Chinese Renminbi to 5.37% for British Pound. The magnitude of this effect depends on the structure as well as the time of issuance.

Keywords: Structured Eurobonds, Exchange Rates, Yield Curves

2.1 Introduction

Since the beginning of the European Monetary Union (EMU), the discussion on deepening the sovereign bond markets is vivid. Many different possibilities have been debated to achieve this. One of these is the common issuance of bonds for all countries in the EMU, the so-called Eurobonds.

The Giovannini Group (2000) published first ideas and they have been evolved until today. The different approaches reach from only issuing a part of the needed debt to a whole refinancing through Eurobonds with varying strengths of liabilities.[11] Besides

[11] For a more in-depth insight into the various possibilities, see Claessens et al. (2012).

strengthening the connection between member countries, the bonds can help reducing interest expenses of countries as well as deepening the market of sovereign bonds. The European Commission (2017) also highlights this in their actual Reflection Paper on the Deepening of the Economic and Monetary Union. Another purpose is to build an equivalent to the US-American T-Bill market to receive more investor attraction and to have bonds that are as liquid as US-American T-Bills. On the other hand, there are some disadvantages. One of them, which is prominent in political and scientific discussions, is "moral hazard". There is a controversial debate about whether common issuance will set negative incentives for countries which have had refinancing problems in the financial and Euro debt crisis. Another issue is the question of liability in case of default. More stable countries, e.g. Germany, fear a situation in which they have to pay for other countries which might use Eurobonds as a cross-financing instrument.

Hild et al. (2014) and Brunnermeier et al. (2016) developed new approaches for Eurobonds to reduce the disadvantages mentioned above. They use structured products - especially Asset-Backed Securities (ABS) - to construct Eurobonds. The advantage of both ABS-approaches is a reduction of the negative aspects that are the reason for moral hazard. Due to the different tranches and the set-up of an ABS, the liquidity will improve and the liability will be reduced. Since ABS products and especially Collateral Default Obligations (CDOs) have been the reason for the recent financial crisis, investors have some aversion against this product. Also, their complexity might prevent institutional investors from investing in them. Therefore, the issuance of structured Eurobonds might be more difficult or connected with significant interest spreads that cancel out the advantages. One of the main differences between both approaches is that the one outlined by Brunnermeier et al. (2016) allows for two tranches, the "European Safe Bonds" (ESBies) and "European Junior Bonds" (EJBies), whereas, Hild et al. (2014) do not have a specific number of tranches.

Some authors, e.g. the European Commission (2011) in their Green Paper, claim that the role of the Euro as an international reserve currency will be strengthened after the introduction of Eurobonds. This paper aims to have a closer look at the impact on the exchange rate between the Euro and relevant foreign currencies.

My calculation is based on the method used by Chen and Tsang (2013), who examined several currency pairs with the Nelson-Siegel model, which was developed by Nelson and Siegel (1987). This model is used to describe yield curves with a non-linear approach. The current yield curve of the Euro Area, which is calculated and published by the European Central Bank (ECB), is the benchmark for the following examinations. Yields of bonds issued by member countries are weighted relative to their capital commitment at the ECB to construct the European yield curve. Using this data from the

European Central Bank, I found a significant link for different factors, mainly slope and curvature, and exchange rate predictability. The slope factor is the most robust of the three factors for the Euro against the US-Dollar (USD) because it is significant for every time horizon above one month whereas for other countries the other two factors are of major interest. After this analysis of the current yield curve and its connection to the exchange rate, I use tranches of Hild et al. (2014) to create a new yield curve for the EMU after the (theoretical) introduction of structured Eurobonds. This new yield curve and the resulting impact on the exchange rate is influenced by the issuance date and the structure of the ABS-model. The introduction represents a shock on the yield curve with an abrupt downward shift and is an extension to the method of Chen and Tsang (2013) because they rely on current yield curves and a shock is not taken into account. I focus on two settings for the structure, a conservative and a progressive one. These are characterised by different assumptions – made by Hild et al. (2014) – regarding the correlation of default probabilities between EMU countries. In the progressive setting, the correlation is lower than in the conservative setting and results in a better average rating of the structure. In a conservative setting, the appreciation of the Euro is 0.53% and ranges to 5.37% in a progressive structure in the following 12 months after introduction against British Pound (GBP), Swiss Franc (CHF), and Chinese Renminbi (CNY). Against US-Dollar, the Euro will face a depreciation of 1.70% to 3.56% in the following 12 months after issuance, again dependent on the structure and issuance date. For Japanese Yen (JPY), no significance can be found for any time horizon and therefore, an impact cannot be determined. These specific five foreign currencies were chosen for several reasons. The USD was selected because the currency pair EUR/USD is the most traded pair worldwide, GBP, JPY, and CHF are the following most traded counter currencies with EUR as involved currency. A list of the most traded currency pairs can be found in the Triennial Central Bank Survey of the Bank for International Settlements (2019). At last, CNY is chosen due to its rising impact on world trade as well as its increasing economic importance. To the best of my knowledge, there is no recent work that tries to measure the impact of structured Eurobonds on exchange rates. My paper attempts to fill this gap.

The next sections are structured as follows. Section 2.2 provides a theoretical background. Section 2.3 describes the data, shows the actual link between the yield curves and the exchange rate and will describe the new yield curve after the issuance of structured Eurobonds. In Section 2.4, the main results, which is the impact of the new yield curve on the exchange rate, are presented, while in Section 2.5, several robustness checks are discussed. Section 2.6 concludes the findings.

2.2 Theoretical Background of Eurobonds

Eurobonds are considered one possibility to solve the debt crisis and to manage the sovereign-bank nexus as outlined by several European institutions, e.g. the ESRB (2018). They could also be helpful to prevent a new adverse situation in the European Monetary Union. The Giovannini Group (2000) first established the idea of a coordinated debt issuance who offer different hypotheses for elaboration. The concepts got more specified by Boonstra (2005) who introduced the possibility to use a special fund for issuance. Several concepts of Eurobonds have been outlined by the Securities Industry and Financial Markets Association (2008) and by Eijffinger (2011). With the start of the financial crisis, the "flight to safety" started and the ideas of Eurobonds got more detailed. As a consequence, de Grauwe and Moesen (2009) proposed a system to challenge the arising liquidity premium with the issuance of Eurobonds by the European Investment Bank. These bonds shall be backed by all EMU countries and can help to reduce interest burdens for high interest-paying countries. They identified the liquidity premium as the main driver for the increasing yield differences between the EMU member countries. Other authors, e.g. Issing (2009), illustrated the negative aspects arising from the implementation of Eurobonds. A greater problem in the concept of Eurobonds is moral hazard which arises of the joint liability. There is a viable risk that some countries might use Eurobonds as an instrument to refinance them at a low-interest rate to have an excessive budget spending. Afterwards, they default and leave the remaining countries on their own to repay the debt.

Delpla and von Weizsäcker (2010) has given a much-noticed approach. They proposed a system with two different types of Eurobonds. Every debt up to a threshold of 60% of the individual national GDP can be issued together through so-called "blue bonds". The threshold has its origin in the Stability and Growth Pact (SGP) of the EMU. Every country will issue needed debt above 60% on its own. They are called "red bonds". The speciality of this construction is a joint liability, higher liquidity, and seniority of blue bonds against red bonds. Due to the features of blue bonds, participating countries have significantly lower interest payments on their debt. Delpla and von Weizsäcker (2010) assume that such a construction will gain positive incentives on discipline because red bonds will admonish countries to get under the threshold. A greater problem in this construction is the "no-bailout" clause of the Maastricht Treaty which will be violated in case of default. This method was picked up by Gopal and Pasche (2011) who assume issuing of blue bonds by an European Central Agency to refinance 80% of every countries debt. The method of Delpla and von Weizsäcker (2010) was also examined by Baglioni and Cherubini (2016) who use 40% as threshold.

They analyse how much cash collateral is needed to construct risk-free senior bonds. The German Council of Economic Experts (2012) discussed a reverse method. They suggest a system where sovereign debt above a threshold of 60% is transferred into a special debt redemption fund with joint liability. The threshold is also chosen with respect to the SGP. The debt will be transferred in a multi-annual process. Every country has to pay a part of its transferred volume to the fund year by year. This mechanism ensures that the fund is closed after a fixed time horizon of 25 years. Every country will be below or just at a 60% debt-to-GDP ratio after the closing of this fund.

Another approach was to use the mechanics of the so-called "Brady Bonds", who played a crucial role in the Latin American debt crisis in the 1980s. Economides and Smith (2011) propose "Trichet Bonds", named after the president of the ECB at this time, as another possibility. The mechanism is identical with the difference that the collateral is not a 30-year zero-coupon US bond but rather a 30-year zero-coupon bond issued by the ECB.

All of the approaches mentioned above to implement Eurobonds would face diverse challenges in the legal framework of the EMU. Another crucial point is the budget authority of the German Parliament over their budget. This authority has been a blocker in the past. Basu (2016) shows how to solve some of the legal obstacles that arise with Eurobonds. A further investigation shall not be included in this work.

One way to deal with some issues, especially the moral hazard problematic, and still reach the benefits of common issuance is to create "Structured Eurobonds". They have the same aim as the above mentioned Eurobonds, e.g. to reduce the interest burden and stabilize bond markets, but they diminish the negative aspects drastically. Two methods have been developed, first by Hild et al. (2014) and then by Brunnermeier et al. (2016). The introduction of Eurobonds through a structured product is also favoured by the European Commission (2017), as mentioned in their current Reflection Paper. Essentially, both approaches use similar techniques with slight but nevertheless fundamental differences. Both use an ABS-approach to creating a new bond. The outstanding and newly raised debt of every country is pooled together by an SPV, e.g. a fund. This SPV restructures the pooled bonds into new tranches with other ratings than the original bond ratings. This effect is attributed to a correlation of less than one between the countries of the EMU. Hild et al. (2014) discusses the correlation effect and structuring through an SPV.

The newly issued tranches have a lower implied default probability than the weighted average of the current default probability of participating countries. One main difference between the two approaches is the number of tranches. While Brunnermeier et al. (2016) present a model restricted to two tranches, European Safe Bonds (ESBies) and European Junior Bonds (EJBies), Hild et al. (2014) have some more possibilities, ranging from two to more tranches. They use a reserve fund or trust fund to absorb first losses in case of default. This fund has a predefined size, e.g. 10% of the nominal volume of issued debt. The trust fund bears interest, and if a country defaults the recovered value is transferred to the trust fund. Losses that extend the size of the trust fund will cause depreciation and default of the junior tranches. Due to this construction with an ABS product, the above mentioned negative aspects of Eurobonds concerning "moral hazard" and joint liability can be prevented. Joint liability is limited to the initial payment to the trust fund. A direct consequence of this approach is the emergence of a new yield curve in the EMU, whose impact on FX rates is of significant interest in this work.

An extended discussion on the topic of structured Eurobonds with several simulations can be found in the feasibility study of the ESRB (2018). Different possibilities of Eurobonds with their advantages and disadvantages are discussed there. These are also thematised by van Riet (2017).

2.3 Data and Methodology

2.3.1 Data

To fit the Nelson-Siegel model, yield data of the associated countries is needed. Also, the exchange rate between the base currency (Euro) and different foreign currencies (US-Dollar, British Pound, Chinese Renminbi, Swiss Franc, and Japanese Yen) measured as foreign currency price per unit of Euro is necessary. The sample consists of end-of-month data from September 2004 to February 2018, resulting in 162 observations. The zero-coupon yields with maturities of 3, 6, 12, 24, 36, 60, 84, 120, 240 and 360 months for the United States and China as well as the exchange rate are downloaded from Thomson Reuters Datastream. For Switzerland, the United Kingdom and Japan, the yield data consists of the same maturities extended by 48, 72, 96, 108 and 180 months and is also downloaded from Thomson Reuters Datastream. Yield data for the same maturities of the EMU are taken from the European Central Bank's statistical database. The yield data for every member country of the EMU is also downloaded from Thomson Reuters Datastream. This data is used for the construction of the yield curves of the EMU after issuance of structured Eurobonds. Figure 2.1 shows several yield curves of EMU member countries and it can be seen that the yield curve of the EMU lies between the French and Spanish one, but is closer to the Spanish. It is representing a nearly AA yield curve. The German yields are the lowest since they have

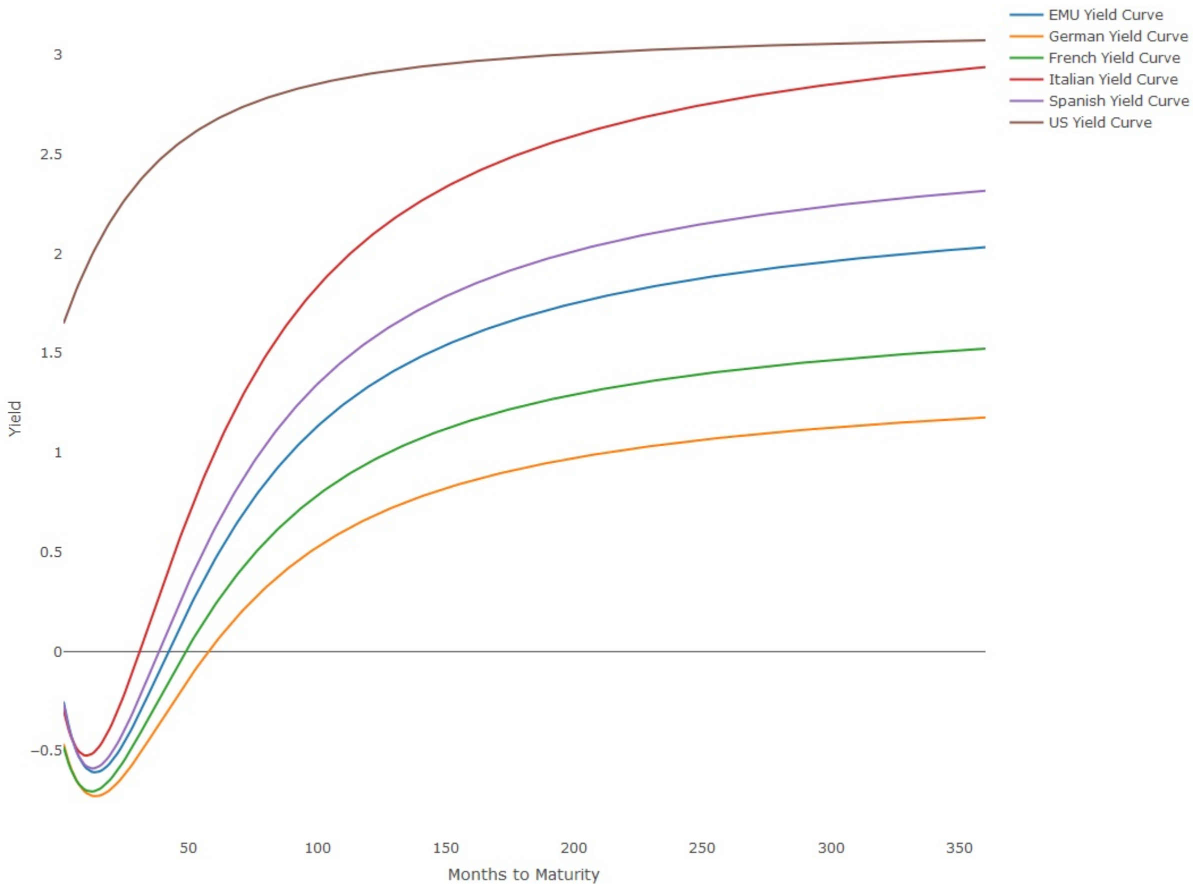

Figure 2.1: EMU member and US yield curves for End of February 2018 Data.

the best possible rating from rating agencies and highest liquidity. For comparison and calculation purposes, the expectations of market participants are also taken into account. Therefore, a Consensus Forecasts dataset is used. To construct these explicit dataset market participants are asked what exchange rates they expect several months in the future.

The next steps also require several macroeconomic data. The GDP and debt statistics of every country in the EMU are taken from Eurostat, the European Statistical Office. The latest available datasets for these two variables are from December 2017. Also, the commitment which every country of the EMU has at the capital stock of the ECB is needed. The value is also provided and calculated by the ECB, and they use two key figures, one is the ratio of GDP of every country to the GDP of the whole EMU, and the other one is the ratio of population. The ECB calculates this ratio of commitment every five years or after the accession of a new country in the European Union and the last time it was adjusted was in January 2014. At last, trade information (imports and exports) for the European Union are downloaded from Eurostat for 2017. This information is later used to calculate a synthetic exchange rate to measure the impact on the Euro weighted by the trade partners.

2.3.2 Current Connection between Yield Curve and Exchange Rates

The yield curve links the yield of bonds to their maturity. Diverse work has shown that information about future macroeconomic conditions can be derived from this curve. Ang et al. (2006) use its information to forecast GDP growth, and Dewachter and Lyrio (2006) can connect it to the business cycle and the central banks' monetary stance. The yield curves of the US and the EMU with the end of February 2018 data can be seen in Figure 2.1. The current US yield curve is above the EMU curve due to a higher interest rate level in the US which is a non-neglecting driver of the level of the yield curve.

Since this curve has a non-linear character, different models have been designed to fit the yields. The model developed by Nelson and Siegel (1987) is a prominent method to describe yield curves. Their model has an exponential character and is of the following form,

$$y(m) = L_t + S_t\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + C_t\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right), \tag{2.1}$$

where $y(m)$ describes the yield to maturity or the continuously compounded zero-coupon nominal yield of a bond with m months to maturity. The variables L_t, S_t and C_t represent level, slope and curvature of the yield curve at an observation time t. One benefit of the model is the feasibility to describe different kinds of yield curves, ranging from normal over humped to inverted curves. The parameter λ is crucial to the strength of exponential decay in S_t and C_t. λ is set to 0.0609 as a standard value in the literature.[12] The impact of a non-fixed value of λ will be discussed in Appendix B.1. There it is shown that the assumption of a fixed value of λ does not have a negative impact on the results. It can be seen from Eq.(2.1) that the components have a different impact over time. While the level factor is a constant linear part, the slope is more relevant in the short term and decays rapidly whereas the curvature gets more relevant in the midterm and decays to zero in the long term. The choice of $\lambda = 0.0609$ implies a maximum impact of the curvature factor at $m = 30$.

[12] A discussion on the choice of λ can be found in Diebold and Li (2006b).

Chen and Tsang (2013) have found a link between the exchange rate predictability and the relative shape of associated yield curves. They make use of the Nelson-Siegel model and its linearity to formulate a model with a small alteration,

$$y(m) - y^{*}(m) = L_t^R + S_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + C_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right) + \epsilon_t, \tag{2.2}$$

where $y(m)$ describes the home yield, in this framework the European yield, and $y^{*}(m)$ the foreign yield. L_t^R, S_t^R, and C_t^R are the relative Nelson-Siegel factors and ϵ_t is the fitting error resulting from the least square model.[13] A discussion of the interaction of the three Nelson-Siegel factors, several macroeconomic indicators and their impact on future movements in the respective yield curve can be found in Diebold and Li (2006).

At first, the relative Nelson-Siegel factors are estimated with Eq.(2.2) for every observation in the sample. Figure 2.2 illustrates the EUR/USD exchange rate against the relative Nelson-Siegel factors. As we can see in this figure, the level and slope factor are hardly varying over time, whereas the curvature factor has a higher volatility. Two interesting breaks can be seen here. The slope factor drops in 2009 which can be explained with the financial crisis because the slope is an indicator of economic growth. Afterwards, it is slowly increasing due to some recovery in the USA. Another break can be seen at the end of the year 2011 for the level factor. There was one of the peaks of the European debt crisis where a higher level is characteristic. Also, the US yield faced a downward shift in this time horizon. Since the US yield curve is subtracted from the EMU curve, the resulting relative factor has a fast growth. However, our attention is not on a current link but rather on the influence on future exchange rate changes.

In the first step, we want to have a closer look at the link between the relative factors and exchange rate changes. To do so, we use the same linear regression as Chen and Tsang (2013),

$$\Delta s_{t+m} = \beta_{m,0} + \beta_{m,1} L_t^R + \beta_{m,2} S_t^R + \beta_{m,3} C_t^R + u_{t+m}, \tag{2.3}$$

where Δs_{t+m} is the annualized relative difference of the exchange rate at time t looking m months into the future. L_t^R, S_t^R, and C_t^R are the parameters resulting from the Nelson-Siegel model presented in Eq.(2.2)[14]. Due to overlapping data when $m > 1$, the error term on the left-hand side will be a moving-average process and the resulting estimates will be biased. To overcome the autocorrelation, one possibility is to use a covariance estimator developed by Newey and West (1987). In small samples this estimator rejects

[13] The relative Nelson-Siegel factors can be used due to the linearity of the Nelson-Siegel model. An empirical evaluation where the three factors are estimated for every country and differences between them are calculated afterwards, can be found in Appendix B.2.

[14] An examination of the impact of every single relative Nelson-Siegel parameter on the exchange rate predictability can be found in Appendix B.3.

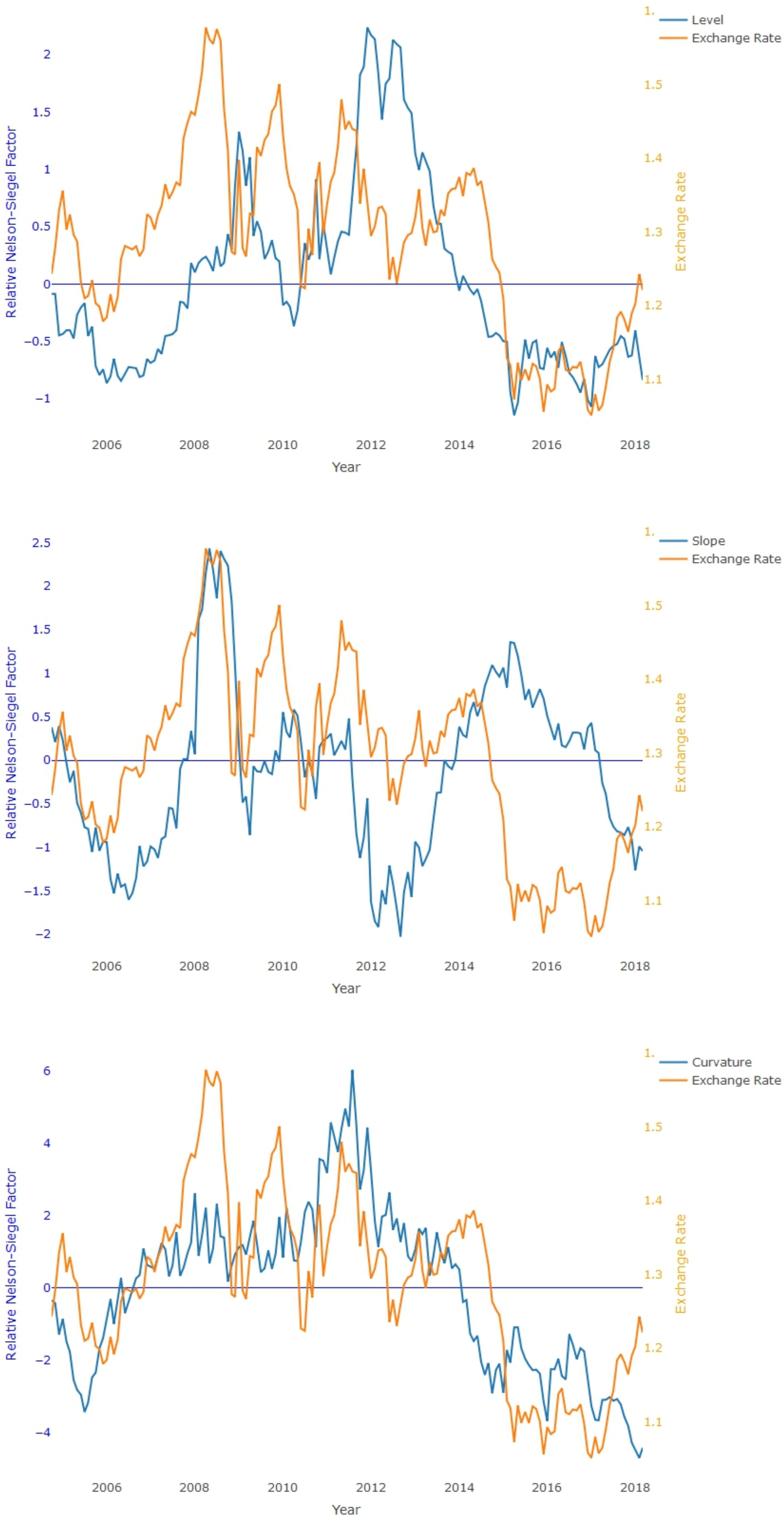

Figure 2.2: Time series of Nelson Siegel factors and exchange rate EUR/USD.

too often and therefore a rescaled t-statistic, where t is adjusted by multiplying with $1/\sqrt{m}$, is used. This method is developed by Moon et al. (2004) and Valkanov (2003).[15] Table 2.1 shows diverse descriptive statistics on the different relative Nelson-Siegel factors for all foreign currencies. It is noticeable that the standard deviation in the Nelson-Siegel parameters grows from level to curvature, as observed in Figure 2.2. This can be explained by the long yield horizon, which is relevant for the level factor. On the other hand, slope and curvature are more influenced by short or medium-term changes.

	Minimum	Median	Mean	Maximum	Std.Dev
Panel A: US-Dollar					
Level	-1.143	-0.175	0.019	2.236	0.808
Slope	-2.025	-0.052	-0.126	2.431	0.955
Curvature	-4.710	0.515	-0.054	6.031	2.246
Panel B: British Pound					
Level	-0.909	0.079	0.204	2.141	0.577
Slope	-3.032	-0.837	-0.815	0.637	0.764
Curvature	-5.764	0.848	-0.461	5.934	2.401
Panel C: Chinese Renminbi					
Level	0.389	2.328	2.214	3.547	0.676
Slope	-3.276	-1.649	-1.285	1.381	1.404
Curvature	-4.009	0.258	-0.288	2.693	1.958
Panel D: Swiss Franc					
Level	0.819	1.532	1.915	3.947	0.739
Slope	-3.417	-0.963	-1.043	0.450	1.003
Curvature	-5.576	-1.926	-1.996	1.373	1.398
Panel E: Japanese Yen					
Level	0.181	2.335	2.219	3.950	0.668
Slope	-3.428	-1.655	-1.292	1.465	1.410
Curvature	-4.311	0.191	-0.323	3.540	1.955

Table 2.1: Descriptive statistics for the three different Nelson-Siegel factors out of 162 observations per factor and foreign currency.

The results for all currency pairs show large differences in the combination which relative Nelson-Siegel factor is relevant for which time horizon. These are displayed in the Tables 2.2 and 2.3. It can be seen that the impact of the different factors on the exchange rates is varying over time and some factors are significant only for special time horizons.

A significant connection between the relative slope factor and future changes of Euro/US-Dollar can be identified in panel A of Table 2.2. This connection can be

[15] A discussion on this challenge can be found by Chen and Tsang (2013).

Panel A: US-Dollar						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-2.836	-4.824	-4.431	-3.818	-3.127	-2.232
	(-0.552)	(-1.129)	(-1.039)	(-0.970)	(-1.030)	(-0.714)
Slope	-4.462	-6.640**	-6.950**	-5.309**	-4.118**	-3.398*
	(-1.341)	(-2.388)	(-2.488)	(-2.035)	(-2.057)	(-1.658)
Curvature	1.638	1.279	0.751	0.199	0.009	-0.387
	(0.912)	(0.837)	(0.480)	(0.133)	(0.008)	(-0.316)
R^2	0.016	0.087	0.184	0.250	0.362	0.355
Panel B: British Pound						
Level	-9.064	-11.436*	-10.917*	-9.915	-7.572	-4.429
	(-0.847)	(-1.730)	(-1.806)	(-1.601)	(-1.189)	(-0.714)
Slope	1.390	-7.018*	-6.805*	-5.756	-4.118	-1.714
	(0.223)	(-1.831)	(-1.953)	(-1.644)	(-1.132)	(-0.504)
Curvature	1.606	1.752	1.504	1.087	0.348	-0.448
	(0.692)	(1.217)	(1.136)	(0.797)	(0.248)	(-0.332)
R^2	0.009	0.054	0.135	0.216	0.238	0.276
Panel C: Non-Overlapping US-Dollar						
Level		-3.626	-4.323			
		(-0.759)	(-0.835)			
Slope		-6.388*	-6.034*			
		(-2.197)	(-1.988)			
Curvature		0.816	0.475			
		(0.493)	(0.264)			
R^2		0.034	0.038			

Table 2.2: The connection of different Nelson-Siegel factors and their predictive power for exchange rate changes with the currency pairs EUR/USD and EUR/GBP. The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

found for each observed period besides the one-month horizon. This delay is a result of some response time on changes in macroeconomic factors in the yield curve as well as the exchange rate. The level and the curvature factor are not significant for any time horizon. The results can be interpreted as follows: A one percentage point increase in the relative slope factor predicts a 3.40% annualized depreciation of the Euro in the following 24 months. This increase in the relative slope factor is equivalent to a steeper US yield curve relative to the European curve. Here, the growth expectations of the United States are increasing. The annualized effect of this factor decreases over time. This decrease can be explained by the declining impact of current expectations and information as well as new effects occurring in longer horizons. For British Pound (panel B of Table 2.2), the only factors that are significant for different time horizons

Panel A: Chinese Renminbi						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	0.529	-0.877	-2.145	-2.265	-1.767	-1.713
	(0.179)	(-0.348)	(-0.835)	(-0.936)	(-0.857)	(-0.772)
Slope	-0.799	-0.506	-0.248	0.423	0.586	0.449
	(-0.494)	(-0.372)	(-0.182)	(0.342)	(0.580)	(0.450)
Curvature	1.005	0.985	1.566*	1.365*	0.643	0.413
	(0.937)	(1.095)	(1.753)	(1.678)	(0.972)	(0.632)
R^2	0.009	0.009	0.101	0.186	0.149	0.126
Panel B: Swiss Franc						
Level	-5.517	0.787	1.687	1.876	2.881	4.418
	(-1.006)	(0.145)	(0.305)	(0.363)	(0.548)	(0.736)
Slope	-5.520	0.394	1.326	1.128	1.979	3.210
	(-1.202)	(0.092)	(0.304)	(0.279)	(0.488)	(0.705)
Curvature	1.381	1.313	1.550	2.134*	1.729	1.064
	(0.992)	(0.966)	(1.125)	(1.696)	(1.419)	(0.820)
R^2	0.011	0.007	0.067	0.255	0.317	0.277
Panel C: Japanese Yen						
Level	-3.800	-4.542	-3.289	-4.099	-3.040	-3.675
	(-0.506)	(-0.596)	(-0.407)	(-0.495)	(-0.365)	(-0.466)
Slope	-4.196	-2.672	-1.816	-2.189	-2.449	-3.574
	(-1.193)	(-0.748)	(-0.479)	(-0.564)	(-0.631)	(-0.981)
Curvature	1.214	1.004	0.304	1.057	1.466	2.425
	(0.491)	(0.400)	(0.112)	(0.370)	(0.493)	(0.830)
R^2	0.011	0.012	0.014	0.027	0.033	0.159

Table 2.3: The connection of different Nelson-Siegel factors and their predictive power for exchange rate changes with the currency pairs EUR/CNY, EUR/CHF and EUR/JPY. The significance levels are *10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

are the relative level and slope factor. In this case, a one percentage point increase in the relative level factor predicts a 10.92% annualized appreciation of the Pound in the following six months. Here, the whole yield curve of the European Monetary Union shifts one percentage point up relative to the UK one. We will focus on the six months time horizon in the upcoming analysis. The results for Chinese Renminbi (panel A in Table 2.3) only imply a significance in the relative curvature factor. This connection shows up for time horizons of six months and twelve months, where it is significant at a 10% level. Swiss Franc (panel B in Table 2.3) is only significant for a 12-month horizon. Here the relative curvature factor is the primary driver of exchange rate predictions. A one percentage point rise in the relative curvature factor will lead to a 2.13% rise of the exchange rate in a 12-month horizon, which is equivalent to an appreciation of the Euro against the Swiss Franc. At last, the Japanese Yen (panel C in Table 2.3) shows no significance for any time horizon. As a consequence, exchange rate changes cannot be predicted by using the yield curve or the Nelson-Siegel factors, respectively. Therefore, an evaluation of the impact of the issuance of structured Eurobonds on the exchange rate EUR/JPY is not possible and the Yen is excluded from further considerations. Nevertheless, the Yen is crucial because it is the third most traded currency in combination with the Euro. The R^2 in the regression of all remaining four foreign currencies show mixed pictures. Whereas it is growing with the time horizon for GBP, it reaches its highest value at 18 months for USD and CHF, and at 12 months for CNY. The maximum of the R^2 is between 0.185 and 0.362, dependent on the foreign currency and the time horizon.

We also look at non-overlapping data to show the robustness of this regression with the rescaled t-statistic. They are constructed for 3 and 6 months in the future by only looking at the end of quarter and semi-annual data. They are displayed in panel C of Table 2.2, but only for US-Dollar as foreign currency. If longer horizons are examined, there are some problems with the number of observations. We have 54 observations for quarterly and 27 observations for semi-annual data. For longer horizons with one year or more, we only have 13 or fewer observations left. This will reduce the explanatory power of the test. The relative slope factor delivers values close to the shown values in panel A. The other currencies are showing the same picture and are therefore not included in the table.

2.3.3 Estimation of New Yield Curves

The next step in finding the impact of structured Eurobonds on exchange rates is to estimate the shape of the yield curve after the introduction of a new bond system through an ABS-approach. We look at three different structures which have been calculated by Hild et al. (2014) and build new yield curves for every one of these. An overview of the structures and their tranches can be found in Table 2.4.

Since the rating of every tranche, as well as the thickness, are fixed, we can utilize them to calculate the new yield curves. The countries with the same rating build a benchmark curve for their respective rating, e.g. Germany, Luxembourg, and the Netherlands are the constituents of the AAA benchmark curve. Their particular impact on this curve is dependent on a macroeconomic factor, e.g. their GDP. The resulting curve for every rating is afterwards connected with the respective tranche in the structure. To construct the new yield curve for structured Eurobonds the yield curves for every tranche – or rating – will be weighted with the thickness of the tranche.

Tranche	Thickness	Rating	Interest Rate
Panel A: Conservative Structure			
Tranche I	56.63%	AAA	2.9%
Tranche II	9.35%	AA-	3.5%
Tranche III	9.42%	A	4.3%
Tranche IV	20.01%	BBB+	5.3%
Tranche V	4.59%	BBB	6.3%
Panel B: Ordinary Structure			
Tranche I	85.07%	AAA	2.9%
Tranche II	7.38%	AA-	3.5%
Tranche III	2.96%	A	4.3%
Tranche IV	4.59%	BBB+	5.3%
Panel C: Progressive Structure			
Tranche I	95.41%	AAA	2.9%
Tranche II	2.94%	AA	3.5%
Tranche III	1.65%	A	4.3%

Table 2.4: The different structures which were calculated by Hild et al. (2014).

To estimate the new curve, we use three different calculation methods or macroeconomic indicators (ECB capital commitment, GDP, and Debt) and show that they deliver similar results. The first method is inspired by the actual way to calculate the European yield curve using the commitment of every country at the capital stock of the ECB. The AAA yield curve, e.g., is built using the yield curves of the corresponding countries. Their weighting in the ECB capital stock is normalized to the sum of the capital stock for all countries having the same rating. With this normalization, we receive their weighting in the benchmark curve for the AAA tranche. The yields for every maturity of a benchmark curve can be calculated using the equation

$$y_R(m) = \sum_{i=1}^{n} \frac{IND_i}{\sum_{j=1}^{n} IND_j} \cdot y_i(m), \tag{2.4}$$

where n is dependent on the number of countries with the same rating. IND_i is the value of the chosen macroeconomic indicator for country i, so $\frac{IND_i}{\sum_{j=1}^{n} IND_j}$ describes the weighting of country i in the benchmark curve. Finally, $y_i(m)$ is the yield of the corresponding country and $y_R(m)$ describes the yield with m months to maturity and rating R. This method can be used to build a benchmark curve for every rating.

This method is also used for the two other macroeconomic indicators. The second indicator is the ratio of the single country GDP to the GDP of the whole EMU. At last, we will use the debt ratio of every country to the entire debt of the EMU. One advantage of the second and third estimator is that they can be adjusted every quarter with the release of new data sets whereas the capital commitment – as mentioned earlier – is only calculated every five years or after the accession of a new country in the European Union.

Now that we have the benchmark curve for every rating, we use the weighted sum of all benchmark curves to calculate the new yield curve. We apply the equation

$$y(m) = \sum_{k=1}^{l} T_R(k) \cdot y_R(m), \tag{2.5}$$

where l is the number of tranches with different ratings and $T_R(k)$ the thickness of the representative tranche with rating R. This method is used for three different structures which are based on varying assumptions, e.g. regarding the correlation of the EMU-countries. Due to the assumptions made to calculate the structures, they are named Conservative, Ordinary, and Progressive throughout the following empirical analysis.

2.4 Impact on Exchange Rates

2.4.1 Conservative Structure

This structure implies a yield curve which is different from the current one, but great swings are not expected. This assumption can also be validated in Figure 2.3, which shows the actual yield curve and a new one build by the method of ECB-commitment for February 2018.[16]

In the short term, both curves are close to each other, but in the long run with maturities above eight years, the values are diverging more.[17] In the previous chapter it was mentioned that a consequence of a steeper yield curve in the Eurozone would mean an appreciation of the Euro against the US-Dollar. Since it is flatter in this case, we will face a depreciation of the Euro against the US-Dollar. In comparison, for the British Pound, the long-term level is lower and the lower relative Nelson-Siegel level delivers an appreciation of the Euro.

Eq.(2.2) is again used to calculate the relative Nelson-Siegel factors with the now emerged EMU yield curve and the US yield curve. In the next step, the relative factors of the new curve need to be compared with the relative factors of the current curve for

[16]Using the three indicators (ECB capital commitment, GDP, and debt) as well as Eqs.(2.4) and (2.5), we get slightly different yield curves which cannot be visualized appropriately. The differences can only be seen in the calculation results.

[17]A consequence will be a relatively higher slope and lower level factor because we take differences as we have seen in Eq.(2.2).

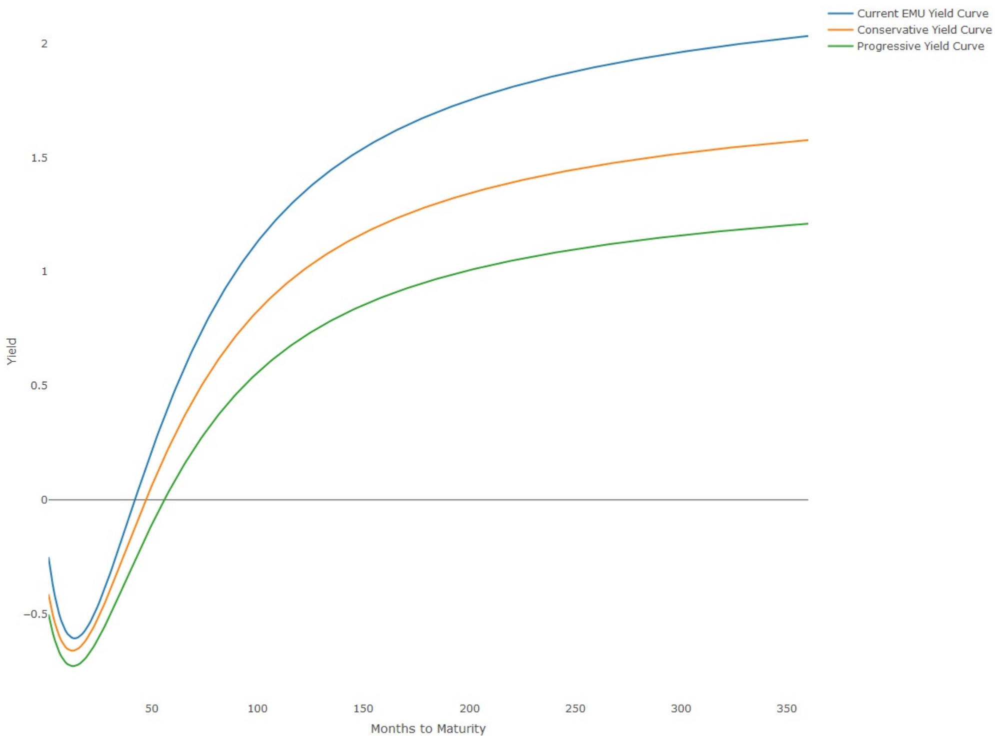

Figure 2.3: The current EMU and two newly constructed yield curves assuming a conservative and progressive structure for End of February 2018 Data.

January and February 2018 using the three methods of construction. This is presented in Tables 2.5 and 2.6.

The first column of the table is listing the different Nelson-Siegel factors and in brackets, the method of calculation for the currency pair EUR/USD. The following column presents the calculated values using the current EMU yield curve from the ECB, followed by the values of our newly calculated yield curve after the introduction of the structured Eurobonds. At last, the difference between both values is highlighted. Panel A shows the results for January and panel B for February 2018. As it can be seen in the calculation results, the difference between the three methods of construction – ECB commitment, GDP ratio and debt ratio – is marginal. The same can be seen for the three other examined currencies and is therefore not included here. Due to this fact, we only evaluate the calculation method regarding the ECB-commitment in this and the following chapter, because it is in line with the actual method of calculation of the EMU yield curve. Table 2.6 displays the other three foreign currencies and has the same setup as Table 2.5.

It is noticeable that the difference in the level is always negative and in the other two factors – slope and curvature – nearly always positive. Only for Swiss Franc, the difference in the slope factor is negative. This picture can be explained with the characteristics of the three structures. Due to the better average rating, the yield curve is

Factor	Current Values	New Values	Difference
Panel A: January 2018			
Level (ECB)	-0.651	-0.969	**-0.318**
Slope (ECB)	-0.991	-0.790	**0.202**
Curvature (ECB)	-4.711	-4.261	**0.449**
Level (GDP)	-0.651	-0.967	**-0.316**
Slope (GDP)	-0.991	-0.793	**0.199**
Curvature (GDP)	-4.711	-4.264	**0.446**
Level (Debt)	-0.651	-0.967	**-0.316**
Slope (Debt)	-0.991	-0.794	**0.198**
Curvature (Debt)	-4.711	-4.264	**0.447**
Panel B: February 2018			
Level (ECB)	-0.836	-1.136	**-0.300**
Slope (ECB)	-1.044	-0.824	**0.220**
Curvature (ECB)	-4.433	-4.052	**0.381**
Level (GDP)	-0.836	-1.135	**-0.299**
Slope (GDP)	-1.044	-0.824	**0.220**
Curvature (GDP)	-4.433	-4.064	**0.369**
Level (Debt)	-0.836	-1.136	**-0.300**
Slope (Debt)	-1.044	-0.824	**0.219**
Curvature (Debt)	-4.433	-4.065	**0.368**

Table 2.5: The effect of introducing structured Eurobonds in a conservative structure on the exchange rate EUR/USD and under different building methods of the new yield curve. The introduction months are January and February 2018.

lower at the long end, so the relative Nelson-Siegel factor decreases. Also, a flatter EMU yield curve explains the increased slope and curvature.

As mentioned above, the predominant parameter for the US-Dollar is the relative slope factor. The other factors do not explain exchange rate movements for any time horizon. The impact of introducing structured Eurobonds is dependent on the issuing month. With issuing month January, the effect on the relative slope factor is about 0.20. This impact can be explained with a lower growth expectation relative to the US economy resulting from the flatter yield curve. As a consequence, the Euro will face a depreciation.

For the following month, the impact of issuing structured Eurobonds on the relative slope factor is greater, and it reaches 0.22. To calculate its impact on the exchange rate, the following formula is used:

$$\Delta s_{24} = \beta_{24,2} \cdot \underbrace{(S_0^{R,New} - S_0^{R,Curr})}_{\text{Difference in Table 2.5}}. \tag{2.6}$$

Panel A: January 2018			
Factor	Current Values	New Values	Difference
Panel A.1: British Pound			
Level	0.221	-0.123	**-0.344**
Slope	-0.930	-0.712	**0.218**
Curvature	-2.471	-1.936	**0.534**
Panel A.2: Chinese Renminbi			
Level	-2.059	-2.394	**-0.335**
Slope	-1.627	-1.411	**0.216**
Curvature	-3.705	-3.214	**0.491**
Panel A.3: Swiss Franc			
Level	1.343	1.255	**-0.087**
Slope	-0.835	-0.934	**-0.100**
Curvature	-2.284	-1.591	**0.693**
Panel B: February 2018			
Panel B.1: British Pound			
Level	0.295	-0.006	**-0.301**
Slope	-1.016	-0.796	**0.220**
Curvature	-3.111	-2.728	**0.383**
Panel B.2: Chinese Renminbi			
Level	-1.995	-2.298	**-0.304**
Slope	-1.484	-1.261	**0.223**
Curvature	-3.561	-3.172	**0.389**
Panel B.3: Swiss Franc			
Level	1.268	1.208	**-0.060**
Slope	-0.852	-0.945	**-0.093**
Curvature	-1.761	-1.144	**0.617**

Table 2.6: The effect of introducing structured Eurobonds in a conservative structure on the exchange rate EUR/GBP, EUR/CNY and EUR/CHF. The introduction months are January and February 2018.

Δs_{24} is the estimated 24-month impact of an issuance of structured Eurobonds on the exchange rate EUR/USD, $\beta_{24,2}$ is the calculated current 24-month connection of a change in the slope factor. This value can be found in Panel A of Table 2.2. We can conclude an annualized impact of -0.69% in January and -0.75% in February on the exchange rate for a two-year horizon. As we have taken the exchange rate in Dollar price per unit, the Euro faces an annualized depreciation of 0.69% respectively 0.75% in the following 24 months. For a better comparison between the four different foreign currencies, the focus will lie on a twelve-month time horizon for USD, CHF, and CNY, because it is the only horizon with significant factors for each of these currencies. GBP is only significant in shorter time horizons, and therefore we focus on the six-month horizon.

This delivers an impact of -1.07% in January and -1.17% in February on the exchange rate EUR/USD in the following year after the introduction of structured Eurobonds. The difference in the calculated impact is due to monthly changes in the yield curves of the US and the EMU countries. This impact is the sole effect of issuing structured Eurobonds. The next step is to combine it with market expectations which are derived from the Consensus Forecasts dataset. It implicates a depreciation of the Euro of -0.80% in January and an appreciation of 0.36% in February 2018 in the upcoming 12 months. To calculate the market expectation after the issuance of structured Eurobonds, we have to add the sole effect, which is a depreciation of the Euro, to the market expectation. In sum, after the (theoretical) introduction of structured Eurobonds a depreciation of -1.87% and -0.81% is being predicted.
When we are evaluating British Pound, the focus is on the change of two Nelson-Siegel factors, level and slope. Eq.(2.6) and the following calculations have to be altered here to fulfil the changed requirements of significant parameters. The impact of issuing structured Eurobonds in January will be an annualized rise of 2.27% of the exchange rate in the following six months, which is equivalent to an appreciation of the Euro. Contrary to the results for the US-Dollar the effect is weaker for February 2018 with an impact of 1.79%.[18] The same direction of impact can be seen for Chinese Renminbi. The effect is weaker than the one for British Pound with 0.67% for January and drops to 0.53% if structured Eurobonds would have been issued in February 2018. At last, we take a closer look at the impact on Swiss Franc, which is only determined by the relative curvature factor. The effect is 1.48% for January and weakens to 1.32% in February. A look at the forecasts from market participants for January and February shows that an appreciation of the Euro against British Pound, Chinese Renminbi, and Swiss Franc was predicted. The issuance of structured Eurobonds would have boosted this estimation. The combination of both observations – the sole effect of the issuance of structured Eurobonds and estimations of market participants – also delivers an appreciation of the Euro. The appreciation is 4.57% (2.17%) / 2.54% (3.05%) / 3.15% (4.22%) for GBP / CNY / CHF in January (February).

The next step is to combine the impacts to get a full effect on exchange rates. Therefore, we use trade statistics of the European Union and exclude the United Kingdom. The imports and exports of these countries for 2017 represent more than 45% of the complete trades of the European Union. The largest share in imports is China, with

[18]The sole influence in the following six months is 1.13% and 0.89% in January and February, respectively. For comparison purposes the annualized value is taken.

19.1% of the whole imports of the EU. USA is the country with the most exports from the EU, amounting to 17.3%. Now, the impact of every single country is weighted by its share on the trade with the EU. As a result, we get an effective trade exchange rate. The individual impact can be seen in Table 2.7.

Country	Imports (in Million)	Exports (in Million)
United States	257,265.1	376,166.8
United Kingdom	186,246.3	295,399.5
China	386,311.3	234,438.9
Switzerland	110,727.4	149,843.0
Total	940,550.1	1,055,848.2
Country	Import (in % of **Total**)	Exports (in % of **Total**)
United States	27.35%	35.63%
United Kingdom	19.80%	27.98%
China	41.07%	22.20%
Switzerland	11.77%	14.19%

Table 2.7: The trade balances of the four countries with significant results for exchange rate changes. The first panel shows the absolute import and export statistics. The second panel sets the individual volume in result to the total of the four countries.

Here the individual value of imports and exports is set in relation to the total values for all four countries. Combining the above-calculated impact on exchange rates with the individual trade-related share of the four countries delivers an appreciation of the Euro. In January the Euro would appreciate 0.61% independent from the weighting scheme and in February between 0.39% and 0.41%. The depreciation from the EUR/USD is not enough to offset the appreciations from the three other currency pairs.

After all, the isolated impact of issuing structured Eurobonds would be that the Euro faces a depreciation against the US-Dollar in a conservative structure whereas it would appreciate against the three others in a range from -1.17% to 2.27%. The different effect for the USD is also a consequence of its unique role as a reserve currency in the global economy.

2.4.2 Progressive Structure

Now, we want to have a closer look at a structure with even lower correlations than in the conservative structure shown in the previous section.[19] In this case, the BBB+ tranche is being dropped, leaving only three tranches. The AAA tranche now represents more than 95% of the total issuance. We can see the structure in panel C of Table 2.4. Due to the structure of the tranches, the new yield curve will be nearly an AAA curve which is close to the current German one. The newly created yield curve in this structure can be seen in Figure 2.3. We see a flattened yield curve with only small changes in the short horizon in comparison to the previous curve but with a larger downward shift for the long horizon. This will influence the different Nelson-Siegel factors and will result in an even stronger exchange rate impact.

This assumption is proven by the numerical results which can be seen in Table 2.8. The setup of the table is the same as in the previous section. The differences in crucial factors are rising in absolute values.

The shock of introducing structured Eurobonds in January or February on the relative slope factor for US-Dollar would be 0.62 and 0.67, respectively. This delivers an effect of -3.31% to -3.57% on the exchange rate against the US-Dollar. In this case, the Euro will again face a depreciation of 3.31% to 3.57% the following 12 months after introducing structured Eurobonds. This is equivalent to an increase of the effect by more than two percentage points compared to the results from the conservative structure. Adding the expectations from market participants does not change the picture, and the Euro is still facing a depreciation of 4.11% in January and 3.21% in February.

As in the previous section, the other foreign currencies would depreciate against the Euro. The effect is stronger than in the previous scenario, now ranging from 4.72% to 5.37% for British Pound, which is more than three percentage points higher than in the conservative structure. For Chinese Renminbi, the impact is 1.74% and 1.43% for an introduction in January and February, respectively. The picture is the same for Swiss Franc with a stronger impact in January (3.27%) and a decline in February (2.74%). This reflects a growth of more than one percentage point for Chinese Renminbi and nearly two percentage points for Swiss Franc. This strong impact can be explained by the drop of nearly 25% in the BBB tranches and an increase of nearly 40% in the AAA tranche. The addition of the market expectations in the Consensus Forecasts dataset delivers 7.67% (5.10%) / 3.61% (3.95%) / 4.94% (5.64%) for GBP / CNY / CHF in January (February).

[19] An examination of an ordinary structure with higher assumed correlations than in this case, but lower than in the conservative structure, can be found in Appendix B.4. The results lie between the ones of the conservative and progressive structure.

Panel A: January 2018 Factor	Current Values	New Values	Difference
Panel A.1: US-Dollar			
Level	-0.651	-1.504	**-0.853**
Slope	-0.991	-0.367	**0.624**
Curvature	-4.711	-3.499	**1.211**
Panel A.2: British Pound			
Level	0.221	-0.669	**-0.890**
Slope	-0.930	-0.291	**0.639**
Curvature	-2.471	-1.098	**1.372**
Panel A.3: Chinese Renminbi			
Level	-2.059	-2.938	**-0.879**
Slope	-1.627	-0.982	**0.645**
Curvature	-3.705	-2.432	**1.273**
Panel A.4: Swiss Franc			
Level	1.343	0.710	**-0.633**
Slope	-0.835	-0.513	**0.321**
Curvature	-2.284	-0.753	**1.531**
Panel B: February 2018			
Panel B.1: US-Dollar			
Level	-0.836	-1.691	**-0.855**
Slope	-1.044	-0.372	**0.672**
Curvature	-4.433	-3.369	**1.064**
Panel B.2: British Pound			
Level	0.295	-0.549	**-0.844**
Slope	-1.016	-0.356	**0.660**
Curvature	-3.111	-2.061	**1.050**
Panel B.3: Chinese Renminbi			
Level	-1.995	-2.841	**-0.847**
Slope	-1.484	-0.818	**0.665**
Curvature	-3.561	-2.516	**1.045**
Panel B.4: Swiss Franc			
Level	1.268	0.665	**-0.603**
Slope	-0.852	-0.505	**0.347**
Curvature	-1.761	-0.477	**1.284**

Table 2.8: The effect of introducing structured Eurobonds in a progressive structure on the exchange rate EUR/USD, EUR/GBP, EUR/CNY and EUR/CHF. The introduction months are January and February 2018.

We again build a trade-weighted exchange rate. As before, the Euro would appreciate, and the other three currency pairs would offset the impact on the exchange rate of EUR/USD. The effect ranges from 1.17% to 1.26% in January and 0.76% to 0.87% in February. On average, the Euro would appreciate more than 1% in this structure and a trade-related exchange rate.

2.5 Robustness check

2.5.1 Introduction Time

We can observe that the impact on the exchange rates crucially depends on the time of introduction of structured Eurobonds. The effect can be more than 0.5 percentage points larger if Eurobonds are issued in January compared to February. Therefore, a historical analysis with several introduction months is examined. The focus lies on the past twelve months and simulates an introduction every month, starting in March 2017. The connection between changes of the relative Nelson-Siegel factors and exchange rates is fixed to the results displayed in Tables 2.2 and 2.3. We again assume an issuance with a conservative structure because the progressive structure does not change the direction of the impact and only makes it stronger.

For the examination, the minimum, mean, and maximum of changes in the relative Nelson-Siegel factors are built. This can be seen in Table 2.9. It can be seen that for US-Dollar, British Pound, and Chinese Renminbi, the changes in the factors are similar. For Swiss Franc, the changes are smaller. This is due to a relatively steady yield curve in Switzerland in the observed time horizon. In other countries, there are larger changes in the yield curves, and they are moving in similar directions.

Now, we evaluate the impact of the changes on the exchange rates. The previous results are confirmed by the changes obtained for more introduction times. It can be concluded that the effect on the exchange rate of EUR/USD is between -0.61% and -1.50% with an average change of -1.16% in the past twelve months. The minimum is reached in April 2017 and the maximum in September 2017. In every month, the Euro would face a depreciation. The results for January and February 2018 are close to the average value.

The Euro will again appreciate against the other three foreign currencies. The minimum annualized impact for British Pound is 1.79% with an introduction in February 2018 and can increase up to 3.67% for an introduction in April 2017 with an average value of 2.79%. It can be seen that the results for January and February 2018 obtained above are at the lower bound for the last twelve months. Therefore, the previous es-

Factor	Min. Change	Mean Change	Max. Change
Panel A: USD			
Level	-0.300	-0.373	-0.411
Slope	0.155	0.218	0.283
Curvature	0.381	0.616	0.893
Panel B: GBP			
Level	-0.301	-0.390	-0.444
Slope	0.117	0.226	0.307
Curvature	0.383	0.675	0.959
Panel C: CNY			
Level	-0.304	-0.389	-0.430
Slope	0.128	0.230	0.297
Curvature	0.389	0.655	0.931
Panel D: CHF			
Level	0.002	-0.057	-0.121
Slope	-0.058	-0.131	-0.258
Curvature	0.303	0.506	0.693

Table 2.9: The effect of introducing structured Eurobonds in a conservative structure on all four significant foreign currencies. The introduction months are March 2017 to February 2018. It displays the minimum, mean, and maximum change in the relative Nelson-Siegel factors.

timation is conservative. The same can be seen for Chinese Renminbi. The minimum effect is 0.53% in February 2018, and it reaches a maximum of 1.27% in April 2017 with an average change of 0.92%. The results for January and February 2018 are again at the lower bound.

Against Swiss Franc, the Euro will also appreciate. The minimum annualized impact is 0.65% for an introduction in August 2017and reaches 1.48% for an introduction in February 2018. The average effect is 1.11%. The results for January and February 2018 are at the upper bound of the last twelve months.

As in the previous chapters, a trade-weighted exchange rate is consulted to measure an effective exchange rate. In every month the Euro will appreciate. The minimum impact in the last twelve months is 0.39%, and the maximum is 1.27%, with an average of 0.74%. Both the maximum and the minimum are reached when the import shares are used as weights. The minimum is reached for an introduction in February 2018 and the maximum for April 2017. Therefore, the results in the previous chapters are conservative because the effect of the issuance of structured Eurobonds would have been stronger in the preceding months.

2.5.2 Uncertainty measure

Besides the impact of the relative yield curve on the exchange rate, we also like to have a view on a global uncertainty measure which can be an additional explanation of the exchange rate shift. For this, the CBOE Volatility Index (VIX), which is calculated daily by the Chicago Board Options Exchange, will be used. It represents the implied volatility of options on the S&P 500 index. The S&P 500 index consists of the 500 largest US companies with respect to their market capitalization.[20] Regression (2.2) is modified by adding the VIX and results in

$$y(m) - y^*(m) = L_t^R + S_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + C_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right) + \delta_1 VIX + \epsilon_t. \quad (2.7)$$

Data for the VIX is taken from Reuters Datastream for the same time horizon as for the yield curves resulting in 162 observations. For the regression, we again use month-end data. This delivers the connection displayed in Table 2.10.

Euro / US-Dollar	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-10.031*	-8.064*	-8.197*	-6.169	-4.224	-3.879
	(-1.851)	(-1.786)	(-1.933)	(-1.567)	(-1.338)	(-1.378)
Slope	-9.391***	-8.893***	-9.561***	-6.917***	-4.865**	-4.531**
	(-2.652)	(-2.994)	(-3.410)	(-2.637)	(-2.325)	(-2.371)
Curvature	0.777	0.865	0.276	-0.076	-0.114	-0.568
	(0.442)	(0.576)	(0.191)	(-0.055)	(-0.100)	(-0.541)
VIX	1.455***	0.659*	0.751**	0.477	0.222	0.336
	(3.366)	(1.828)	(2.261)	(1.518)	(0.884)	(1.464)
R^2	0.082	0.138	0.317	0.366	0.416	0.532

Table 2.10: The effect of uncertainty, measured by the index VIX, beside the Nelson-Siegel factors on the exchange rate changes. The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

The relative slope factor is now highly significant for every time horizon, and the values for this factor are higher than in the baseline case presented in Section 2.3.2. Also, the VIX and the level factor are significant for every time horizon below 12 months. The VIX can describe the higher significance of both factors. As the level and the slope of a yield curve are explaining long-term interest rate and country growth expectations, an uncertainty measure smooths out these disturbances. The R^2 reaches a maximum of 0.532 in this particular scenario with the VIX as an additional parameter.

[20] Although it is a specific US index, the VIX can be used as a measure for global uncertainty due to spillover effects from the US stock market to others.

The higher values of the slope factor are a hint that the first evaluations are too conservative. The connection between the Nelson-Siegel factors and the exchange rate becomes stronger, and thus, the shocks after introducing structured Eurobonds will have a stronger impact. To emphasize this, the exchange rate influence in a conservative structure is calculated. The effect on the other structure is the same. We can see an effect of the slope factor of -6.917 in the regression results displayed in Table 2.10 for a 12-month horizon. Since this does not influence the change of the EMU yield curve, the difference in Table 2.5 to find the change in the slope factor can be used. The impact on the factor is between 0.20 and 0.22. This leads – with Eq.(2.6) – to an exchange rate effect between -1.38% and -1.52% which is higher than in the setting of the previous chapter. Again, this sole effect predicts a depreciation of the Euro against the US-Dollar. The regression for the other three foreign currencies delivers similar results. R^2 is growing compared to the examination in the baseline scenario. For CHF and CNY, the curvature factor loses its significance for every time horizon. In the 12-month horizon, the rescaled t-statistic shows that the curvature factor is close to the 10% significance level. Also, the sign and the strength of value are nearly the same compared to the baseline scenario. Therefore, the effect on the exchange rate stays the same.

The level factor loses its significance for GBP but is close to the 10% significance level in a 6-month horizon. With this included, the direction of the impact on the exchange rate stays the same. When the focus is only on the slope as the remaining significant factor, the impact reverts and the Euro will depreciate.

We can conclude that the inclusion of this uncertainty measure increases the explanatory power of the regression.[21] Besides, it supports our findings and the direction of the effect on exchange rates. However, the effect is stronger in this scenario which might be explained by the impact of the uncertainty measure on the three Nelson-Siegel factors. For a more conservative estimation, the baseline scenario shall be consulted.

2.5.3 Post Lehman Default

In the previous results, we have seen that the Euro is only depreciating against the US-Dollar. This is counter-intuitive to the original assumption that the Euro will be strengthened in its role as a reserve currency. A subsample with a shorter time horizon that drops the more distant past might be more accurate in this context. To shed some

[21] An alternative uncertainty measure with the Global, European, and US EPU by Baker et al. (2016) was also tested. It supports the previous results gained by including the VIX, but the R^2 is slightly lower. Nevertheless, the sign of impact is not changing and is therefore not discussed in more detail.

light on this issue, the focus now lies on the time after Lehman Brothers defaulted in September 2008. We start the time horizon in December 2009 and again end in February 2018. Since the time horizon starts in December 2009 with more than a year past the Lehman default, the negative effects that arise solely from the financial crisis can be nearly omitted. In December 2009 the interest rates in the USA were low, but they rose to an ordinary value by February 2018. In contrast, the interest rates in the Eurozone continue to be low. The analysis is again run for all five foreign currencies. JPY is included to check whether it gains some significance in the smaller sample.

The connection between the exchange rate and the yield curve needs to be recalculated. The results can be seen in Tables 2.11 and 2.12.

Panel A: US-Dollar						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-16.499*	-14.065*	-12.812	-12.641	-7.136	-1.231
	(-1.830)	(-1.757)	(-1.629)	(-1.492)	(-0.853)	(-0.121)
Slope	-17.716**	-14.924**	-14.218**	-14.794*	-9.226	-2.488
	(-2.416)	(-2.222)	(-2.068)	(-1.855)	(-1.176)	(-0.264)
Curvature	2.901	1.943	1.259	0.615	0.050	-0.206
	(1.317)	(1.010)	(0.692)	(0.364)	(0.033)	(-0.13)
R^2	0.029	0.111	0.199	0.313	0.243	0.015
Panel B: British Pound						
Level	-22.742**	-14.473	-14.470*	-12.922	-9.233	-3.263
	(-2.099)	(-1.608)	(-1.685)	(-1.472)	(-1.011)	(-0.340)
Slope	-17.947**	-13.297*	-12.998*	-11.940	-7.688	-2.159
	(-2.034)	(-1.792)	(-1.796)	(-1.551)	(-0.946)	(-0.256)
Curvature	1.192	0.230	0.314	-0.071	-0.389	-0.953
	(0.621)	(0.141)	(0.196)	(-0.040)	(-0.213)	(-0.550)
R^2	0.019	0.073	0.166	0.265	0.242	0.256

Table 2.11: The connection of different Nelson-Siegel factors and their predictive power for exchange rate changes with the currency pair EUR/USD and EUR/GBP. The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

There are several changes in the connection of the Nelson-Siegel parameters to the exchange rate in comparison to Tables 2.2 and 2.3. For USD, the level factor gets significant for a one- and three-month horizon and the slope factor is now significant for every time horizon up to 12 months. The factors are larger in absolute values than in the previous setting, e.g. the impact of the slope changes from -5.309 to -14.794 in a 12-month horizon. In the case of GBP, the level and slope factor gain significance for the one-month horizon, but the level factor loses its significance in the 3-month horizon. The factors are again larger in absolute values. CNY loses its significance

Panel A: Chinese Renminbi						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-7.929	-5.026	-3.205	-2.995	-2.324	-2.146
	(-1.445)	(-0.947)	(-0.576)	(-0.571)	(-0.553)	(-0.511)
Slope	-5.278	-2.515	-0.494	1.119	1.798	1.497
	(-1.023)	(-0.512)	(-0.098)	(0.235)	(0.479)	(0.413)
Curvature	4.749	2.671	1.897	2.943	2.234	1.424
	(1.554)	(0.946)	(0.674)	(1.130)	(1.088)	(0.694)
R^2	0.028	0.035	0.047	0.215	0.388	0.348
Panel B: Swiss Franc						
Level	-9.573	-2.580	1.029	0.961	1.591	3.296
	(-1.218)	(-0.317)	(0.127)	(0.141)	(0.261)	(0.578)
Slope	-10.084	-2.934	1.430	1.069	1.543	3.302
	(-1.345)	(-0.366)	(0.186)	(0.167)	(0.275)	(0.652)
Curvature	2.347	3.085	3.170*	3.177**	2.619*	1.829
	(1.263)	(1.608)	(1.664)	(2.003)	(1.895)	(1.454)
R^2	0.029	0.048	0.145	0.365	0.471	0.508
Panel C: Japanese Yen						
Level	-17.262	-3.579	3.650	1.381	8.867	6.042
	(-1.407)	(-0.375)	(0.208)	(0.079)	(0.523)	(0.412)
Slope	-25.062	-6.630	-1.778	-5.403	3.290	2.785
	(-1.465)	(-0.375)	(-0.097)	(-0.290)	(0.184)	(0.180)
Curvature	-0.001	-0.030	-1.847	-1.351	-0.505	1.500
	(-0.003)	(-0.010)	(-0.525)	(-0.375)	(-0.139)	(0.295)
R^2	0.025	0.007	0.027	0.080	0.135	0.293

Table 2.12: The connection of different Nelson-Siegel factors and their predictive power for exchange rate changes with the currency pair EUR/CNY, EUR/CHF, and EUR/JPY. The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

for every time horizon and is therefore excluded from the examination of this subsample. Nevertheless, it is observable that the signs of the factors are the same and as a consequence, it would deliver an appreciation of the Euro. For CHF, the curvature factor grows in significance and reaches the 10% significance level also for a 6- and 18-months horizon. JPY stays insignificant for every time horizon and is therefore again excluded in the further examinations. For better comparability, we again focus on the 12 months horizon for USD and CHF, and on the 6-months horizon for GBP. An introduction of structured Eurobonds in a conservative structure is assumed. The differences in the Nelson-Siegel factors are the same as before and can be seen in Table 2.5 and 2.6.

An introduction in January delivers an impact of -2.99%. This is again equivalent to a depreciation of the Euro against the US-Dollar. The effect increases for an introduction in February where the impact -3.25%.[22] The other two foreign currencies – GBP and CHF – show the same picture as in the full-sample case. The Euro would face an annualized appreciation of 2.15% (1.50%) in January (February) against British Pound. This is slightly lower than in the baseline case. The effect is stronger for CHF. Here the Euro would appreciate 2.20% (1.96%) in January (February), which is nearly one percentage point larger than in the baseline case. Ultimately, it is notable that the direction of the effect stays the same for every of the foreign currency.

For a complete examination of this sample split, we also focus on the era where the Lehman default is included. The sample now starts in September 2004 and reaches to November 2009, which is contrary to the first part. Again, the first examination is on the connection between the Nelson-Siegel factors and the exchange rate predictability. Every currency pair loses significance for several time horizons besides of Japanese Yen compared to the full sample. The US-Dollar stays significant in the level factor, but only in a three and six-month horizon. The sign stays the same and implies a depreciation of the Euro. In the case of British Pound, the level factor is not significant for any time horizon, and the slope is significant for one month and curvature for three months. The factors are positive and an introduction of structured Eurobonds would shift the yield curve in a way that both factors rise and as a result, the Euro would appreciate. Swiss Franc is not significant for any time horizon. Nevertheless, the sign for the curvature factor stays the same compared to the baseline case for a twelve-month horizon, which is an indicator that the Euro would again appreciate. The curvature factor remains significant for Chinese Renminbi in a twelve-month horizon but is insignificant for the six-month horizon. The sign stays the same as before, but the factor decreases. Therefore, the Euro would again appreciate, but the effect is smaller than before.

A contrary picture can be seen for the Japanese Yen. In the previous cases, the currency has not been significant for any time horizon. In this subsample, the level factor is significant for every time horizon beside one month. The focus lies on the twelve-month horizon for a better comparison to the previously obtained results. Here, the regression delivers a connection of -23.267 between changes in the relative level factor and exchange rate changes. Therefore, if the relative level decreases by one percent, the Euro would appreciate by 23.267%. In the conservative structure, the level factor

[22] An introduction through the other structure also delivers a depreciation of the Euro. The maximum value is -9.94% in a progressive structure with an introduction in February.

would change by -0.55 and -0.48 in January and February, respectively. This delivers an impact of 12.77% and 10.19% on the currency pair EUR/JPY and is equivalent to an appreciation of the Euro. The effect is again stronger for the progressive structure with an appreciation of the Euro of 20.42% and 19.52%. Due to the large impact and the fact that the currency pair shows only significance in this special subsample, the validity is to be seen as critical and is therefore not displayed in detail. Nevertheless, it indicates that the Euro would also appreciate against Japanese Yen when structured Eurobonds will be issued.

2.6 Conclusion

An issuance of structured Eurobonds through an ABS-approach would not only influence the European sentiment but would also have a severe impact on capital markets. The fragmentation of sovereign bond markets in the European Monetary Union would vanish, and the interest burden of every single country would be reduced. The impact of an introduction on the FX market has not yet been examined although the strengthening of the Euro resulting from a new bond system was mentioned in recent research references.

Using the Nelson-Siegel model and previous methods established by Chen and Tsang (2013), we can find a significant connection between exchange rate predictability and the relative yield curve of the European Monetary Union and other countries. This connection is significant in all Nelson-Siegel factors. When Eurobonds are issued with an ABS-approach, an issuance on a country level is not necessary and individual yield curves are no longer existing. As a consequence, a new yield curve on an EMU-level would replace the current one. Dependent on the structure of issuance, the shape will be different, ranging from a nearly AAA yield curve to a mixed yield curve near to an AA curve, e.g. like for Belgium or Spain. The new yield curve causes a shock by influencing the relative Nelson-Siegel factors. This shift has an impact between -1.07% and -3.57% on the exchange rate of Euro against US-Dollar, dependent on the structure and time of introduction of structured Eurobonds. The impact describes a depreciation of the Euro against US-Dollar in the following 12 months. The other three examined foreign currencies – British Pound, Chinese Renminbi and Swiss Franc – will face depreciation, ergo the Euro will appreciate. The strength of this effect also depends on issuing time and structure. It will reach an impact between 0.53% and 5.37% in the following 12 months after the issuance of structured Eurobonds. The Japanese Yen was also examined, but it has not shown significance for any time horizon.

A post-crisis sample is also analysed. Here, the direction of the effect stays the same and grows in its strength. In the end, it seems very likely that the Euro faces an appreciation against every foreign currency, but not the US-Dollar. This might be a consequence of the unique position of the US-Dollar in the exchange rate market and as a reserve currency.

Appendix B

B.1 Modified Lambda

The choice of λ as 0.0609 in the Nelson-Siegel model is influenced by Diebold and Li (2006b) and Chen and Tsang (2013). At this value, the maximum impact of λ on the curvature is at 30 months. As we see in the regression results, the curvature factor is only relevant in predicting the exchange rate for CNY and CHF. Other authors such as Afonso and Martins (2012) do not choose a global value, but a local value for every observation time. This value is a result of the least square model. Using this approach for the model from Eq.(2.2) with the European and US yield curve, the median value for λ is 0.0380. This value implies a maximum loading of the curvature factor at a maturity of 47 months, and it is slower increasing than in the baseline choice. Also, the loading of the slope factor is less rapidly decreasing, which implies a longer effect of this factor. Both factors are getting similar after 80 months in the baseline case and after 145 months in this case. This can also be seen in Figure B1. Here, the different impact on the Nelson-Siegel factor is presented.

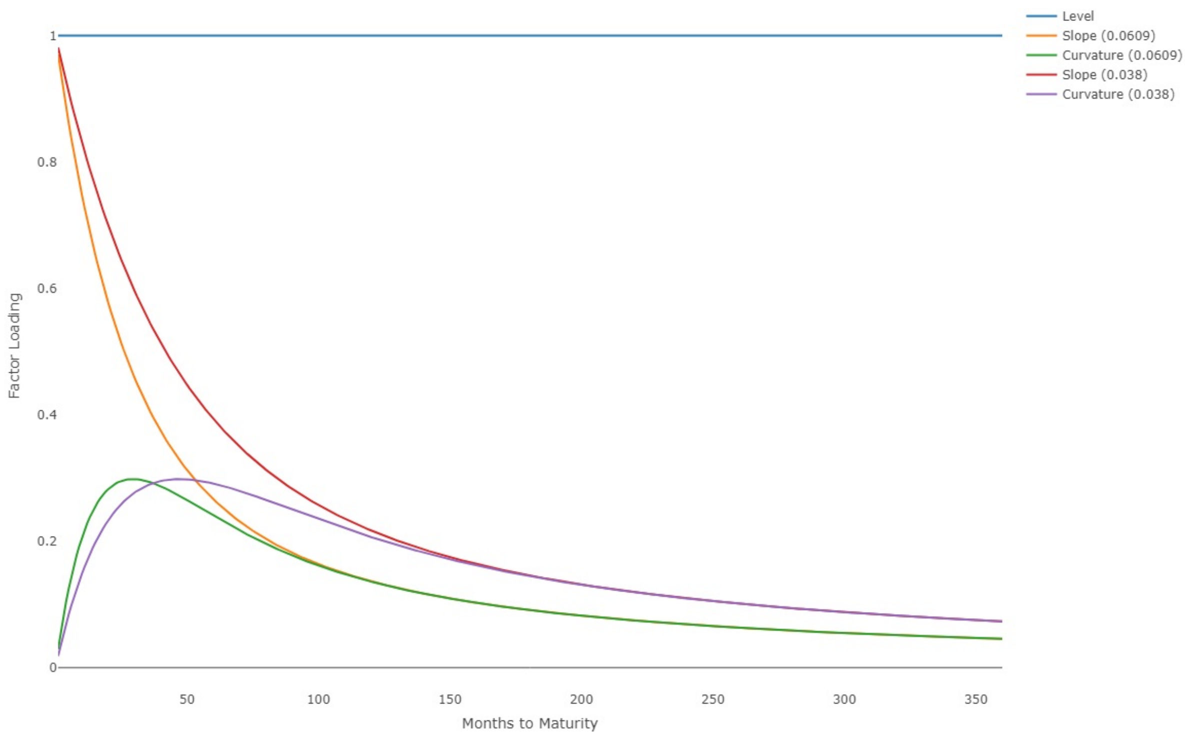

Figure B1: Factor impact with $\lambda = 0.0609$ and $\lambda = 0.038$.

Now the value of 0.0380 is used to calculate the three Nelson-Siegel factors for Eq.(2.2) with the European and US yield curve. Following this, we use the linear regression (2.3) with the new λ and the exchange rate to find a link between the predictability of exchange rates and the change of the three factors. The results are displayed in Table B1. We focus on the examination of US-Dollar as foreign currency because the results for the other currencies are showing the same characteristics and are therefore not explicitly displayed here.

Euro / US-Dollar						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-4.712	-5.835	-5.538	-4.565	-3.526	-2.503
	(-0.906)	(-1.354)	(-1.295)	(-1.158)	(-1.158)	(-0.792)
Slope	-3.522	-5.779**	-6.328**	-5.016*	-3.977**	-3.482*
	(-1.099)	(-2.147)	(-2.329)	(-1.951)	(-2.011)	(-1.705)
Curvature	2.548	2.055	1.673	0.879	0.474	0.026
	(1.592)	(1.525)	(1.226)	(0.676)	(0.461)	(0.024)
R^2	0.022	0.098	0.202	0.268	0.376	0.360

Table B1: The connection of different Nelson-Siegel factors and their predictive power for exchange rate changes for EUR/USD. Now there is a new chosen λ=0.0380. The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

Comparing these results with the results from Table 2.2, we can see that the significance in the slope factor is getting weaker for 12 months, but it remains significant. We will again mainly focus on the slope factor as a predictive instrument. Although the values are changing, the value for our main significant factor – slope – is close to the one generated with the original value of λ in the longest horizon of 24 months. Also, the sign of the factor is the same, so a change in the yield curve will have the same effect on the exchange rate as we have examined before. The focus lies only on the conservative and progressive structure since they are representing two extreme positions in the model. The yield curves are again built using the ECB-capital commitment as macroeconomic indicator. This calculation method ensures consistency and comparability between the computed outcomes.

As before, the λ needs to be recalculated when the new yield curve evolves after the issuing of structured Eurobonds. The non-linear least square model delivers an optimal value of λ for the composed yield curve in the conservative structure of 0.0426 for January and 0.0458 for February. Results for the conservative structure are presented in panel A of Table B2.

In the first columns, the current (without structured Eurobonds) and new values (with structured Eurobonds) calculated with the optimal values of λ are shown. As above mentioned, the values are 0.0426 (Jan.) and 0.0458 (Feb.) for these columns. After this, the difference between both values is shown and for comparison purposes in the last column, the results from the initial λ presented in Table 2.5. Comparing

Factor	Current Values	New Values	**Δ: Mod. λ**	**Δ: $\lambda = 0.0609$**
Panel A: Cons. Structure				
Panel A.1: Jan. 2018				
Level	-0.359	-0.923	**-0.564**	**-0.318**
Slope	-1.569	-1.059	**0.510**	**0.202**
Curv.	-4.064	-3.676	**0.388**	**0.449**
Panel A.2: Feb. 2018				
Level	-0.570	-1.136	**-0.566**	**-0.300**
Slope	-1.581	-0.986	**0.595**	**0.220**
Curv.	-3.741	-3.607	**0.134**	**0.381**
Panel B: Prog. Structure				
Panel B.1: Jan. 2018				
Level	-0.359	-1.381	**-1.022**	**-0.853**
Slope	-1.569	-0.625	**0.944**	**0.624**
Curv.	-4.064	-3.312	**0.752**	**1.211**
Panel B.2: Feb. 2018				
Level	-0.570	-1.591	**-1.021**	**-0.855**
Slope	-1.581	-0.586	**0.995**	**0.672**
Curv.	-3.741	-3.199	**0.542**	**1.064**

Table B2: The effect of introducing structured Eurobonds in a conservative and progressive structure on the exchange rate EUR/USD. The introduction months are January and February 2018.

the values and differences, it can be seen that the relative factors are changing, but the sign of the differences stay the same. Finally, the shift in the relative slope factor is getting stronger with the modified λ. Since the sign of the shift and the regression parameters presented in Table B1 are the same as in the baseline case, there will also be a depreciation of the Euro against US-Dollar in the following 12 months after the introduction of structured Eurobonds. When the focus lies on the results for a 12-month horizon, the original impact is a 1.07% depreciation for January. With the modified λ, the depreciation reaches 2.56% due to the relative slope factor for an introduction in January. The same pattern can be observed for the following introduction month. The difference in the slope factor is higher than in January. As a consequence, the impact will be higher. The original λ has a depreciation of 1.17% as a consequence. With the modified λ, a depreciation of 2.98% in the following 12 months can be concluded. Now we want to examine the progressive structure and test whether the modification has the same effect on the exchange rate.

In this case, the non-linear least square model delivers an optimal value of λ for the composed yield curve in the progressive structure of 0.0464 for January and 0.0483 for

February. The values for λ change – compared to the conservative structure – due to a new composition of the yield curve and therefore a new shape. Panel B of Table B2 shows the results. When the results of the original and modified λ – as highlighted in the last two columns of Table B2 – are compared, the differences in the level factor are close to each other and for the curvature factor are vast. The difference in the slope factor is still our main factor of interest. Since the signs are the same, there will also be a depreciation of the Euro against the US-Dollar. We initially found a depreciation of 3.31% for January and 3.57% for February in the following 12 months after introducing structured Eurobonds. In the modified setting, the effect is even stronger. Here, an impact of 4.74% and 4.99% on the exchange rate can be observed.

Therefore, the original choice of λ as 0.0609 delivers more conservative results since the absolute value of the annualized depreciation is lower than in the case with a modified λ.

B.2 Linearity of relative Nelson-Siegel Factors

The impact of an alternative way to calculate the relative Nelson-Siegel factors – when not exploiting the linearity of the model – shall be presented here. In Section 2.3.2 the equation

$$y(m) = L_t + S_t\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + C_t\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right)$$

is introduced as Nelson-Siegel model. Chen and Tsang (2013) make use of the linearity of the model to construct

$$y(m) - y^*(m) = L_t^R + S_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + C_t^R\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right) + \epsilon_t$$

and connect it to exchange rate changes by a linear regression. To check for robustness of the calculation of the relative factors, another way shall be discussed here.

The factors for the two involved countries, e.g. L_t and L_t^*, are calculated using Eq.(2.1) and $L_t^{R,L}$ is determined by forming the difference,

$$L_t^{R,L} = L_t - L_t^*, \tag{B1}$$

where $L_t^{R,L}$ is the linear relative level factor. It has the same meaning as the level factor in the previous sections but the notation is slightly changed to avoid confusion. The focus of the examination lies on the currency pair EUR/USD, with US-Dollar again in the role of the foreign currency marked with the asterisk. Descriptive statistics for the different Nelson-Siegel factors and the difference $\Delta = L_t^R - L_t^{R,L}$, where L_t^R is a result of Eq.(2.3), can be found in Table B3.

	Minimum	Median	Mean	Maximum	Std.Dev
Panel A: Factors from Eq.(B1)					
Level	-1.210	-0.148	0.062	2.411	0.812
Slope	-1.777	0.015	0.052	2.662	0.958
Curvature	-5.158	-0.259	-0.580	5.191	2.269
Panel B: $\Delta = L_t^R - L_t^{R,L}$					
Level	-0.213	-0.059	-0.043	0.162	0.083
Slope	-0.694	-0.183	-0.179	0.165	0.137
Curvature	-0.265	0.458	0.826	2.054	0.418

Table B3: The descriptive statistics for the three different Nelson-Siegel factors out of 162 observations.

Panel B shows that the difference between both methods is minimal for the level factor, is growing – but still small – for slope and has its peak for the curvature. The results give a hint that especially the slope is often higher when using the approach from Eq.(B1), and the curvature, on the other hand, is higher when applying Eq.(2.2). The same regression as in Eq.(2.3) is run with $L_t^{R,L}$, $S_t^{R,L}$, and $C_t^{R,L}$ instead of the original relative Nelson-Siegel factors. The results of this calculation can be seen in Table B4.

Euro / US-Dollar						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	-3.612	-4.394	-4.499	-3.295	-2.091	-1.216
	(-0.706)	(-1.029)	(-1.056)	(-0.838)	(-0.689)	(-0.398)
Slope	-5.023	-6.566**	-6.880**	-5.367**	-4.157**	-3.489*
	(-1.530)	(-2.379)	(-2.475)	(-2.066)	(-2.085)	(-1.748)
Curvature	1.997	1.169	0.824	0.005	-0.413	-0.818
	(1.117)	(0.764)	(0.523)	(0.003)	(-0.346)	(-0.676)
R^2	0.002	0.087	0.186	0.254	0.364	0.386

Table B4: The relative Nelson-Siegel factors are calculated the alternative way as displayed in Eq.(B1). The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

In comparison to the results in panel A of Table 2.2, it is noticeable that the slope factor does not lose any significance. It is significant for every time horizon besides one month. The main point of interest is that neither the sign nor the size of the slope factor – which is the main factor in this evaluation – is changing compared to the previous results. The difference in the slope factor to the previous results is not larger than 0.1 points for any significant time horizon. Also, R^2 has a similar size as in the baseline case. Thus, we can conclude that the direction of the impact of the introduction of structured Eurobonds stays the same. Nevertheless, it is a bit stronger because the connection is slightly larger in absolute values.[23]

B.3 Regression results of single Nelson-Siegel factors

The connection of every relative Nelson-Siegel factor with the exchange rate shall be examined isolated here. For a detailed analysis, we focus on the connection between them and the currency pair Euro and US-Dollar, as displayed in Figure 2.2. The focus again lies on the impact on the exchange rate change.

Therefore, Eq.(2.3) is split into

$$\begin{aligned}\Delta s_{t+m} &= \beta^1_{m,0} + \beta_{m,1} L^R_t + u^1_{t+m},\\ \Delta s_{t+m} &= \beta^2_{m,0} + \beta_{m,2} S^R_t + u^2_{t+m},\\ \Delta s_{t+m} &= \beta^3_{m,0} + \beta_{m,3} C^R_t + u^3_{t+m}.\end{aligned} \tag{B2}$$

The time horizons for m are the same as in the previous analysis. The results are presented in Table B5.

Euro / US-Dollar						
	1 Month	3 Months	6 Months	12 Months	18 Months	24 Months
Level	1.544	-0.331	-0.671	-1.447	-1.512	-1.483
	(0.412)	(-0.101)	(-0.194)	(-0.432)	(-0.532)	(-0.503)
R^2	0.001	0.001	0.001	0.008	0.025	0.036
Slope	-4.077	-5.758**	-6.032**	-4.333*	-3.298	-2.719
	(-1.293)	(-2.166)	(-2.240)	(-1.682)	(-1.593)	(-1.265)
R^2	0.004	0.076	0.158	0.181	0.238	0.214
Curvature	1.115	0.423	0.087	-0.315	-0.425	-0.659
	(0.821)	(0.348)	(0.066)	(-0.236)	(-0.367)	(-0.551)
R^2	0.004	0.002	0.000	0.004	0.009	0.043

Table B5: The relative Nelson-Siegel factors are calculated the alternative way as displayed in Eq.(B2). The significance levels are * 10 percent; ** 5 percent and *** 1 percent. In parenthesis below the factor, the rescaled t-statistic is displayed.

They support the observed connection, which can be seen in panel A of Table 2.2, for the multi-linear regression from Eq.(2.3) between the relative yield curve – or rather the relative Nelson-Siegel factors – and the exchange rate trend. The only

[23] The result and its implications remain the same for the three other foreign currencies. The results can be delivered upon request.

parameter, which remains significant, is the slope, but it is only significant for three time horizons (3 months / 6 months / 12 months). The other two parameters do not show any significance and their R^2 is very low. In the single parameter connection, the slope factor seems to be the main parameter in driving the exchange rate trend for the currency pair Euro and US-Dollar.

The results for the other foreign currencies are mixed. Whereas GBP and CNY lose all of their significance for every time horizon and factor, CHF remains significant in the curvature parameter for a 12- and 18-months horizon. The curvature is also the main describing parameter to calculate the impact of the introduction of structured Eurobonds on the exchange rate EUR/CHF in our baseline case. Besides the loss of significance for some relevant parameters, their sign stays the same compared to the previous regression, when all Nelson-Siegel parameters were included. This is another confirmation that the results and the direction of the impact on the exchange rates are correct.

B.4 Evaluation of Ordinary Structure

The ordinary structure is the baseline scenario in the work of Hild et al. (2014). This structure is built with less rigorous restrictions on the correlation than the conservative structure. The structure is presented in panel B of Table 2.4. Compared with the conservative structure, the BBB tranche drops out and the AAA tranche grows above 85% thickness. Figure B2 presents the yield curve shape in this structure compared to the actual one.

Due to the greater thickness of the AAA part, this yield curve has a lower long-term level and when the other yield curves are subtracted from it, the relative level factor will decrease. The similar values in the short term can be explained with the expansive monetary policy of the ECB, which suppresses individual risk premium. Because the level factor is again lower than in the actual yield curve, we expect an appreciation of the Euro against British Pound. Since the level factor is also lower than in the conservative structure, the effect will be stronger than before. The relative Nelson-Siegel factors and the results for January and February can be seen in Table B6.

It can be seen that the differences in the relative Nelson-Siegel factors are getting greater than when using the conservative structure. They range between 0.55 and 0.60 for the relative slope factor with US-Dollar as foreign currency. Following this result, an impact of -2.92% to -3.16% on the exchange rate can be concluded. This also represents a depreciation of the Euro against the US-Dollar the following 12 months after introducing structured Eurobonds.

Panel A: January 2018 Factor	Current Values	New Values	Difference
Panel A.1: US-Dollar			
Level	-0.651	-1.408	**-0.757**
Slope	-0.991	-0.441	**0.550**
Curvature	-4.711	-3.636	**1.075**
Panel A.2: British Pound			
Level	0.221	-0.572	**-0.793**
Slope	-0.930	-0.363	**0.567**
Curvature	-2.471	-1.249	**1.221**
Panel A.3: Chinese Renminbi			
Level	-2.059	-2.841	**-0.782**
Slope	-1.627	-1.057	**0.570**
Curvature	-3.705	-2.572	**1.133**
Panel A.4: Swiss Franc			
Level	1.343	0.807	**-0.536**
Slope	-0.835	-0.586	**0.249**
Curvature	-2.284	-0.904	**1.380**
Panel B: February 2018			
Panel B.1: US-Dollar			
Level	-0.836	-1.592	**-0.755**
Slope	-1.044	-0.449	**0.595**
Curvature	-4.433	-3.496	**0.937**
Panel B.2: British Pound			
Level	0.295	-0.452	**-0.748**
Slope	-1.016	-0.430	**0.586**
Curvature	-3.111	-2.190	**0.921**
Panel B.3: Chinese Renminbi			
Level	-1.995	-2.744	**-0.749**
Slope	-1.484	-0.894	**0.589**
Curvature	-3.561	-2.641	**0.921**
Panel B.4: Swiss Franc			
Level	1.268	0.761	**-0.507**
Slope	-0.852	-0.579	**0.273**
Curvature	-1.761	-0.606	**1.155**

Table B6: The effect of introducing structured Eurobonds in an ordinary structure on the exchange rate EUR/USD, EUR/GBP, EUR/CNY and EUR/CHF. The introduction months are January and February 2018.

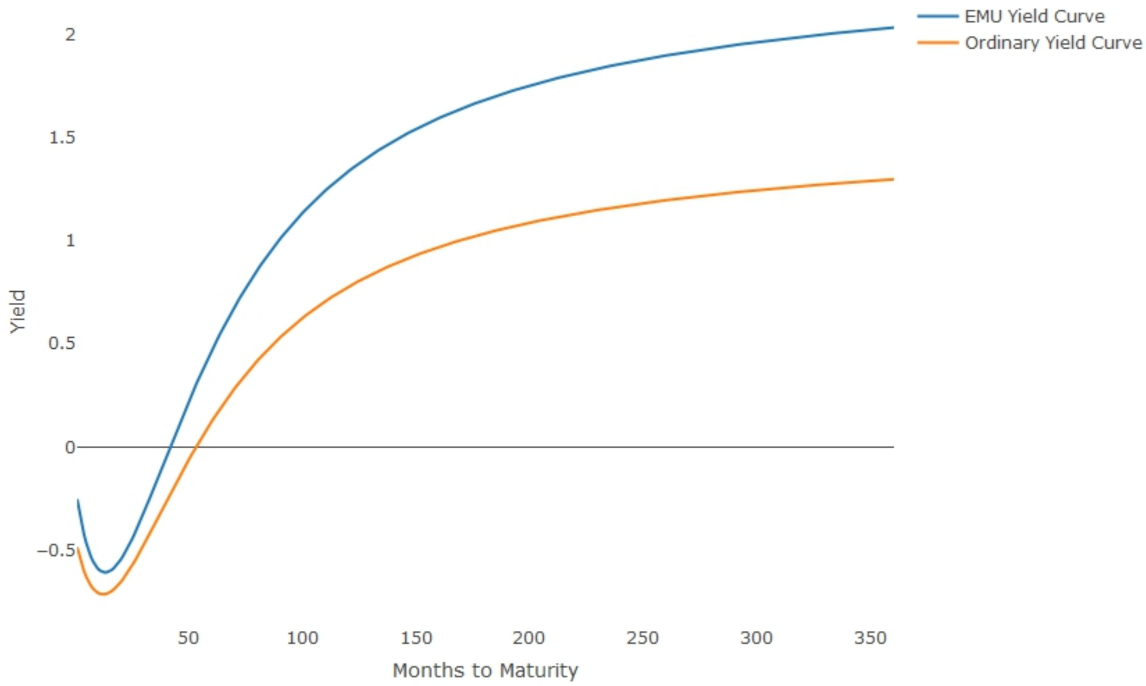

Figure B2: The European and newly constructed Yield Curve with an ordinary structure for End of February 2018 Data.

The same can be seen for the other three foreign currencies where the direction of impact is the same as before, but now the impact is stronger. It grows to 4.80% in January and 4.17% in February for British Pound, which is more than two percentage points greater than in the conservative structure. The impact on CNY is rising to 1.55% in January and 1.26% in February, which is about 0.5 percentage points higher than for a conservative structure. As before the Euro would face an appreciation. The same picture is drawn for Swiss Franc with impacts of 2.94% and 2.47%. Several factors drive the drop on the impact from January to February, which can be seen for every foreign currency. One is the change in the foreign yield curves, and the other is changes in the yield curves of the components of the new curve in the EMU. The two factors sum up and result in this drop in the impact. The same pattern can be seen for the conservative structure and the progressive structure.

As in the previous sections, we also look at the trade-related exchange rate. Combining the trade shares of the four relevant countries with their impact on the exchange rate delivers an appreciation of the Euro. This appreciation is in a range of 1.06% to 1.13% in January and 0.67% to 0.77% in February. On average, it is slightly below 1%. Nevertheless, in this combination, the depreciation of the Euro against the US-Dollar is offset.

3 The Extent of Jensen's Inequality

Evaluating the Approximation Bias in Forward-Looking DSGE Models

Abstract

Occurring in non-linear, forward-looking models, we evaluate a source of error of the type: $\mathrm{E}[f(X)] \approx f(\mathrm{E}[X])$. Since the difference is negligible in typical DSGE models, we explore settings in which this can become a liability. For that purpose, an illustrative model containing growth rates, calibrated via Consensus Forecasts (CF), is utilized in a way that the magnitude of the inequality can be determined in basis points. In a simulation-based analysis, we investigate the accuracy depending on the empirical standard deviation and the function's curvature. Our findings show that the difference is reaching up to 25 or 30 basis points. Also, we analytically solve for a baseline case, showing that the difference depends on the standard deviation in a quadratic way. Finally, the number of variables and the correlation between them is taken as a further influence on the approximation bias. In cases with over five highly-correlated variables, a difference of over 25 basis points can be reached. As a general result, the bias' relation is nearly linear to the data's (co)variance and exponential to both the curvature and the number of variables. Taking this source of error into account, the economist's attention should also be on how a model is built and not only on the data itself. This can be important, especially for large-scale models.

Keywords: Consensus Forecasts, Expectations, Growth Rates, Jensen's Inequality, Non-Linearity, Uncertainty.

3.1 Introduction

In a recent article, Lindé (2018) discusses the usefulness of DSGE models in policy analyses in the aftermath of the 2007–2008 financial crisis. Despite weaknesses in these models, he argues that improved versions will play an important role for a long time to come—at least for smaller policy institutions. He explains that DSGE models are advantageous for smaller entities because more sophisticated models are extremely costly, requiring both advanced human resources and full data access and availability. This usefulness extends to central banks in developing and emerging countries, yet to a lesser degree to central banks in developed countries. Therefore, smaller institutions are reliant upon well-known workhorses in the process of building up their technical analysis apparatus. In the same vein, by discussing challenges when evaluating these models, Fernández-Villaverde et al. (2016), Schorfheide (2013), and Christiano et al. (2011) highlight their (future) importance. This underpins the potential of further scrutinizing models from the "simpler" DSGE family.

In our work, we add another puzzle piece by examining situations in which forward-looking behaviour encircles a non-linear transformation. In the time dimensional context of DSGE models, the conditional expectation usually spans from one period to the next after solving for the first-order conditions. Moving on by log-linearizing, the non-linearities typically resolve and only the function's parameters—multiplied by an expectation value containing growth rates—remain.[24] This method is utilized in many studies, whereby Fernández-Villaverde (2010) and Sbordone et al. (2010) provide a good introduction. As a consequence, models obtain the certainty equivalence property. This sophisticates or suppresses the impact of second (and higher) moments like risk, volatility, or uncertainty.[25] These are measures that play an important role in financial crisis scenarios. To fix this, higher-order approximations are a practical way, but how exactly to incorporate the additional moments can be arbitrary. Avoiding this caveat, in a scenario alike, is equivalent to considering Jensen's inequality.[26] To account for this, we label any changes that are made to the originally derived model equations, *approximation bias*.

To the best of our knowledge, there is no literature dealing with this specific problem. There are, however, a large number of articles dealing with workarounds, e.g. Sargent (1987) and Ljungqvist and Sargent (2012), or sophisticated approximation methods, e.g. Judd (1998) and Aruoba et al. (2006).[27] Two main reasons could account for Jensen's inequality being underrepresented in macroeconomics. First, the resulting error (or approximation bias) is very small in the context of DSGE models. Nonetheless, we aim to see the whole picture and examine several scenarios with a wide variety of parameter values. This gives a good impression in which situations caution is advised. Second, it is challenging (or almost impossible) to find a whole

[24]E.g., $\mathrm{E}_t[(X_{t+1}/X_t)^\eta] \Rightarrow \log \mathrm{E}_t[(X_{t+1}/X_t)^\eta] \approx \eta \mathrm{E}_t[\widehat{x_{t+1}}]$.

[25]See the technical paper by Straub and Ulbricht (2019) for a discussion on the connection between theoretical and empirical second moments.

[26]Note that ignoring Jensen's inequality as an approximation technique is different from (non-linear) Taylor expansion. We show this in Section 3.2.

[27]Sargent (1987, 32) resolves the future consumption (in the context of the Euler condition) at time $t+1$ into variables in time t, with the interest rate as a stochastic element that can be overcome with the unconditional expectation value. Aruoba et al. (2006, 2484) compare several approximation methods for Euler equations, among them perturbation up to order five.

distribution of future values to bring the model to the data as originally intended. In this case, we argue that there is no necessity to have raw data at hand. It is sufficient to calibrate certain parameters concerning the data and to translate model assumptions into parameters. Therefore, we build a small-sized model that could be part of a Euler condition or a similar intertemporal connection, which can be found in every DSGE model—for instance in Christiano et al. (2011, 290) and Schorfheide (2013, 219). In this framework, we derive an analytical solution describing the approximation bias, interpreted in basis points. To assess the variability of the theoretical results we also conduct Monte Carlo (MC) simulations.

Emerging from a variant of Jensen's inequality—for a cleaner economic interpretation—the illustrative model consists of two terms of weighted (or transformed) growth rates, $f(\mathrm{E}_t[gr_{t+1}])$ and $\mathrm{E}_t[f(gr_{t+1})]$.[28] The first term contains the expected growth rate transformed by a non-linear function. The same function weights the future growth rates in the second term and only afterwards, the expectation value is calculated. Subtracting these terms results in the approximation bias measured in basis points. Also, the future growth rates follow a log-normal distribution in our baseline case and an inverted-beta distribution in a robustness check. We will cover this in more detail in the next section. In a final step, we increase the number of variables to emulate large-scale models. Prominent references in this context are the IMF's Global Projection Model (Carabenciov et al. 2013) and the ECB-Global (Dieppe et al. 2018). The latter contains over 800 parameters.

In our model, the different parameters can be assigned to five categories: (i) first moments of the future growth rates, (ii) second moments of the future growth rates, (iii) the curvature of the non-linear function, (iv) the number of variables or the model size, and (v) the number of possible future states or the number of drawn random variables (RV's) per repetition in the MC simulation. Although the mean plays an important role in the analytical solution, the impact of realistic values is negligible. The second moments, consisting of the standard deviation in a univariate case and the correlation in a multivariate case, show a mixed picture. While the latter plays a minor role, even counteracting the bias for negatively correlated variables, large values for the standard deviation produce a serious approximation bias. The third category, the degree of curvature, can be associated with risk aversion or elasticity measures in an economic context.[29] In our model, the outcome is very similar to the impact of the standard deviation. Augmenting the model in a multiplicative way, we add

[28]Note that the growth rates, gr, will be centered around one.

[29]See Bollerslev et al. (2011), Chetty (2006), and Morin and Suarez (1983) for a discussion of the practical application and estimation of the level of risk aversion.

variables to check the influence of the model's size. By including ten variables when assuming a weak, positive correlation in the data, we find a similar result to that of large values for standard deviation and curvature. Except for the last category (the number of future states), the simulated distributions of the approximation bias draw a similar picture, with a constant increase in their second moment and a weak, right-tail property. Increasing the number of (potential) future states, however, decreases the third moments of the initially heavily skewed distributions toward zero. Also, the distributions contract slowly to ideal values of the analytical solution.

As a general result, the approximation bias' relation is nearly linear to the data's (co)variance and exponential to both the curvature and the number of variables. Moreover, the mean of the simulated distributions converges to the analytical result from below when allowing for more possible future states. In most setups, the bias is smaller than ten basis points, which confirms the insignificant role of this issue in the literature. However, taking up the focus on frontier markets as a cost-efficient usage for DSGE models, some of the discussed parameters can become significantly large and, therefore, justify a closer examination.

The remainder of this paper is organized as follows. Section 3.2 explains Jensen's inequality more carefully and how it is used to establish our illustrative model. Also, the non-linear function and the utilized distributions are introduced. Section 3.3 derives an analytical solution for the baseline case, subsequently discussing and interpreting the results. Section 3.4 presents the data and the calibration method. Section 3.5 shows the simulation results, depending on uncertainty (standard deviation), the degree of non-linearity (curvature), number of future states (number of drawn RV's), correlation (covariance), and model size (number of variables). Section 3.6 concludes.

3.2 Theoretical Framework

3.2.1 Preliminary Consideration

Apart from the actual, numerical difference, Jensen's inequality is well studied in the sole mathematical context.[30] Shifting into economic terrain, decision theory, in particular, examines how the expectation value has to be altered such that an equal sign can be applied:

$$\mathrm{E}[f(X)] = f(\mathrm{E}[X] + \tilde{x}), \tag{3.1}$$

where $\tilde{x}$ stands for the risk premium, which can be positive or negative, depending on the curvature of the non-linear function, f. In contrast, we aim to describe the actual degree of the inequality. To account for this difference, we define

$$\mathrm{E}[f(X)] = f(\mathrm{E}[X]) + \Delta_X \tag{3.2.1}$$

$$\Leftrightarrow \quad \Delta_X = \mathrm{E}[f(X)] - f(\mathrm{E}[X]), \tag{3.2.2}$$

where the LHS of Eq.(3.2.1) shows the actual value and the RHS shows the approximated value plus an error term. Later, we will refer to this term as *bias*. When f is convex (concave) the residual Δ is positive (negative).

Considering approximation techniques for f in general, only first-order Taylor expansion makes Jensen's inequality redundant. The second-order version already leaves a distinction between $\mathrm{E}[f(X)]$ and $f(\mathrm{E}[X])$ for non-linear functions. Although—after a quadratic approximation—the original and the proxied expressions contain measures for the curvature, the latter is always smaller for convex functions.[31] For this reason, to not dilute the results by other approximations, our model originates from the basic inequality.

[30] Jensen's inequality for convex functions is: $f(\mathrm{E}[X]) \leq \mathrm{E}[f(X)]$. See, for example, Mitrinović et al. (1993) for an examination of continuous and multivariate versions and the connection to other inequalities.

[31] See Appendix C.1 for the proof, taking advantage of the inequality of arithmetic and geometric means (AM-GM).

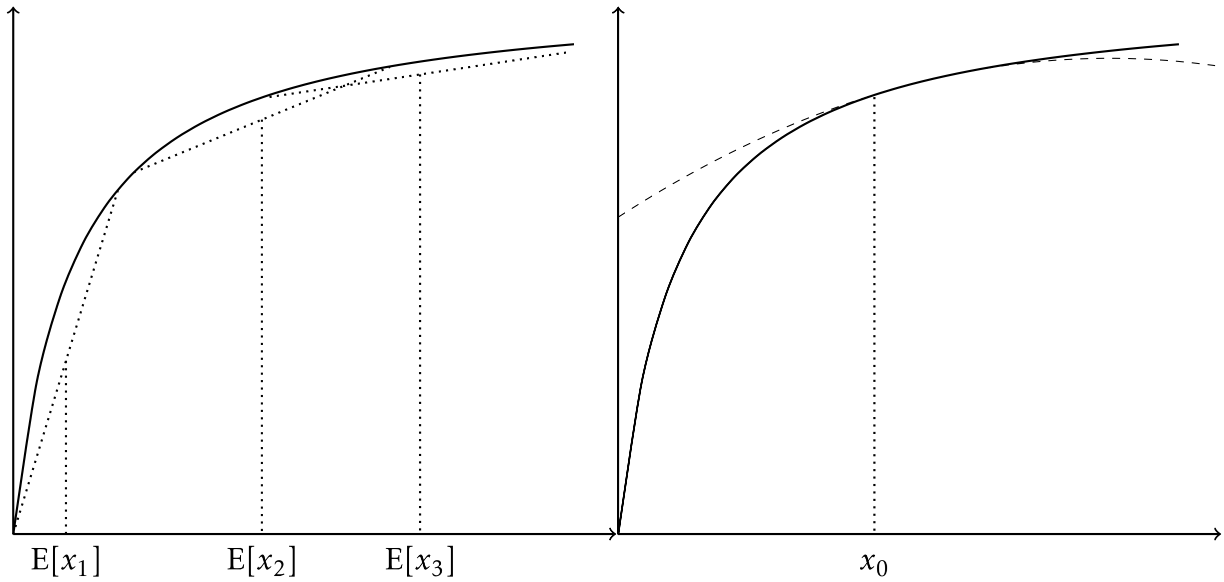

Figure 3.1: On the left side (Jensen), the approximation bias (Δ) is displayed as the vertical difference between the non-linear function (black line) and the intersection of the compatible dotted lines. On the right side (2^{nd}-order Taylor), the difference between the black and dashed line shows the inaccuracy. (The three expectation values and the evaluation point, x_0, are arbitrarily chosen.) The black line is the graph of the function $f(x) = (-1) \cdot (\frac{1}{3} + x)^{-1} + 3$. The dashed line represents the 2^{nd}-order Taylor around x_0 of this function.

Going one step further, a juxtaposition of Jensen and Taylor helps to visualize the methodology. Therefore, Figure 3.1 compares both sources of inaccuracy. Due to graphical clarity, the expectation values are stemming from only two points on the function, respectively. However, taking a (continuous) distribution does not change the basic result. To further depict the difference, one could imagine an additional curve (as the dashed line on the right side) passing through the intersections of the compatible dotted lines, showing a smaller difference the less curved the black line is (left to right). In this example, this holds true even when the range of the x_i increases from left to right. Also, in contrast to the second-order Taylor polynomial (Figure 3.1, right side, dashed line), the Δ are always negative with the sign depending on the function's concavity. In the Taylor example, the sign of the deviation is ambiguous, having an exact result only around x_0, with equal slope and curvature. On the other hand, when Jensen's inequality is applied, a rule of thumb for accuracy is more complicated, such as the "small deviation" around the steady state (x_0) or ±5% growth rates when log-linearizing.

3.2.2 An Illustrative Model

As typically found in Euler equations, we isolate the non-linear, forward-looking part and, for the purpose of generality, apply growth rates to it.[32] Also, even when renouncing log-linearization in DSGE models, the growth rates can be implemented in a straightforward way.[33] This implementation goes hand in hand with the data availability of future rates from the CF data for the subsequent calibration. Moreover, we assume a log-normal distribution, following the approach by Black and Scholes (1973) and Merton (1973).[34] As an alternative, the inverted beta distribution is used, also being restricted with a lower bound (i.e., being non-symmetric), which is appropriate for the data structure. Notwithstanding the usage of growth rates, the expression $\mathrm{E}[X]$ as the arithmetic mean (instead of the geometric mean) is reasonable since the rates are not consecutive but cross-sectional.

In line with most (consumption) Euler conditions, the CRRA utility function provides a flexible, functional form for the non-linearities. As a result of this and because the expressions typically stem from first-order conditions, we use the marginal utility for $f(X)$:

$$f(X) = \frac{1}{(1+X)^{\gamma}}, \quad \text{where} \quad F(X) = \frac{1}{1-\gamma} \cdot \left((1+X)^{1-\gamma} - 1\right) + C_0, \quad \gamma > 0. \tag{3.3}$$

The general antiderivative $F(X)$ in Eq.(3.3) is formulated in a way that for $\gamma = 1$ the function collapses to log-utility, a case common to the literature (see, e.g. Clarida et al. 2000, 170, Galí 2015, 67, Yun 1996, 359).

Figure 3.2 captures these assumptions in a single coordinate system, showing the approach to evaluate Jensen's inequality. It displays the marginal utility $f(X)$ and a probability function from which the random variables X are drawn. Starting next to the lower bound, the marginal utility or revenue from growth rates is the highest, monotonically decreasing for larger values. Not depending on the curvature parameter, the intercept for $X = 0$, which is 1 for every γ, equals the neutral value of one

[32]A simple version, including the marginal utility with respect to consumption and the interest rate as an intertemporal connection is as follows: $\mathrm{d}U/\mathrm{d}C_t = i_t \cdot \mathrm{E}_t[\mathrm{d}U/\mathrm{d}C_{t+1}]$.

[33]E.g., $\mathrm{E}_t[(X_{t+1}/X_t)^{\eta}] = \mathrm{E}_t[(1+\widehat{x_{t+1}})^{\eta}]$.

[34]More precisely, the Black-Scholes formula requires log-normally distributed returns to price options in a relatively simple manner. In a similar context, the distribution is first mentioned by Samuelson (1965). The obvious normal distribution is problematic since there is no lower bound (for growth rates) and, defined on $\mathbb{R}$, the expression $\mathrm{E}[X^p]$ is hard to solve and requires many cases or complex functions. This is not practical for an analytical solution.

describing a situation without growth. In the following, we refer to the transformation result as weighted, inverse growth rates.[35]

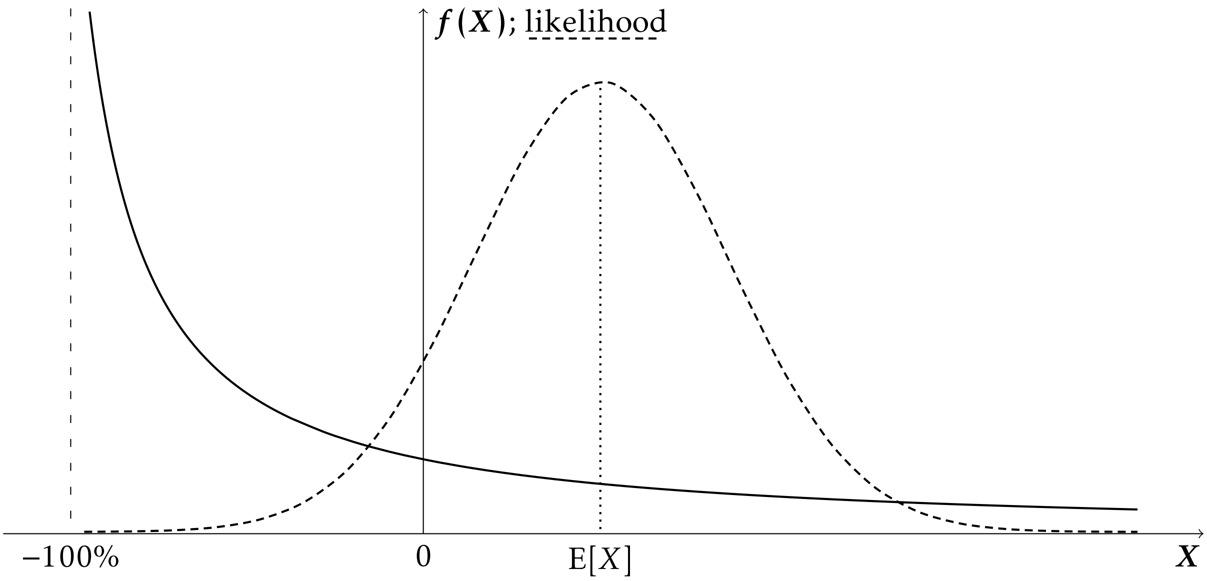

Figure 3.2: The model is tailored to growth rates due to a lower bound. Shown as schematic representation, values are drawn from a distribution (dashed curve) to insert into the non-linear function (black curve), $f(X)$.

Three of the model's decisive parameters are visualized in the graphics: the curvature in the hyperbola and the mean and variance in the density function. The latter predefines the probability by which values X, the growth rates, are drawn to insert into $f(X)$. Therefore, the image of X needs to be a subset of the domain of f.

Based on Eq.(3.2.2), the plain concept, we alter the equation for a more intuitive and meaningful economic interpretation. At first, we have to show that $\frac{1}{\mathrm{E}[f(X)]}$ is the growth rate in the model. As stated in the introduction, $\mathrm{E}[f(X)]$ is the expected value of weighted future growth rates. They are weighted by the above introduced $f(X)$. Due to the structure of the marginal utility, building the inverse is just a transformation back to growth rates.[36]

For the alteration of the concept, we make use of Jensen's inequality and the marginal utility:

$$f(\mathrm{E}[X]) \leq \mathrm{E}[f(X)] \tag{3.4.1}$$

$$\Leftrightarrow \quad \mathrm{E}[f(X)]^{-1} \leq f(\mathrm{E}[X])^{-1} \tag{3.4.2}$$

[35]Weighting growth (as a relative number, expressing the change in absolute numbers) by means of an exponent works analogously to weighting absolute numbers multiplicatively with coefficients. Here, the approximate log-transformation can be misleading since growth rates often appear to have a coefficient or to be addable.

[36]The interpretation as growth rate difference can also be derived with first-order Taylor expansion and, thus, would be similar. See Appendix C.2 for the proof and numerical examples.

$$\Leftrightarrow f(\mathrm{E}[X])^{-1} - \mathrm{E}[f(X)]^{-1} \geq \ 0 \tag{3.4.3}$$

$$\Leftrightarrow f(\mathrm{E}[X])^{-1} - \mathrm{E}[f(X)]^{-1} \geq \ \underbrace{bias(X)}_{\geq 0} \tag{3.4.4}$$

$$\Leftrightarrow \qquad bias(X) = \ (1 + \mathrm{E}[X])^{\gamma} - \mathrm{E}[(1 + X)^{-\gamma}]^{-1} \tag{3.4.5}$$

Pouring the preceding considerations into one equation results in

$$bias(X) = 10^4 \cdot \left((\mathrm{E}[1 + X])^{\gamma} - \mathrm{E}[(1 + X)^{-\gamma}]^{-1}\right), \quad (1 + X) \sim \log \mathcal{N}\left(\mu, \sigma^2\right), \tag{3.5}$$

the approximation bias measured in basis points (bp) when multiplying the RHS of Eq.(3.5) by 10^4. $f(X) = (1 + X)^{-\gamma}$ is the derivative (marginal utility) of the CRRA (isoelastic) utility function with γ as the curvature and γ^{-1} as the elasticity of intertemporal substitution (EIS). X follows a horizontally shifted log-normal distribution with a lower bound of -1. This meets the characteristics of growth rates with their minimum possible value of -100% and no upper bound. However, the actual parameter constellation concentrates the probability mass relatively tight around their mean value slightly larger than zero. Ultimately, using a continuous distribution enables us to derive simple, analytical results for the approximation bias.

3.3 Analytical Solution

In this section, we derive a function, depending on the first two moments and the curvature-parameter, which is able to predict the approximation bias. Therefore, we use the framework of Eq.(3.5) with the X now labeled as *growth* to address the economic setting. This distinction is also made to first draw from a standard-normal distribution and subsequently transform into log-normally distributed RV's. The advantage is to keep track of the parameters of the latter distribution. The conditional expectation operator accounts for the time series context. Initially, we specify the expected bias,

$$bias(\mu,\sigma,\gamma) = \mathrm{E}\Big[10^4 \cdot \Big(\underbrace{(1+\mathrm{E}_t[growth_{t+1}])^{\gamma}}_{\text{(1): approximated}} - \underbrace{\mathrm{E}_t\left[(1+growth_{t+1})^{-\gamma}\right]^{-1}}_{\text{(2): unbiased}}\Big)\Big], \tag{3.6}$$

where $growth = \exp(\alpha + \beta Z) - 1$ and $Z \sim \mathcal{N}(0,1)$. This makes *growth*, the growth rates, a log-normally distributed RV with a lower bound of -1 (or -100%). Three notational aspects are worth mentioning. First, we drop the time indices for more clearness. Second, we further simplify by setting $m = 1 + \mu$, defining m as the *centered mean*, thus, centering the growth rates around 1. Third, there is the risk to confound the transformation parameters, α and β, and the targeted moments, μ and σ, since they are approximately the same size.[37] The following formulas show the connection between log-normal parameter and moments:

$$\alpha = \log(m) - \log\left(\sqrt{1+(\sigma/m)^2}\right) \tag{3.7.1}$$

$$\beta = \sqrt{\log\left(1+(\sigma/m)^2\right)} \tag{3.7.2}$$

$$m = \exp(\alpha + \beta^2/2) \tag{3.7.3}$$

$$\sigma^2 = \exp\left(2\alpha + \beta^2\right) \cdot \left[\exp\left(\beta^2\right) - 1\right]. \tag{3.7.4}$$

The key step relies on the moment generating function of the normal distribution.[38] By inserting the distribution expression in the approximated (1) and the unbiased (2) terms of Eq.(3.6), the stochastic source Z becomes apparent but immediately cancels out:

$$(1): \mathrm{E}[\exp(\alpha+\beta Z)]^{\gamma} \quad = \exp(\gamma\alpha)\cdot \mathrm{E}[\exp(\beta Z)]^{\gamma} \quad = \exp(\gamma\alpha)\cdot\exp(\gamma\beta^2/2) \tag{3.8.1}$$

[37]Note that the assignment of Greek letters is different from most sources for the reason mentioned above and for being in accordance with the parameter designation of the inverted beta distribution, also used in this article.

[38]$\mathrm{E}\left[e^{tZ}\right] = e^{t^2/2}, t \in \mathbb{R}$ (see Appendix C.3 for the proof).

$$(2): \mathrm{E}[\exp(\alpha+\beta Z)^{-\gamma}]^{-1} = \exp(\gamma\alpha)\cdot \mathrm{E}[\exp(-\gamma\beta Z)]^{-1} = \exp(\gamma\alpha)\cdot\exp(-(\gamma\beta)^2/2). \tag{3.8.2}$$

This replaces all RV's by parameters only and ensures that no stochastic part is remaining. Re-merging the function without inserting μ/m and σ yet using summarizing parameters gives a first impression of the functional form regarding the curvature:

$$bias(\alpha,\beta,\gamma) = 10^4\cdot\left(\exp(\alpha+\beta^2/2)^{\gamma} - \exp(\alpha-\gamma\beta^2/2)^{\gamma}\right). \tag{3.9}$$

When using Eqs.(3.7) to replace α and β, we can take advantage of the inverse function (exponential and logarithm, square and square root) to arrive at the main function:

$$\begin{aligned} &bias(m,\sigma,\gamma)\\ &\quad= 10^4\cdot\left(m^{\gamma} - \exp\left(\log(m) - \log\left(\sqrt{1+(\sigma/m)^2}\right) - \gamma\log\left(\sqrt{1+(\sigma/m)^2}\right)\right)^{\gamma}\right) && (3.10.1)\\ &\quad= 10^4\cdot\left(m^{\gamma} - m^{\gamma}\cdot\left(\frac{1}{\sqrt{1+(\sigma/m)^2}}\right)^{\gamma}\cdot\left(\frac{1}{\sqrt{1+(\sigma/m)^2}}\right)^{\gamma^2}\right) && (3.10.2)\\ &\quad= 10^4\cdot m^{\gamma}\cdot\left(1-\left(\frac{m}{\sqrt{m^2+\sigma^2}}\right)^{\gamma(\gamma+1)}\right). && (3.10.3)\end{aligned}$$

Factoring out m^{γ}, such that the first term in the outer brackets becomes one, focuses on the second term stemming from the unbiased expression. To get more insight, Eq.(3.10.3) can be separately examined from the perspective of both moments and curvature. The *bias* depending on m and σ with $\gamma = 1$ (corresponds to log-utility) heavily reduces the complexity:

$$bias(m,\sigma \mid \gamma = 1) = 10^4\cdot m\left(1-\frac{m^2}{m^2+\sigma^2}\right) = 10^4\cdot\frac{m\sigma^2}{m^2+\sigma^2}. \tag{3.11}$$

Eq.(3.11) shows a nearly quadratic relationship with regard to σ (since $m^2 \gg \sigma^2$), corrected by m, the centered mean. This non-trivial result is plausible considering the schematic representation in Figure 3.2. Shifting the distribution to the right (increasing m) decreases the approximation bias since the function becomes less curved. A wider distribution (large σ) spreads the probability mass over the non-linear function in a manner that the approximation bias becomes larger. As mentioned above, there is no stochastic element left in the formulas and it can be calculated with deterministic values. Finally, an upper bound can be found by applying l'Hôpital's rule, $\lim_{\sigma\to\infty} bias(\sigma \mid m) = 10^4\cdot m$, revealing a maximum deviation of 100 percentage points, again, corrected by m.

Consulting an economic interpretation, term (1) from Eq.(3.6) stays at the mean value since it does not consider any uncertainty. However, term (2), the inverse of the "marginal utility," converges towards zero. This shows, via a roundabout route, the negative relationship between uncertainty and utility. Moreover, it shows that in an economic model a function's curvature artificially replaces the actual real-world higher-order moments, stemming from the data.

Rearranging Eq.(3.10.3) with $r = \sqrt{m^2 + \sigma^2}$ delivers

$$bias(\gamma) = 10^4 \cdot m^\gamma \cdot \left[1 - (m/r)^{\gamma(\gamma+1)}\right]. \tag{3.12}$$

This reveals the fraction as the crucial term, approaching zero when γ is increasing. γ is the exponent in two parts of the equation, but due to the multiplication in the exponent of the fraction and the smaller basis (m/r), its impact is larger in the fraction. Eq.(3.12) has some simple properties. When σ approaches zero then r approaches m and the *bias* vanishes. Also, for positive growth rates, there is no upper bound since $\lim_{\gamma\to\infty} bias(\gamma \mid m > 1)$ does not exist. It is easy to show that $bias(0) = bias(-1) = 0$ and for $\gamma \in]-1, 0[$ the *bias* becomes negative.[39]

Putting everything together, Figure 3.3 graphically presents the findings concerning the function $bias(\mu, \sigma, \gamma_i)$, with γ as a parameter in levels (i.e., different opacity-levels). Taking the mathematical curvature instead of the EIS highlights the theoretical aspect. The ranges for the parameters are relatively large to show a broader picture. The realistic ranges will be introduced in the next sections.

[39] $\gamma > 0$ is required for a meaningful economic interpretation. See Appendix C.4 for additional investigations concerning this function.

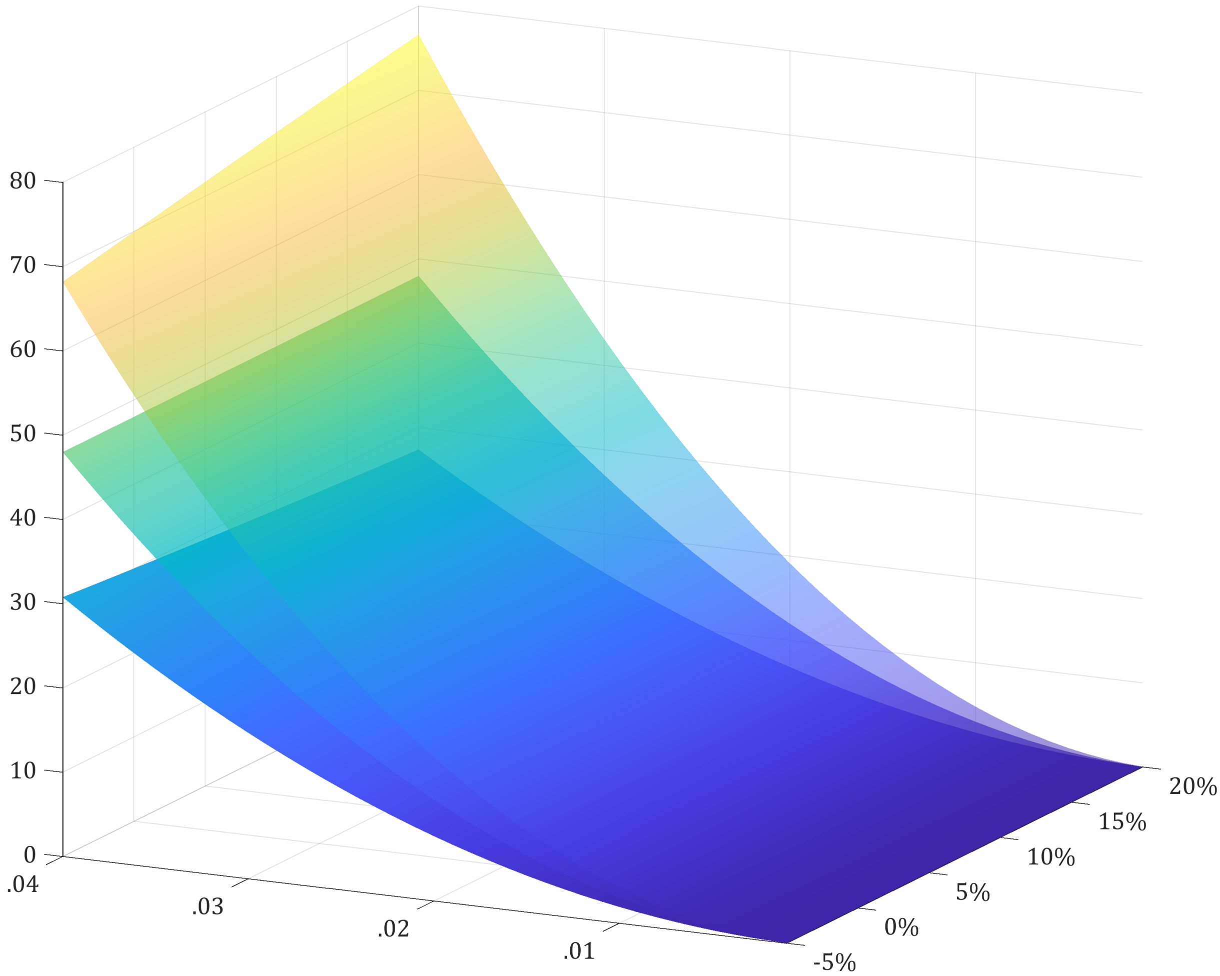

Figure 3.3: Theoretical approximation bias depending on all parameters with $\gamma_i \in \{1, 1.5, 2\}$ represented in the different planes. The higher the layer, the larger is γ_i. Horizontal axes: $\sigma \in [0.001, 0.04]$ and $\mu \in [-0.05, 0.2]$. Vertical axis: Growth rate difference in basis points.

Figure 3.3 shows how the impact of m and σ on the *bias* changes with γ. The undermost plane reveals a slightly negative relationship between m and *bias*, whereas in the intermediate plane a constant relationship between m and *bias* is observable. It remains close to a 50bp difference for the maximum $\sigma = 0.04$ for all m. In contrast, for $\gamma = 2$ (uppermost plane) the relationship between m and the *bias* becomes positive for larger σ. In each case, the absolute effect of m (conditional on large σ and γ) is rather small. The effect of σ is strictly positive for all γ_i and increases for larger values of γ_i.

3.4 Data and Calibration

After the analytical solution, the next step is to check whether the assumptions are also fulfilled in the empirical data. This can be done by checking whether the RV's, which is *growth* in the previous section, are log-normally distributed. The final aim is to get a realistic range of parameters/moments to work with when there is no or insufficient raw data available. For the empirical tests, we use CF data from the *Consensus Economics* surveys. They provide projections on a series of macroeconomic indicators (e.g., GDP growth and inflation measures) that are mainly collected from companies in the finance sector. The data sets are issued monthly and display the forecasts for the current and upcoming year. Initially, the survey started in October 1989, whereas our data reach to June 2019, resulting in a total of 357 months. The observations start with estimations for the G-7 countries. From 1995 on, the number of countries for which there are forecasts available is successively expanded. Therefore, the time horizon for these countries is significantly shorter. The method and calibration results are introduced for US data due to high availability in the number of observations. For the US, the minimum number of forecast observations per month is 19, the maximum is 33, and on the average is 27 per month.[40] Subsequently, we use existing data for frontier or emerging markets with a sufficient amount of observations (Egypt, Nigeria, and South Africa) to account for the possible application of DSGE models as stated in the introduction.

Since the variables' meaning is changing from month to month due to a different forecast horizon, the values from the current period and the next year shall be combined into another variable:

$$\mathrm{E}_t[x_{t+1}|m] = \frac{13-m}{12} \cdot \mathrm{E}_{t,m}[x_t] + \frac{m-1}{12} \cdot \mathrm{E}_{t,m}[x_{t+1}]. \tag{3.13}$$

Consequently, a weighting scheme as in Eq.(3.13) is constructed to combine the forecasts. With the weighting scheme, it is also ensured that the viewed future time horizon is always 12 months. The examined variable of the CF dataset is the consumer price (%-change) forecast, which is equivalent to an inflation measure.

Several normality tests are run on the adjusted data for consumer price. This is done due to the requirement in Section 3.3 that Z is normally distributed. The observed variables x_t need to be rearranged to

$$\tilde{x}_t = \log(x_t + 1) \tag{3.14}$$

[40] Based on these numbers, N—which represents the number of drawn variables—is initially fixed to 30 in Section 3.5.

and $\tilde{x}_t$ is checked on normality. If the normality test approves the hypotheses and $\tilde{x}_t$ is normally distributed, the observed variables are log-normally distributed.

We use four different tests, including Jarque-Bera (J-B), Shapiro-Wilk (S-W), Anderson-Darling (A-D), and Lilliefors (LF). The results can be seen in Table 3.1. In 85% or 55 of the 357 observations, the J-B test cannot reject the H_0 of non-normally distributed variables.

Norm. Test	**p-value** < 10%	< 5%	< 1%
J-B	55	39	24
S-W	73	44	15
A-D	86	57	15
LF	93	60	18

Table 3.1: Normality test results for inflation forecasts (US). It displays the number of observation points that reject the H_0 of non-normally distributed variables out of 357 observations.

As the normality tests show sufficient results, the observed variables shall be log-normally distributed. Now, the calibration of the distribution on the weighted observations can be run. We use a least square model with a grid of possible parameters to solve this issue. The drawn variables x_{draw}, which are random variables from the log-normal distribution, are shifted by a subtraction of one to fit the observations:

$$x_{\text{shift}} = x_{\text{draw}} - 1, \tag{3.15}$$

where $x_{\text{draw}} \sim \log\mathcal{N}(\mu, \sigma^2)$. Forecasts can also contain negative values, i.e., negative growth for consumer prices or other macroeconomic indicators, which cannot be drawn in a log-normal distribution. This can be solved by the shift. For this we calculate the mean squared error, MSE, between the observations and the shifted variables by

$$MSE = \frac{1}{N} \cdot (x_{\text{obs}} - x_{\text{shift}})^2, \tag{3.16}$$

where x_{obs} are the observed variables or the variables from the CF dataset. N is the number of observations or the number of variables x_{obs} and x_{shift}, respectively. To match the two sets of variables, they are sorted by their value. The target is to minimize MSE and to find the optimal parameters μ and σ^2 of the distribution. The start-parameters of the log-normal distribution in the calibration are chosen in respect of

μ_{obs} and σ^2_{obs} of the observed variables. From every parameter we go ten predefined equidistant steps in every direction. This builds a grid

$$\mathcal{C}_1 = \{\mu_{1,\text{lower}},..,\mu_{1,\text{obs}},..,\mu_{1,\text{upper}}\} \times \{\sigma^2_{1,\text{lower}},..,\sigma^2_{1,\text{obs}},..,\sigma^2_{1,\text{upper}}\} \in \mathbb{R}^2, \tag{3.17}$$

where $\mu_{1,\text{lower}} = \mu_{1,\text{obs}} - 10 \cdot \text{step}_1$ and $\mu_{1,\text{upper}} = \mu_{1,\text{obs}} + 10 \cdot \text{step}_1$ and step_1 is the equidistant step. The same calculation is done with the variance, but the step size can differ from the one used for the mean. As a result, there are $21^2 = 441$ possible parameter combinations $(\mu, \sigma^2) \in \mathcal{C}_1$.

At every combination, as many random variables are drawn as observations we have in these respective periods. The MSE of the drawn and observed variables at every combination point is calculated. The parameter combination with the lowest MSE is chosen as the new optimal point $(\mu_1^*, \sigma_1^{2*}) \in \mathcal{C}_1$.

In the next step, we again go from this optimal point ten equidistant steps in every direction with the difference that the steps are with a factor 10 smaller than before. They can be calculated by the following method:

$$\text{step}_i = \frac{\text{step}_1}{10^{i-1}}. \tag{3.18}$$

The new grid is of the form:

$$\mathcal{C}_2 = \{\mu_{2,\text{lower}},..,\mu_1^*,..,\mu_{2,\text{upper}}\} \times \{\sigma^2_{2,\text{lower}},..,\sigma_1^{2*},..,\sigma^2_{2,\text{upper}}\} \in \mathbb{R}^2. \tag{3.19}$$

So, we assure that the grid is getting finer. The least MSE in this step forms the new optimal point $(\mu_2^*, \sigma_2^{2*}) \in \mathcal{C}_2$. These steps are repeated until we reach a sufficiently predefined small error term and the resulting parameters are $(\mu^*_{\text{opt}}, \sigma^{2*}_{\text{opt}})$. The results are the specific parameters of both distributions, which are transformed to μ and σ^2 using Eqs.(3.7) from Section 3.3 and Eqs.(C16) from Appendix C.6. The calibration results for two distribution functions (log-normal and inverted beta) are displayed in Table 3.2, whereas the calibration method for the inverted beta is nearly the same as for the log-normal distribution. The only difference is the shift of the drawn variables from Eq.(3.15) by 0.01. This ensures that the inverted beta distribution also generates negative values.[41]

[41] Due to the characteristics of the inverted beta distribution a larger shift is not feasible. A significant part of the probability mass is concentrated at the lower bound, which is in this case the shift, and this would mainly result in negative values. The mass can be stretched by changes in the parameters of the distribution but this would deliver an unreasonable large σ.

Para-meter	Min	25th centile	Median	75th centile	Max	Obs. Median	Mean Error
log-norm.							
μ_{opt}	−0.0032	0.0191	0.0246	0.0320	0.0420	0.0239	$1.09{\cdot}10^{-6}$
σ_{opt}	0.0001	0.0001	0.0002	0.0007	0.0060	0.0027	$1.09{\cdot}10^{-6}$
inv. beta							
μ_{opt}	0.0061	0.0233	0.0275	0.0322	0.0556	0.0239	$4.56{\cdot}10^{-5}$
σ_{opt}	0.0011	0.0023	0.0028	0.0036	0.0262	0.0027	$4.56{\cdot}10^{-5}$

Table 3.2: Calibration results (US) when assuming CF data follow a log-normal and inverted beta distribution, respectively. The calibration is run on a monthly basis for the rolling window-adjusted observations on inflation forecasts.

In the first column, the two variables, which are the focus of our calibration, are shown. The following five columns focus on the results of the calibration and show descriptive statistics for the 357 months. The next column switches from the calibrated to the observed variables in the CF data set. In each of the 357 months, the mean and the standard deviation of the observed variables are calculated. This delivers 357 empirical values for both moments. The median of the two calculated moments is displayed in the respective row. Consequently, the results for the median of the observed variables are the same for both distribution methods because the viewed data set is the same. The mean error that arises from the calibration can be found in the last column. Focusing on the mean error, the log-normal distribution is better in fitting the observed variables. Regarding the estimation of μ_{opt}, the log-normal distribution gives a better fit than the inverted beta distribution. This is mainly due to the concentration of probability mass at the lower bound of the inverted beta distribution. The picture changes when fitting σ_{opt}. Here the log-normal distribution underestimates the standard deviation, whereas the inverted beta distribution gives a better fit. Nonetheless, it is noticeable that the inverted beta distribution sometimes generates relatively large values for the standard deviation.

For comparison purposes and due to a potentially high interest in DSGE models for emerging/frontier markets, we now focus on forecasts of consumer prices for Egypt, Nigeria, and South Africa.[42] Other countries that are classified in these markets are not available in the data set. Due to a low number of forecasts per month—in contrast to developed countries, e.g., the US—quarterly data is used. The observations start in Q1 2008, which results in a time horizon of 46 quarters. The minimum number of

[42] The classification regarding frontier and emerging markets is taken from MSCI. This is a provider of equity market indexes that uses a classification for different markets.

forecast observations per quarter is six for Nigeria in Q1 2008, the maximum is 57 for South Africa in Q4 2016, and the average is 22. The average is built considering all three countries.

The previous adjustment to rearrange all variables on a 12 months time horizon is also done for the monthly variables as mentioned in Eq.(3.13). In the following normality test, the adjusted variables in a whole quarter are checked for normal distribution. The results can be seen in Tables 3.3–3.5. In all countries, the J-B test results support normally distributed random numbers in more than 65% of the cases. In Egypt and Nigeria, normally distributed random numbers are supported in even more cases, amounting to 90%.

Norm. Test	**p-value** < 10%	< 5%	< 1%
J-B	4	3	1
S-W	15	14	4
A-D	19	14	10
LF	22	14	9

Table 3.3: Normality test results for inflation forecasts (Egypt). It displays the number of observation points that reject the H_0 of non-normally distributed variables out of 46 observations.

Norm. Test	**p-value** < 10%	< 5%	< 1%
J-B	4	3	3
S-W	16	11	6
A-D	18	14	11
LF	19	14	8

Table 3.4: Normality test results for inflation forecasts (Nigeria). It displays the number of observation points that reject the H_0 of non-normally distributed variables out of 46 observations.

Norm. Test	p-value < 10%	< 5%	< 1%
J-B	16	12	10
S-W	23	18	14
A-D	25	19	15
LF	24	19	16

Table 3.5: Normality test results for inflation forecasts (South Africa). It displays the number of observation points that reject the H_0 of non-normally distributed variables out of 46 observations.

The normality tests again show sufficient results for the three countries analyzed. So the same calibration can be run for the log-normal distribution and inverted beta is again used as a robustness check. The method of calibration stays the same. The results can be seen in Tables 3.6–3.7.

Parameter	Min	25^{th}centile	Median	75^{th}centile	Max	Obs. Median	Mean Error
Egypt							
μ_{opt}	0.0804	0.0965	0.1075	0.1193	0.2033	0.1057	$2.88 \cdot 10^{-4}$
σ_{opt}	0.0054	0.0073	0.0085	0.0106	0.0179	0.0134	$2.88 \cdot 10^{-4}$
Nigeria							
μ_{opt}	0.0591	0.0972	0.1094	0.1224	0.1507	0.1088	$1.30 \cdot 10^{-4}$
σ_{opt}	0.0055	0.0080	0.0106	0.0131	0.0341	0.0096	$1.30 \cdot 10^{-4}$
S. Africa							
μ_{opt}	0.0464	0.0523	0.0576	0.0612	0.0852	0.0571	$4.03 \cdot 10^{-5}$
σ_{opt}	0.0047	0.0062	0.0072	0.0088	0.0136	0.0035	$4.03 \cdot 10^{-5}$

Table 3.6: Calibration results when assuming CF data follow a log-normal distribution. The calibration is run on a quarterly basis for the rolling window-adjusted observations on inflation forecasts.

Para-meter	Min	25^{th}centile	Median	75^{th}centile	Max	Obs. Median	Mean Error
Egypt							
μ_{opt}	0.0899	0.1029	0.1135	0.1321	0.2176	0.1057	$3.57{\cdot}10^{-4}$
σ_{opt}	0.0060	0.0107	0.0143	0.0209	0.0361	0.0134	$3.57{\cdot}10^{-4}$
Nigeria							
μ_{opt}	0.0763	0.1002	0.1162	0.1284	0.1519	0.1088	$1.41{\cdot}10^{-4}$
σ_{opt}	0.0004	0.0070	0.0099	0.0135	0.0269	0.0096	$1.41{\cdot}10^{-4}$
S. Africa							
μ_{opt}	0.0492	0.0547	0.0588	0.0632	0.0875	0.0571	$6.19{\cdot}10^{-5}$
σ_{opt}	0.0022	0.0030	0.0035	0.0047	0.0125	0.0035	$6.19{\cdot}10^{-5}$

Table 3.7: Calibration results when assuming CF data follow a inverted beta distribution. The calibration is run on a quarterly basis for the rolling window-adjusted observations on inflation forecasts.

The results deliver the same picture as for the US. The log-normal distribution generates lower error terms compared to the inverted beta distribution. However, all errors are relatively close to each other. Additionally, the median of μ_{opt} is quite close to the median of the observed variables. In the inverted beta distribution, μ_{opt} is not as good as in the log-normal distribution, but σ_{opt} is better fitted.[43]

Summarizing, none of the distributions is clearly better in fitting the CF data set. Due to the slightly better error term for the log-normal distribution in the US and other aspects, e.g. multivariate distributions, we focus on the log-normal distribution for further analysis. Due to the above results for four different countries, the baseline scenario in the next section consists of $\mu = 0.06$ and $\sigma = 0.01$ as fixed parameters.

[43]Similar results can be seen for the calibration of the GDP forecasts in the CF data set. This, and α and β for the distributions for both variables can be found in Appendices C.7 and C.8.

3.5 Monte Carlo Simulation

The simulation section takes up the calibration results and conducts several simple MC experiments. Thereby, expectation value and variance become stochastic, enabling us to check how this variability influences the findings of Section 3.3. We orientate at the following sequence.

1. Specify the parameters in Eq.(3.5): μ, σ, γ, and N (in the multivariate case also ρ and n).

2. Calculate the *bias*, also Eq.(3.5), by drawing N random variables, following a log-normal distribution. Repeat this 10^5 times to obtain a *bias*-distribution.

3. Alter one of the parameters and repeat step 2. As soon as the parameter has covered a certain range, jump to step 4.

4. Graphically show the results as boxplots in a x-y-diagram with the varying parameter on the x-axis and the *bias* on the y-axis.

There is little variation when the first moment takes different values with a change in *bias* significantly lower than 1bp.[44] Hence, μ is constantly set to 0.06, a happy medium regarding the calibrated means. We set N to 30 since this matches the typical number of firms participating in the CF survey. Increasing this number will not result in a substantial difference. However, we examine this in more detail in Appendix C.15. We use 10^5 repetitions to make sure the obtained distributions already converged.

The boxplots, which are uncommon in this context, depict the non-parametric characteristics of a distribution as described in Mcgill et al. (1978). The lower (upper) hinge of the box presents the first (third) quartile, while the middle line presents the median, the second quartile. This gives a good impression concerning the distribution's skewness. The lines extending vertically from the boxes (whiskers) expand both hinges by the interquartile range, multiplied by 1.5. As an orientation, when using a normal distribution, outliers larger (smaller) than the upper (lower) extreme account for only 0.35% of the probability mass. This value will be somewhat larger since $bias > 0$ and, thus, the simulated distributions are likely to be asymmetric with the mean not equaling the median. Although outliers are excluded for graphical clearness in the following figures, we check for their share not being too large ($< 2\%$) as justification to use the standard boxplots.[45]

[44]See Appendix C.9 examining the derivative $\partial bias/\partial \mu$ in detail.

[45]This keeps the interpretation simple and is in line with Hubert and Vandervieren (2008, 5191). In their article, they propose adjusted boxplots accounting for skewed data when outliers exceed 5%.

Throughout this section, the following scheme is used to describe the results. The simulated distributions are analyzed in terms of the first three moments, the best fitting parametric distribution relating to outliers, and the relationship between the medians and the varying parameters. Regressing the median-*bias* on the parameters extents the analytical results and accounts for the approximation bias' immanent skewness.[46]

3.5.1 Standard Deviation

According to Eq.(3.11), we take the standard deviation, a measure for the data's uncertainty, as the first varying parameter, whereas γ is held to one. The calibrated σ's range from 0.001 to 0.032. We choose 20 values starting from nearly zero up to a maximum value of 0.04, which is larger than calibration suggests accounting for the possible bias in long-term forecasts. In this extreme scenario, with $\mu = 0.06$, growth rates of 10% are quite realistic since the 68–95–99.7 rule applies approximately for the log-normal distribution.[47] Figure 3.4 shows the results.

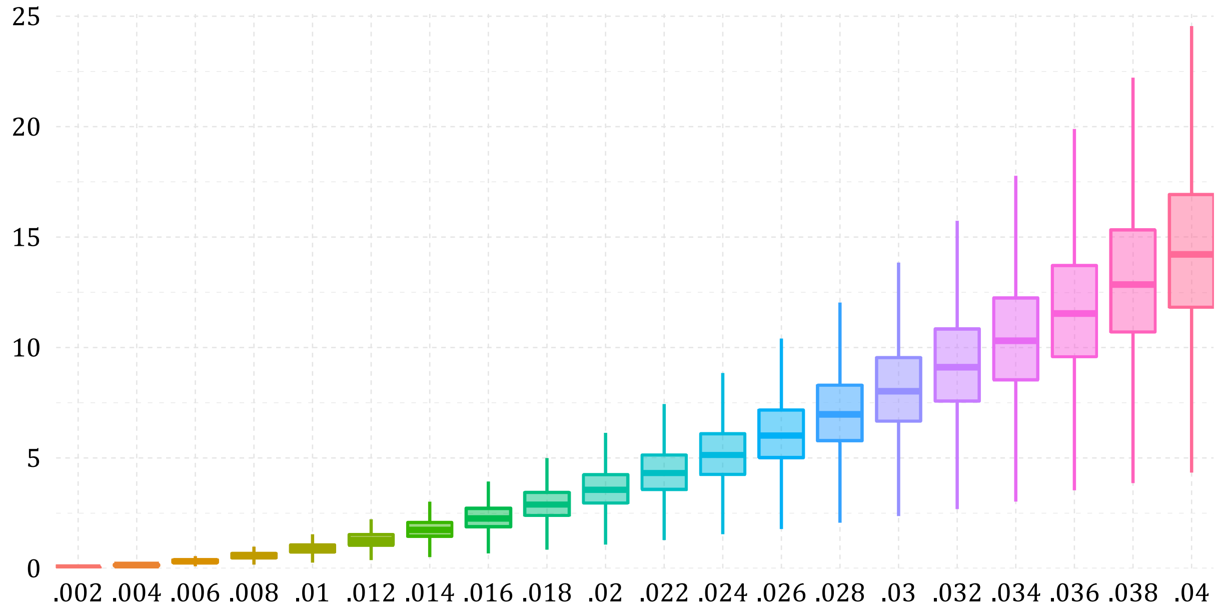

Figure 3.4: Simulations with 10^5 repetitions each (log-normal distribution with $\mu = 0.06$, $N = 30$, and a lower bound of -1). Resulting distributions of the approximation bias are shown as boxplots. Horizontal axis: Standard deviation (σ). Vertical axis: Growth rate difference in basis points (*bias*).

[46]To not overload Section 3.3, we include the median analysis in the simulation part only and, at the same time, using the advantage of the graphical analysis.

[47]Despite the distribution being truncated and skewed, this approximation holds very accurate, e.g., when $\sigma = 0.06$, the probability mass inside the 3σ-interval lowers by only 0.1 percentage point to 99.6%.

The values originating from Eq.(3.11) are augmented by distributions for each σ, illustrating the sensitivity to small samples. The first twelve boxplots (including $\sigma = 0.024$) are of marginal relevance being strictly under 10bp. However, with a larger standard deviation the *bias* is increasing and averages 15bp for the extreme scenario, even reaching up to 25bp. Figure 3.4 also reveals that *bias*-predictions with increasing σ become more and more inaccurate (increasing interquartile range). The distributions' skewness is consistently positive with the deviation of mean and median located slightly over 2% (i.e., always less than one basis point).[48] The right skewness was expected due to the usage of a log-normal distribution. Cullen and Frey (1999) analysis shows that the resulting distributions best fit to a log-normal distribution, which can be expected by construction, but also to a gamma distribution. This means, in turn, that for every σ there are theoretically around 1% outliers larger than the upper extreme. This also holds empirically. Most interesting, the relationship between σ and the approximation bias (median) is quadratic with an R^2 of almost 100% when running regression analysis. The detailed results are shown in Appendix C.10.[49]

The intuition behind the relationship can be outlined by a simple case where f from Eq.(3.2.1) is the convex function $f(X) = X^2$:

$$\mathrm{E}[X^2] = (\mathrm{E}[X])^2 + \mathrm{Var}[X] = (\mathrm{E}[X])^2 + \sigma_X^2. \tag{3.20}$$

Here, the residual (σ_X^2), which was Δ_X in Eq.(3.2.1), consists of the squared standard deviation.[50] For this example, in contrast to the analytical derivation, X does not necessarily follow a specific distribution. Nevertheless, we also check for the accuracy of a second-order Taylor expansion with regard to Eq.(3.11):

$$T_2^{bias}(\sigma \mid \sigma_0 = 0) = 10^4 \cdot \sigma^2 / m. \tag{3.21}$$

The isolated quadratic part describes the relationship sufficiently enough up to $\sigma = 0.16$.[51]

[48] $\mathrm{E}[(mean - median)/median \mid \sigma] = 2.35\%$. In line with our own calculations, the medcouple, a normalized or robust measure for the skewness reaching from -1 to 1, is only around 0.08.

[49] Also in Appendix C.10, we show similar results for the CARA function but hereinafter, for the lack of substantial difference, we stick to the CRRA function.

[50] With the binomial formula: $\mathrm{Var}[X] = \mathrm{E}[X^2] - \mathrm{E}[X]^2$. Note the close connection between Jensen's inequality and our model (Appendix C.2).

[51] Appendix C.11 shows this in more detail. See also Appendix C.12, further examining the approximation bias in terms of a ratio between the approximated and the un-biased term.

3.5.2 Curvature – Elasticity – Risk Aversion

The second varying parameter accounts for the degree of non-linearity of f from Eq.(3.3). In an attempt to interpret this property as general as possible, this can be described by the curvature, typically defined as the amount by which a curve deviates from being a straight line. By defining γ as curvature of $f(1+x)$, the relative risk aversion equals:

$$RRA_f = -(1+x) \cdot f''/f' = 1+\gamma. \tag{3.22}$$

Concerning this interpretation of γ, Meyer and Meyer (2005) assemble slightly different versions of risk aversion to make them comparable and Chiappori and Paiella (2011) conduct an in-depth analysis of risk aversion using panel data.

As mentioned earlier, x is understood as growth and $f(1+x)$ is understood as the marginal utility, satisfying the economical situation. In this case, the inverse, γ^{-1}, can be interpreted as elasticity. Additionally, in a time-varying context, γ^{-1} stands for the EIS. For the sequence of γ, applied to the study, we orientate at Meyer and Meyer (2005, 260) by starting slightly above zero (0.25) and going up to 5 (i.e., $\gamma^{-1} \in [0.2, 4]$). Figure 3.5 shows the simulation results analogous to the previous subsection.[52]

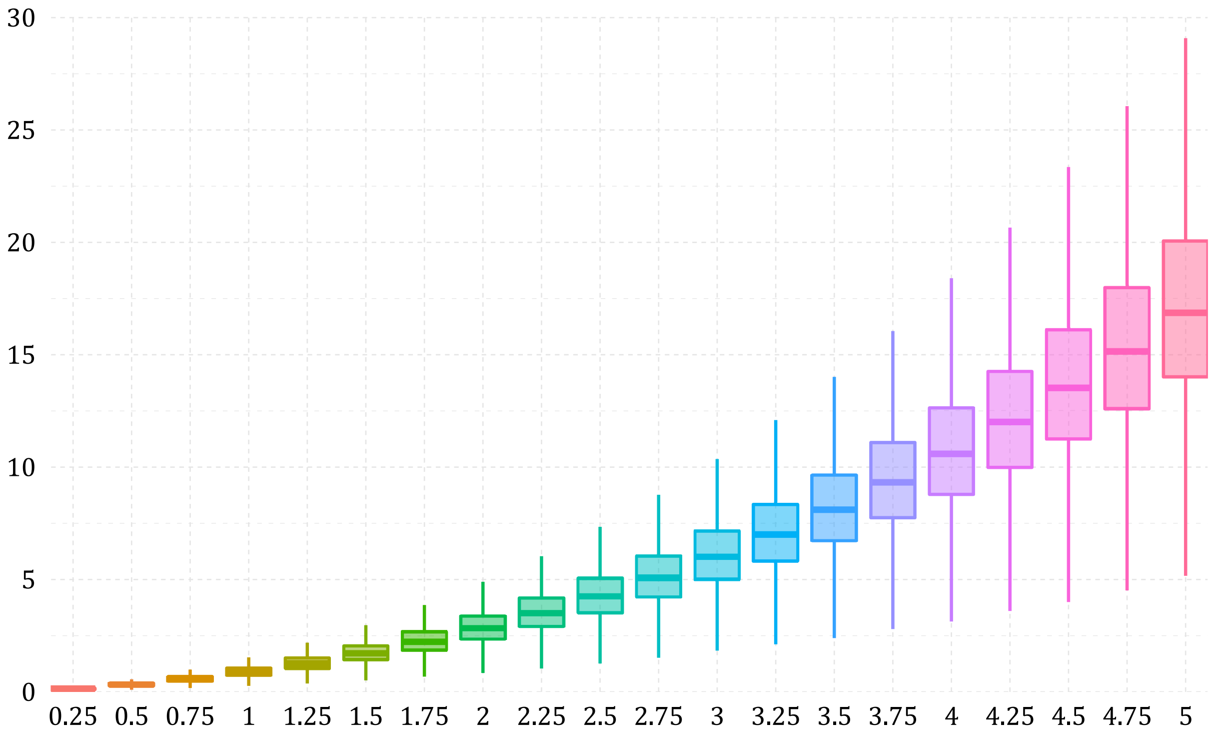

Figure 3.5: Horizontal axis: Curvature (γ). Vertical axis: Growth rate difference in basis points.

[52]From an economic point of view, the horizontal axis should increase with the EIS, which is shown in Appendix C.14.

Almost identical to Figure 3.4, boxplots one to twelve are hardly exceeding the 10bp line. The outcomes are breaking this line for an EIS of 1/3 and lower. The average bias is increasing over 15bp for a curvature of 5 (relative risk aversion of 6), with almost 30bp at the upper extreme. The interquartile range is increasing with larger curvature. Similarly, the variance is proportionate to the curvature. Investigating skewness and possible parametric distributions show the same results as with a varying σ. However, for the largest value of γ, outliers make up for almost 2%. The relationship of x and y is approximately $bias \sim \gamma^2$.[53] Also, in terms of pure elasticities, $\log(bias) \sim \log(\gamma)$, the regression's goodness-of-fit edges up to $R^2 = 0.99$. This allows for the interpretation of a %-change caused by a 1% increase. In this case, the system is relatively elastic with a log-coefficient of around 1.67. Analogous to the σ-version but less accurately, the quadratic relationship can be described by a second-order Taylor expansion for $\gamma \leq 0.9$:

$$T_2^{bias}(\gamma \mid \gamma_0 = 0) = 10^4 \cdot \log(r/m)\Big[\log\Big(\mathrm{e}\sqrt{m^3/r}\Big)\gamma^2 + \gamma\Big]. \tag{3.23}$$

In contrast to Eq.(3.21), with the standard deviation as variable, Eq.(3.23) should only be used for a certain range of realistic values.[54] Overall, an exponential link is preferable when characterizing the relation between curvature and approximation bias.

3.5.3 Number of States – Sample Size

The parameter N, in terms of the simulation procedure, is the sample size per repetition. So far, N equalled 30 to represent the available number of forecasts in the CF data sets. For the generic case of $N = 1$, there is only one outcome and, therefore, no uncertainty. This makes Jensen's inequality redundant. When N is approaching infinity, the approximation bias converges to the analytical finding. Since both scenarios are not reasonable, we vary N from 1 to 20 and examine how the result is driven by uncertainty regarding different prospective outcomes.[55]

When N is increasing, the variance in the repetitions vanishes. However, for small N, the variance is not exploding. Figure 3.6 depicts the baseline case ($\mu = 0.06$, $\sigma = 0.01$, $\gamma = 1$) for a sample size reaching from 1 to 20.

[53] See Appendix C.10 for more detail by using regression analysis.

[54] See Appendix C.13 for more detail.

[55] In a different context, for example, N can be seen as the number of political parties at an election. When N is small, the future outcome of their economic policies is still not clear but easier to anticipate.

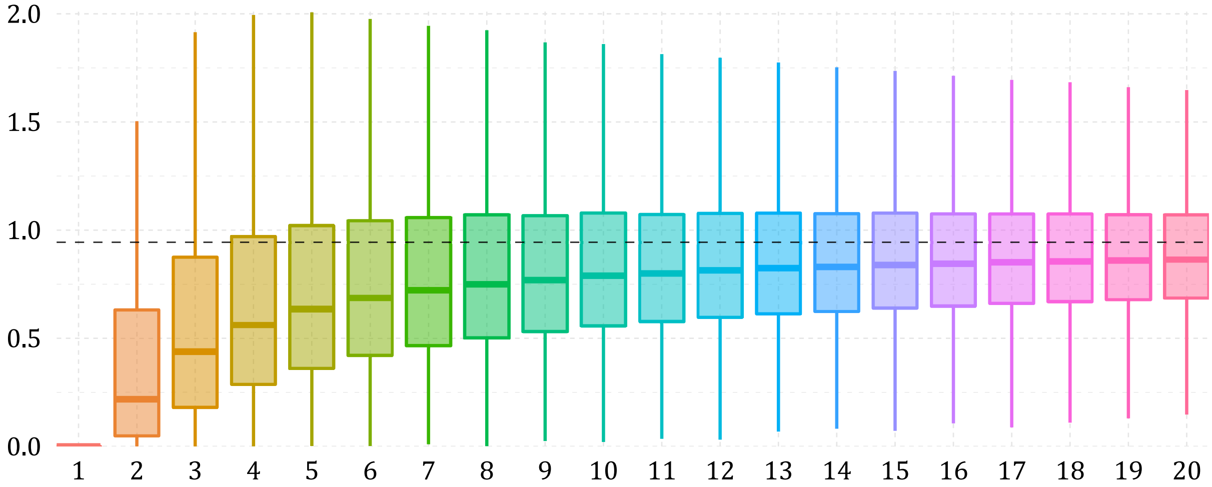

Figure 3.6: Baseline case. Dashed line depicts the theoretical mean value. Horizontal axis: Sample size (i.e., number of states N). Vertical axis: Growth rate difference in basis points.

With an increasing number of states, the distributions are contracting around the dashed line—the theoretical value—slightly below 1bp. For $N = 1$ there cannot be any difference, being in a situation with no uncertainty. Due to the zero lower bound, the distributions are initially heavily skewed. This becomes apparent for $N = 2$ when the median is close to 25%, but the mean stays at 50% of the dashed line, which can be seen in the results of the simulation. Interestingly, until four states are reached, the upper extreme is increasing and, thereupon, slowly decreasing. As to be expected, the magnitude is extremely small.

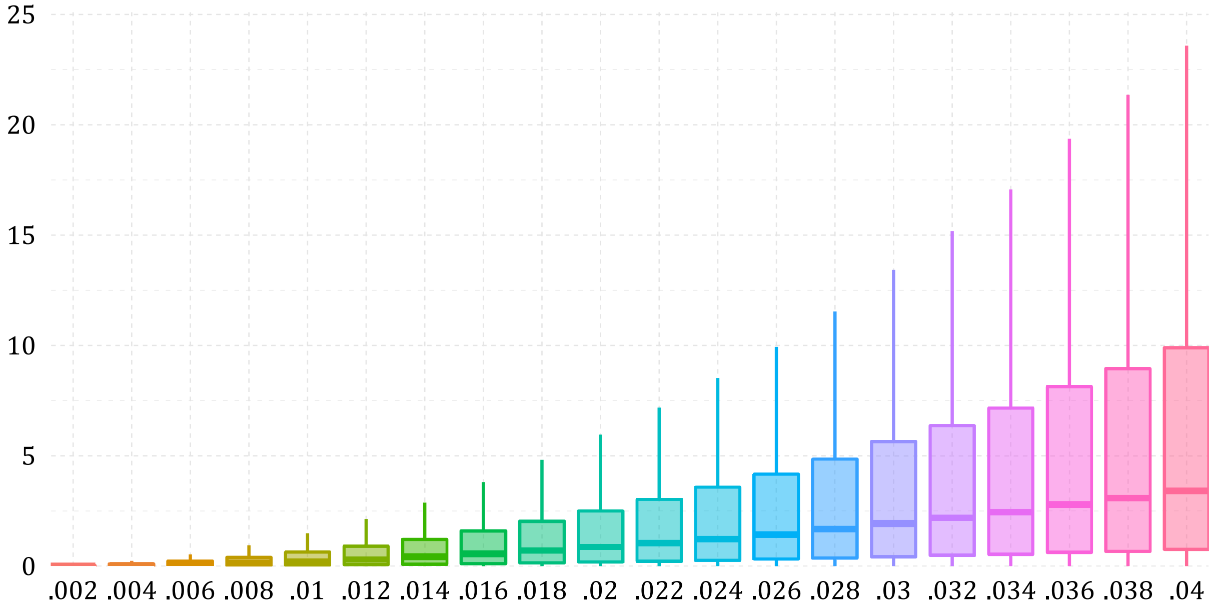

Figure 3.7: Baseline case with varying σ and two states, i.e. $N = 2$. Horizontal axis: Standard deviation (σ). Vertical axis: Growth rate difference in basis points.

Figure 3.7 replicates Figure 3.4 (varying σ) but only drawing two RV's, respectively. All distributions are skewed with a median-*bias* reaching only up to 3bp. An important question comes up, that is, which measure (mean or median) to choose when describing the approximation bias in a situation with few outcomes. On the one hand, most scenarios are negligible, on the other hand, (right-tail) outliers are still present. For $\sigma = 0.04$, the upper extreme is almost as large as in Figure 3.4.

In the model context, N is not interacting with other parameters or variables. Put differently, a parameter constellation other than the baseline case in Figure 3.6 does not alter the behavior for increasing N. In Appendix C.15, we explain in more detail how the convergence to the theoretical value solely depends on the sample size.

3.5.4 Multivariate Functions

To further exploit our model, we increase the number of variables (n), which also leads to the correlation (ρ) as an additional parameter. Simple Euler equations already include variables like consumption (growth) and price (growth). They should also (theoretically) contain information about how these variables are interacting (i.e., their co-movement).[56] Large-scale DSGE models such as the ECB-Global, the IMF's Global Projection Model, and further adjusted versions can contain dozens of variables and hundreds of parameters.

For higher-order Taylor expansions, Collard and Juillard (2001) examine models of the form: $\mathrm{E}_t[f(x_{t+1}, y_{t+1}, \ldots)] = 0$, for non-linear f. Still, our approach focuses on Jensen's inequality.[57] Taking up on Eq.(3.5) for the baseline case ($\gamma = 1$) and multiplicatively expanding by another variable gives

$$bias(X,Y) = 10^4 \cdot \left(\mathrm{E}[1+X]\cdot\mathrm{E}[1+Y] - \mathrm{E}\left[(1+X)^{-1}(1+Y)^{-1}\right]^{-1}\right), \qquad (3.24)$$

assuming that both X and Y are log-normal. Multiplying the new variable Y, instead of a different transformation, preserves the model's structure and replicates the way most (un-approximated) Euler equations work. The factor to produce growth rate differentials stays at 10^4 since multiplying centered growth rates results in a new growth rate, combining the others. This works analogously to adding level data in the same unit of measure. Figure 3.8 explores how the correlation between two variables affects the approximation bias.

[56] See, e.g., An and Schorfheide (2007, 118), including four variables in the context of a medium-scale DSGE model.

[57] See also Mitrinović et al. (1993, 4), dealing with multivariate functions, $f(\mathbb{R}^n)$, in the context of Jensen's inequality.

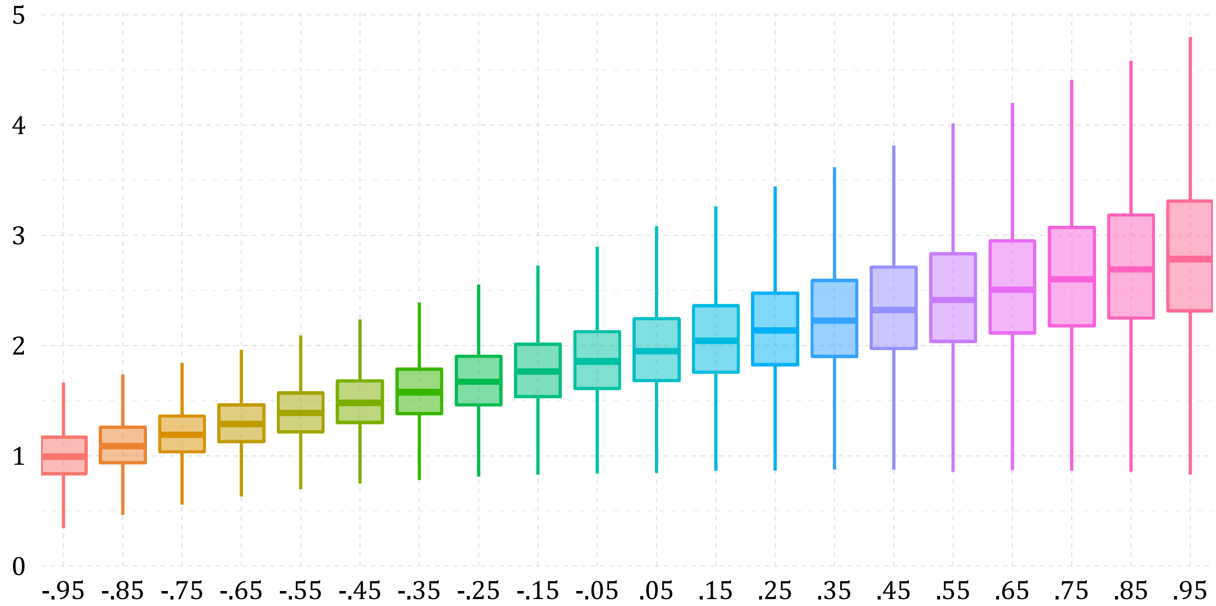

Figure 3.8: Baseline case with two variables. Horizontal axis: Correlation (ρ). Vertical axis: Growth rate difference in basis points. Technical note: The correlation between RV's $\sim \mathcal{N}(0,1)$ is preserved when transforming into the log-normal form. Also, when drawing from a $\mathcal{N}(0,1)$, the correlation equals the covariance.

The bias' magnitude is extremely small, reaching from 1 to 3bp. For negative correlation values up to -0.5, the standard deviation of the simulated distributions remains approximately the same, linearly increasing thereafter. The skewness is rather low with 0.46 on average. A gamma distribution can describe the individual results, however, with changing shape and scale parameters for different correlation values. Outliers account for approximately 1% in the case with maximum correlation. The relationship appears to be linear with negative correlation counteracting the bias.

Similar to Eq.(3.20), a special case can heuristically illustrate the rationale behind the linear relation. Again, take f from Eq.(3.2.1) to a bivariate environment: $f(X,Y) = X \cdot Y$.

$$\mathrm{E}[X \cdot Y] = \mathrm{E}[X] \cdot \mathrm{E}[Y] + \mathrm{Cov}[X,Y] = \mathrm{E}[X] \cdot \mathrm{E}[Y] + (\sigma_X \sigma_Y) \cdot \mathrm{Corr}[X,Y]. \tag{3.25}$$

In this case, the residual Δ_{XY} comprises the correlation times a coefficient (the standard deviations).[58]

[58] With the binomial formula: $\mathrm{Cov}[X,Y] = \mathrm{E}[X \cdot Y] + \mathrm{E}[X] \cdot \mathrm{E}[Y]$. Note that the correlation is the standardized covariance: $\mathrm{Corr}[X,Y] = \mathrm{Cov}[X,Y] / \left(\sqrt{\mathrm{Var}[X]} \cdot \sqrt{\mathrm{Var}[Y]}\right)$.

As a last step, pushing forward to a more comprehensive form, the quantity of variables ought to be reflected in the formula. Thus, modifying the model the same way as accomplished in Eq.(3.24) leads to

$$bias(X_1, ..., X_n) = 10^4 \cdot \left(\prod_{i=1}^{n} \mathrm{E}[1 + X_i] - \mathrm{E}\left[\prod_{i=1}^{n} (1 + X_i)^{-1} \right]^{-1} \right), \tag{3.26}$$

a generalized version with n variables. Specifying $n = 5$ and, for simplicity, assuming the same correlation between all these variables, Figure 3.9 reveals a similar pattern as in the bivariate case.[59]

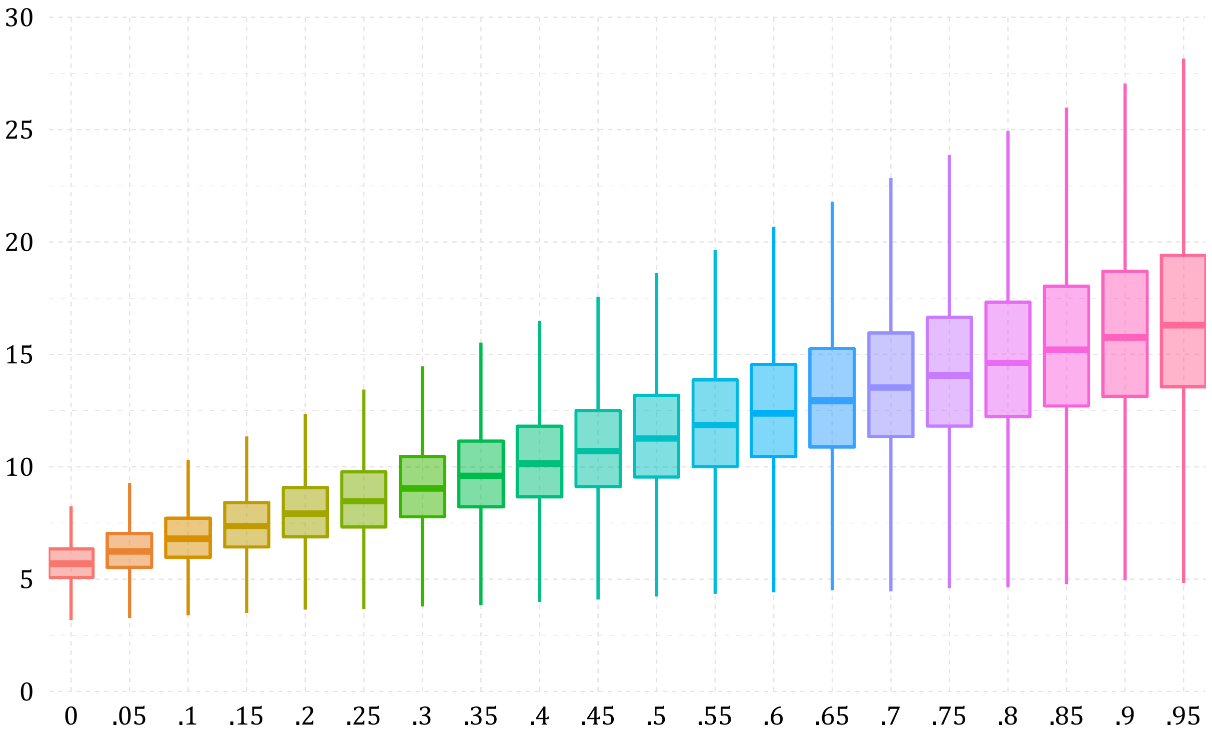

Figure 3.9: Baseline case with $n = 5$ variables. Horizontal axis: Correlation (ρ). Vertical axis: Growth rate difference in basis points.

The distribution indicators are similar and a nearly linear relationship reaches from the minimum to the maximum correlation value. For five highly correlated variables ($\rho = 0.95$), the bias roughly averages at 15bp, with outliers over 25bp. In this scenario, compared to Figure 3.8, the relationship is still linear, but the slope is larger (approximately 1.15bp per 0.1 ρ-step).

Finally, Figure 3.10 varies the number of variables while slightly deviating from the baseline case. Since high growth rates for a large number of macroeconomic vari-

[59]It is interesting to note that in this case a covariance matrix cannot be positive definite, which is required. In other words, there is no combination of values possible where the overall correlation is always negative.

ables are unrealistic, we switch to the calibrated mean for the US ($\mu = 2.5\%$).[60] As an additional assumption, the variables are mildly correlated ($\rho = 0.1$).

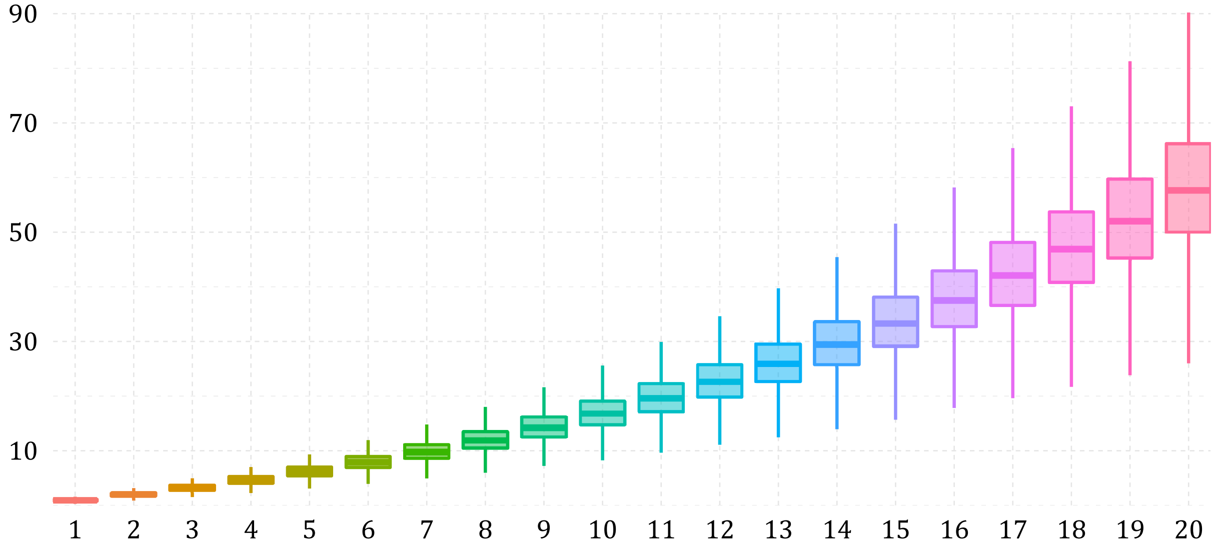

Figure 3.10: Baseline case with on overall correlation of $\rho = 0.1$. Horizontal axis: Number of variables (n). Vertical axis: Growth rate difference in basis points.

For ten or fewer variables, the average bias is not surpassing the 25bp mark. Thus, these situations can be compared to the extreme scenarios in Figures 3.4 and 3.5. For $n = 20$, an average bias of over 50bp is reached, and outliers of almost one percentage point are possible. The standard deviation increases over proportionately and the skewness stays at 0.5.[61] Distribution analysis, again, points towards gamma-distributed residuals and outliers account for 1%. There is a predominant quadratic relationship since the exponential coefficient is significant but basically zero as shown in the regression table in Appendix C.10.

To recapitulate, Appendix C.16 provides a general overview of all figures shown in this section and the multiple-planes figure in Section 3.3.

[60]For $\mu = 6\%$, values of over 2 percentage points will be reached.

[61]Medcouple averages at 0.08 and the %-deviation of mean and median averages at 2%.

3.6 Conclusion

This mostly theoretical paper explores a source of error in the context of macroeconomic models. Occurring in intertemporal Euler equations, Jensen's inequality is typically ignored when bringing the model to the data. Therefore, we set up an illustrative framework to compare the expected outcome of a non-linear function with the functional value of the expected argument consisting of growth rates. Being interested in the magnitude, this difference is constructed in a way that it can be measured in basis points, thereupon designated as approximation bias. Since in the prevalent DSGE family, this bias is rather small, we evaluate parameter constellations in which the difference becomes apparent.

First, we derive analytical solutions with assumptions typical for DSGE models and growth rates, subsequently calibrating first and second moments for the US and emerging markets from forward-looking Consensus Forecasts data. Second, we test the variability in a simulation-based analysis and examine resulting distributions for a wide range of parameter values. Third, we further extend the model to check for the multivariate influence and, thus, the correlation among variables. Throughout the article, we focus on translating model parameters into economic factors.

To generalize the results, we track down five separate factors, describing the functional relationship relative to the bias. The approximation bias increases (i) quadratically to uncertainty, (ii) exponentially to both the overall risk aversion and (iii) the model size, (iv) inversely proportional to the number of future states, and (v) linear to variables' co-movement. On the other hand, the first moments, mean and median, march to a different drummer by switching the sign of their influence depending on the curvature. However, this influence is negligible.

In absolute terms, when uncertainty is high, growth rates are overestimated up to 25 basis points. Consequently, a corresponding interest rate, adjustable by the central bank, should generally be lower when accounting for the approximation bias. Expecting a future scenario consisting of three possible states only, the bias' mean remains low, yet its distribution will be heavily skewed. Accordingly, when including only a few variables, the correlation among them will not be an issue, with negative values even counteracting the bias. Lastly, considering a large number of variables, overestimation mattered the most, with outliers even reaching one percentage point.

Our findings are important for large-scale model users such as central banks in major economies where a possible error can add up and significantly bias the predictions. They also matter for institutions in emerging economies, which are more and more adopting DSGE models. We showed, in particular, that in situations with large uncertainty the bias cannot be ignored.

This groundwork provides a rich field for future research. To avoid the approximation when the bias is potentially large, examining density forecasts as in Rich and Tracy (2010) are of particular interest. Finally, putting all findings together, a Kalman-like filter to transform times series could be established to circumvent the issue. An expected difference to the actual values depending on the identified factors could be derived, including simulation-based confidence intervals.

Appendix C

C.1 Approximating Jensen's Inequality by Quadratic Taylor Series

Since linearizing causes the inequality to vanish, we check for its relevance after conducting second-order (multivariate) Taylor expansion. In contrast to the first-order version, the function's curvature is not ignored. Our proof is presented for the convex version in an illustrative, special case with two real numbers a and b, where $a < b$:

$$\frac{f(a)+f(b)}{2} \geq f\left(\frac{a+b}{2}\right) \tag{C1}$$

We set $\mu(a,b) = (a+b)/2$ as the arithmetic mean and, therefore, $\mu_0(a_0,b_0)$ as the center point. The LHS is additive separable and can be piece-wise differentiated. Interpreting the RHS as a composite function $f(\mu(a,b))$ helps to keep track after the first step since cross-derivatives have to be considered. Carrying out a quadratic Taylor expansion on both sides of Eq.(C1), by using arguments a and b, yields:

$$\begin{aligned} T_2^{LHS}(\mu(a,b)) = \frac{1}{2}\Big[& f(a_0) + f'(a_0)(a-a_0) + \frac{1}{2}f''(a_0)(a-a_0)^2 \\ & + f(b_0) + f'(b_0)(b-b_0) + \frac{1}{2}f''(b_0)(b-b_0)^2 \Big] \end{aligned} \tag{C2.1}$$

$$\begin{aligned} T_2^{RHS}(\mu(a,b)) = & f(\mu_0) + \mu'(a_0)\cdot f'(\mu_0)(a-a_0) + \mu'(b_0)\cdot f'(\mu_0)(b-b_0) \\ & + \frac{1}{2}(f\circ\mu)''_{aa}(\mu_0)(a-a_0)^2 + \frac{1}{2}(f\circ\mu)''_{bb}(\mu_0)(b-b_0)^2 \\ & + (f\circ\mu)''_{ab}(\mu_0)(a-a_0)(b-b_0) \end{aligned} \tag{C2.2}$$

Without a loss of generality the approximation can be centered at the origin: $a_0 = b_0 = 0$ and $f(0) = 0$. Rearranging—by using the binomial theorem—and simplifying the expressions—by setting $f'(0) = f_0'$, $f''(0) = f_0''$, and $\mu(a,b) = \mu$ to save space—gives:

$$T_2^{LHS}(\mu(a,b)) = \frac{1}{2}\left[f_0'\cdot(a+b) + \frac{1}{2}f_0''\cdot(a^2+b^2)\right] = f_0'\cdot\mu + f_0''\cdot\mu - f_0''\frac{ab}{2} \tag{C3.1}$$

$$T_2^{RHS}(\mu(a,b)) = \frac{1}{2}f_0'\cdot a + \frac{1}{2}f_0'\cdot b + \frac{1}{8}f_0''\cdot a^2 + \frac{1}{8}f_0''\cdot b^2 + \frac{1}{4}f_0''\cdot ab = f_0'\cdot\mu + \frac{1}{2}f_0''\cdot\mu^2 \tag{C3.2}$$

Bringing back both expressions in the inequality form reveals the difference by means of the convexity property and the AM-GM inequality:

$$f_0'\cdot\mu + \frac{1}{2}f_0''\cdot\mu^2 + \frac{1}{2}f_0''\underbrace{(\mu^2 - ab)}_{>0} \geq f_0'\cdot\mu + \frac{1}{2}f_0''\cdot\mu^2. \tag{C4}$$

After a quadratic approximation on both sides, the inequality still holds.

C.2 Definition of the *Approximation Bias*

Modifying Eq.(3.2.2) will not fundamentally change the model's results but will lead to a clearer interpretation of Δ. The idea is to show the resemblance of (i) the difference of two growth rates and (ii) the reversed difference of their inverses. The latter can be approximated by first-order Taylor expansion to result in the actual difference. In the context of a DSGE model's first-order condition (i.e., the Euler equation) combined with the derivative of the CRRA function, we calculate the inverse of growth rates, weighted by the parameter γ:

$$\text{un-approximated:} \quad gr_w^{-1} = \mathrm{E}\left[\frac{1}{(1+X)^{\gamma}}\right]; \quad \text{biased:} \quad gr_{w,b}^{-1} = \frac{1}{(1+\mathrm{E}[X])^{\gamma}} \tag{C5}$$

We use the indices w for *weighted* and b for *biased* in connection with growth rates gr. Additionally, gr is centered around one, representing negative (positive) growth for values smaller (larger) than one. Expressed as its magnitude, the plain Jensen's inequality gives a differential of inverted growth rates:

$$\widetilde{bias} = gr_w^{-1} - gr_{w,b}^{-1} = \frac{1}{gr_w \cdot gr_{w,b}}(gr_{w,b} - gr_w), \tag{C6}$$

which is positive for convex functions. Also, the first-order (multivariate) Taylor expansion of this expression is equivalent to the simple difference $gr_{w,b} - gr_w$:

$$T_1^{\widetilde{bias}}(gr_w, gr_{w,b}), \quad \text{at the center point:} \quad \mathbf{gr_0} = (1,1) \tag{C7.1}$$

$$\Rightarrow \quad \widetilde{bias}(1,1) + \widetilde{bias}'_{gr_w}(1,1)\cdot(gr_w - 1) + \widetilde{bias}'_{gr_{w,b}}(1,1)\cdot(gr_{w,b} - 1) = gr_{w,b} - gr_w. \tag{C7.2}$$

Therefore, we use these re-inverses directly for a cleaner interpretation. This changes Eq.(3.2.2) in a way that the plain difference is not only a linearized approximation but the actual research subject. To draw a closer connection, several numerical examples illustrate the similar outcome. Figure C1 reveals the discrepancies depending on the gr_w–level and an approximation bias of 10/25/50bp. The deviations, stemming from the fraction in Eq.(C6), are multiplied by 10^4, thus, being measured in bp.

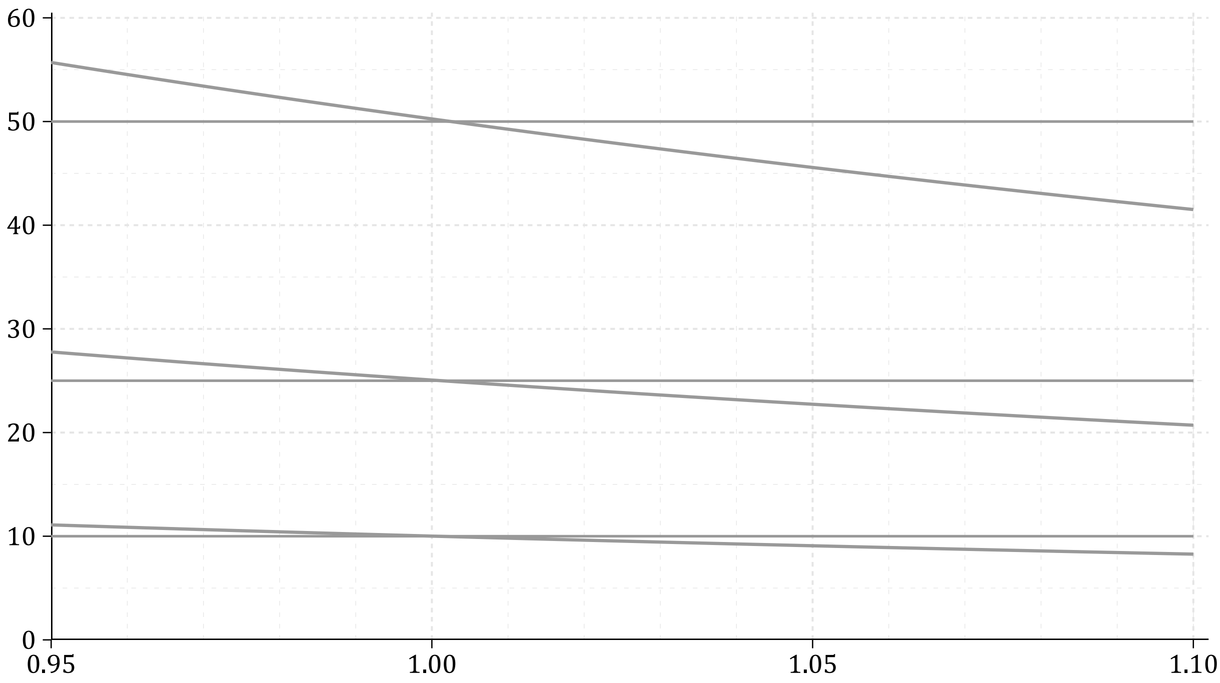

Figure C1: Comparing the difference ($gr_{w,b}-gr_w$) for 10/25/50bp and the corresponding bias in Eq.(C6) depending on the level of growth. Horizontal axis: Weighted growth rate gr_w. Vertical axis: Growth rate difference in basis points.

C.3 Moment Generating Function: Proof

Performing the critical step $\mathrm{E}[\exp(-\gamma\beta Z)] \Rightarrow \exp((\gamma\beta)^2/2)$ to drop the stochastic source, the moment generating function for the (standard) normal distribution is utilized:

$$M(t) = \mathrm{E}\left[e^{tZ}\right] = e^{t^2/2}, \quad Z \sim \mathcal{N}(0,1). \tag{C8}$$

To proof this relationship, the law of the unconscious statistician (LOTUS) is needed to write out the composition regarding the expectation value in terms of an integral:

$$\mathrm{E}\left[e^{tZ}\right] = \int_{\mathbb{R}} e^{tx} \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{1}{2}(x^2-2tx)} dx. \tag{C9}$$

Expanding the exponent for a binomial formula and factoring out the constant gives

$$\mathrm{E}\left[e^{tZ}\right] = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{1}{2}(x^2-2tx+t^2-t^2)} dx = e^{t^2/2} \cdot \underbrace{\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{1}{2}(x-t)^2} dx}_{=1}. \tag{C10}$$

Finally, since the area of a horizontally shifted (by t) standard normal distribution is still one, Eq.(C8) emerges.

C.4 Analytical Solution – Additional Inspection

To obtain a clearer picture of the auxiliary parameter r, the geometric approach in Figure C2 can help.

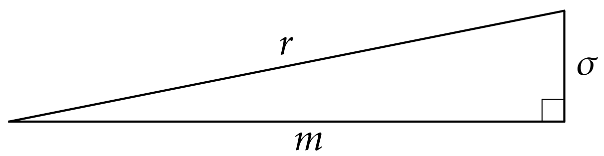

Figure C2: Graphical representation of the two moments and the auxiliary parameter r.

Without factoring out m^γ, the connection to the originating Eq.(3.6) becomes unmistakable. Simultaneously increasing the curvature,

$$\lim_{\gamma\to\infty} bias(\gamma \mid m > 1) = 10^4 \cdot \Big[\underbrace{m^\gamma}_{\to\infty} - \underbrace{m^\gamma \cdot (m/r)^{\gamma^2+\gamma}}_{\to 0} \Big], \tag{C11}$$

shows at the first underbrace the bias growing exponentially, whereas the second expression, after a maximum at $\gamma = \log(m^2/r)/\log(r^2/m^2)$, converges quadratic–exponentially towards zero. In a special case with $m = 1$ and therefore $\mu = 0$ and $r = \sqrt{1+\sigma^2}$, the average growth equals 0%. Here, the second term, stemming from the unbiased expression can be transformed into sums for $\sigma < 1$ and $\gamma \in \mathbb{N}$ by means of the geometric series and triangular numbers:

$$bias(\sigma,\gamma) = 10^4 \cdot \left[1 - (1/r)^{\gamma^2+\gamma}\right] \tag{C12.1}$$

$$= 10^4 \cdot \left[1 - \left(\frac{1}{1+\sigma^2}\right)^{\frac{\gamma(\gamma+1)}{2}}\right] \tag{C12.2}$$

$$= 10^4 \cdot \left[1 - \left(\sum_{i=0}^{\infty} (-1)^i \sigma^{2i}\right)^{\sum_{i=1}^{\gamma} i}\right]. \tag{C12.3}$$

C.5 Algebraic Formula for the Variance

Typically shown by the binomial theorem, an alternative way to point out equality of the variance formula,

$$\mathrm{Var}[X] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2, \tag{C13}$$

works analogously to the approach in Section 3.3. Inserting the distribution formula and its mean, μ, leads to

$$\mathrm{Var}[X] = \mathrm{E}[\exp(\alpha + \beta Z)^2] - \mu^2 \tag{C14.1}$$

$$= \exp(2\alpha)\cdot \mathrm{E}[\exp(2\beta Z)] - \mu^2 = \exp(2\alpha)\cdot\exp((4\beta^2)/2) - \mu^2$$
$$= \left(\frac{\mu}{\sqrt{1+(\sigma/\mu)^2}}\right)^2 \cdot \exp(\beta^2)^2 - \mu^2 = \left(\frac{\mu^2}{1+(\sigma/\mu)^2}\right)\cdot (1+(\sigma/\mu)^2)^2 - \mu^2$$
$$= \mu^2 \cdot (1+(\sigma/\mu)^2) - \mu^2 = \sigma^2, \tag{C14.2}$$

confirming the centered second moment for the log-normal distribution.

C.6 Inverted Beta-Distribution – Parameters

Similarly to Eqs.(3.7), log-normal case, we aim to solve for the parameters of the inverted beta (or beta prime) distribution. Being more unknown than the log-normal distribution, we show this in more detail. The moment-generating function (see, e.g., Keeping 1962, 84),

$$\mathrm{E}[X^p] = \prod_{i=1}^{p} \frac{\alpha+i-1}{\beta-i}, \quad p \in \mathbb{N} \text{ and } p < \beta, \tag{C15}$$

draws the connection between raw moments and parameters. The latter have to be written in terms of μ and σ to translate the grid in Eq.(3.17) into α and β. As claimed in Eq.(C15), p has to be smaller than β and, therefore, β has to be sufficiently large. Indeed, this is given in the calibration. For $p = 1$ and $p = 2$, the mean and the second central moment are

$$\mathrm{E}[X] = \mu = \frac{\alpha}{\beta-1} \quad \text{and} \tag{C16.1}$$
$$\mathrm{E}[X^2] - \mathrm{E}[X]^2 = \sigma^2 = \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2}, \tag{C16.2}$$

respectively. First, replacing α in Eq.(C16.2) with Eq.(C16.1) and solving for β leads to

$$\sigma^2 = \frac{\mu(\beta-1)(\mu(\beta-1)+\beta-1)}{(\beta-2)(\beta-1)^2} = \frac{\mu(\mu+1)}{(\beta-2)} \tag{C17.1}$$
$$\Leftrightarrow \quad \beta = 2 + \frac{\mu+1}{\sigma^2}\mu. \tag{C17.2}$$

Second, solving Eq.(C16.1) for α and replacing β with Eq.(C17.2) leads to

$$\alpha = \mu\left(\frac{\mu(\mu+1)}{\sigma^2} + 2 - 1\right) = \mu + \frac{\mu+1}{\sigma^2}\mu^2. \tag{C18}$$

C.7 Parameter Results for US Inflation Forecasts

To obtain a better insight into the connection of the parameters, Table C1 displays the results of the calibration before conversion. The conversion of α and β is done by using Eqs.(3.7) and Eqs.(C16).

Parameter	Min	25^{th}centile	Median	75^{th}centile	Max
log-norm.					
α	−0.0032	0.0189	0.0243	0.0315	0.0411
β	0.0001	0.0001	0.0002	0.0007	0.0057
inv. beta					
α	0.1000	61.408	101.75	157.17	444.60
β	14.908	2301.5	3587.5	5372.9	18505.8

Table C1: Calibration results (US) when assuming CF data follow a log-normal and inverted beta distribution, respectively. The calibration is run on a monthly basis for the rolling window-adjusted observations on inflation forecasts.

For the log-normal distribution, the resulting values are close to the values after conversion. The α and β values for the inverted beta distribution are more widespread. This is a direct consequence of two characteristics of the distribution. The first one is that negative values are not drawn which can be overcome by a shift. The second—more crucial—point is the high probability mass at the left end of the distribution which can only be overcome by large values for α and β. A side effect of these values is a large spread of drawn variables which results in the higher error values compared to the log-normal distribution.

C.8 Calibration Results for US GDP Forecasts

The same tests and calibration as in Section 3.4 is run with the GDP growth forecast as the observed variable. The time horizon is the same, but the focus is only on the US and not on the frontier markets.

As before, we start with a normality test to check whether the log GDP forecast variables are normally distributed. The results can be seen in Table C2. In more than 75% of the observations, the J-B test cannot reject the H_0 of non-normally distributed variables.

Norm. Test	*p*-value < 10%	< 5%	< 1%
J-B	78	64	39
S-W	90	54	11
A-D	98	62	19
LF	102	73	29

Table C2: Normality test results for GDP growth forecasts (US). It displays the number of observation points that reject the H_0 of non-normally distributed variables out of 357 observations.

As the results of the normality tests are sufficient, the calibration can be run. The results are presented in Table C3.

Parameter	Min	25^{th}centile	Median	75^{th}centile	Max	Obs. Median	Mean Error
log-norm.							
μ_{opt}	−0.0178	0.0192	0.0259	0.0320	0.0420	0.0260	$1.18{\cdot}10^{-5}$
σ_{opt}	0.0001	0.0001	0.0001	0.001	0.0022	0.0027	$1.18{\cdot}10^{-5}$
inv. beta							
μ_{opt}	0.0052	0.0255	0.0293	0.0329	0.2832	0.0260	$2.26{\cdot}10^{-4}$
σ_{opt}	0.0013	0.0022	0.0029	0.0039	0.1397	0.0027	$2.26{\cdot}10^{-4}$

Table C3: Calibration results (US) when assuming CF data follow a log-normal and inverted beta distribution, respectively. The calibration is run on a monthly basis for the rolling window-adjusted observations on GDP growth forecasts.

Compared to the results from the calibration of inflation forecasts, we can identify the same pattern. Whereas the log-normal distribution fits μ really good, the error in σ is larger. The inverted beta distribution shows a contrary picture with a good fit in σ and bad fit in μ. Nevertheless, the log-normal distribution again has a lower error. At last, the direct parameters resulting from the calibration before the conversion into μ and σ are presented in Table C4.

Parameter	Min	25^{th}centile	Median	75^{th}centile	Max
log-norm.					
α	−0.0179	0.0190	0.0256	0.0315	0.0411
β	0.0001	0.0001	0.0001	0.0001	0.0022
inv. beta					
α	0.1021	62.681	120.30	182.10	676.98
β	15.079	2117.6	3919.9	6210.9	19782.2

Table C4: Calibration results (US) when assuming CF data follow a log-normal and inverted beta distribution, respectively. The calibration is run on a monthly basis for the rolling window-adjusted observations on GDP growth forecasts.

The structure of the results is close to the one from the inflation forecasts. The resulting values from the log-normal distribution are close to the values after conversion and the α and β values for the inverted beta distribution are more widespread.

C.9 Impact of the Mean on the Approximation Bias

In the following, derivatives are used to evaluate the effect of μ on the approximation bias. Sticking to Eq.(3.10.3), the centered growth (m) is utilized. Nevertheless, the constant transformation, $m = 1 + \mu$, ensures equivalent derivatives:

$$\frac{\partial bias}{\partial \mu} \equiv \frac{\partial bias}{\partial m}. \tag{C19}$$

Since the absolute values of the approximation bias heavily depend on σ and γ (see Figure 3.3), it is difficult to conclude the impact of μ. We differentiate the *bias*-function to account for the change depending on a change in the mean growth rate. Using the product and chain rule, (...) refers to the part not changing:

$$\frac{\partial bias}{\partial m} = 10^4 \cdot \left[\gamma m^{\gamma-1}(...) + m^{\gamma}\left(\frac{\gamma(\gamma+1)}{2}\left(1+\frac{\sigma^2}{m^2}\right)^{-\frac{\gamma(\gamma+1)}{2}-1} \cdot (-2\sigma^2 m^{-3})\right)\right] \tag{C20.1}$$

$$= 10^4 \cdot \gamma m^{\gamma-1}\left[(...) - (\gamma+1)\left(1+\frac{\sigma^2}{m^2}\right)^{-\frac{\gamma(\gamma+1)}{2}-1} \cdot \frac{\sigma^2}{m^2}\right] \tag{C20.2}$$

$$= 10^4 \cdot \gamma m^{\gamma-1}\left[1 - \left(1+\sigma^2/m^2\right)^{-\frac{\gamma(\gamma+1)}{2}}\left(1+\frac{(1+\gamma)\sigma^2}{m^2+\sigma^2}\right)\right]. \tag{C20.3}$$

Testing the baseline scenario for extreme μ-values:

$$\frac{\partial bias}{\partial m}(\mu = -5\% \mid BL) \approx -1.2 \;\widehat{=}\; -0.012bp/pp \tag{C21.1}$$
$$\frac{\partial bias}{\partial m}(\mu = 20\% \mid BL) \approx -0.8 \;\widehat{=}\; -0.008bp/pp \tag{C21.2}$$

The interpretation is a change in basis point (e.g., an increase of the bias) caused by a 1 percentage point increase of the mean growth. For the BL case, the change at the upper and lower bound is basically zero. Testing maximum values for σ and γ:

$$\frac{\partial bias}{\partial m}(\mu = -5\% \mid \sigma = 0.04, \gamma = 1) \approx -17.6 \;\widehat{=}\; -0.176bp/pp \tag{C22.1}$$
$$\frac{\partial bias}{\partial m}(\mu = 20\% \mid \sigma = 0.04, \gamma = 1) \approx -11.1 \;\widehat{=}\; -0.111bp/pp \tag{C22.2}$$
$$\frac{\partial bias}{\partial m}(\mu = -5\% \mid \sigma = 0.01, \gamma = 5) \approx 40.6 \;\widehat{=}\; 0.406bp/pp \tag{C22.3}$$
$$\frac{\partial bias}{\partial m}(\mu = 20\% \mid \sigma = 0.01, \gamma = 5) \approx 64.8 \;\widehat{=}\; 0.648bp/pp \tag{C22.4}$$

Even for the maximum values, the change is significantly under 1bp. This makes it uninteresting to further examine μ as a varying parameter in the simulations.

C.10 Regression Outputs

Accompanying the MC simulations in Section 3.5, we regress the median-*bias* on the varying parameters, respectively. Using the median instead of the mean is interesting when facing skewed data and since there are already theoretical solutions in some cases for the mean. Also, this goes hand in hand with Figures 3.4–3.10 which display the standard boxplots. However, despite the mean being always larger, there are no substantial differences. As a robustness check, we also present regression results—in the same vein—for both the CARA function and the inverted beta distribution. Starting with the main results, Table C5 shows five regressions for four parameters.

Term	$bias(\sigma)$	$bias(\gamma)$	$\log bias(\gamma)$	$bias(\rho)$	$\log bias(n)$
const.	−0.002	−0.265***	−0.121***	1.903***	−0.146***
	(0.004)	(0.062)	(0.002)	(0.000)	(0.001)
linear	0.003	−0.109*		0.009***	0.066***
	(0.004)	(0.054)		(0.000)	(0.001)
quadr.	0.890***	0.679***		−0.000	−0.000***
	(0.001)	(0.010)		(0.000)	(0.000)
log			1.575***		1.022***
			(0.002)		(0.002)
log-quadr.			0.155***		
			(0.002)		
$\overline{R}^2$	1	0.99	1	1	1
Std.Err.	$5.46 \cdot 10^{-3}$	$8.31 \cdot 10^{-2}$	$6.39 \cdot 10^{-3}$	$1.18 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$
Obs.	20	20	20	20	20

Table C5: Second moment measures (σ and ρ) are multiplied by 100. ***/**/* denote significance at the 1%/5%/10% level.

Multiplying σ and ρ by 100 accounts for the typical step size when referring to one increment. Therefore, the first regression column predicts an increase in the *bias* by $2 \cdot 0.89 \times 100 \cdot \sigma_1$ when increasing the standard deviation by 0.01 at a level of σ_1. Since the other estimates are extremely small and insignificant, they are not absorbing explanatory power from the quadratic term. Also, explaining basically 100% of the dependent variable's variation confirms the (nearly) quadratic relationship discussed in Section 3.3.

The second column draws a different picture since the model artificially shifts the parabola's vertex to the forth quadrant. In addition, the quadratic term is smaller than in the σ-case, probably underestimating the effect for larger values. Alternatively, in the third column, estimating the elasticity (1.575) increases the explanatory power. However, the effect cannot be isolated and the relationship is still more complex, expressed by the significant squared log-coefficient. The intercept comes into play if $\gamma = 1$ (log-utility) and indicates a positive *bias* close to zero. Without the squared term, the elasticity would be somewhat larger (1.665), the intercept insignificant, and $\overline{R}^2 = 0.99$.

Column four confirms the linear relationship regarding the correlation between two variables. Staying at 1.9bp without any correlation, a 0.01-step results in a significant change that is basically zero. However, for more than two variables, Figure 3.9 points out an increase in the slope while preserving the linear connection. In the last column, a roughly unit elastic relationship emerges, reinforced by the linear term, which effects an increase of 6.6% for every additional variable. For $n = 1$, $\log bias = -0.08$, which perfectly matches the baseline result of just below 1bp visible in Figure 3.6.

As seen in Table C6, the above numbers are mostly confirmed in our robustness check. Therefore, for both the inverted beta (instead of log-normal) and the CARA (instead of CRRA) we renounce examining the multivariate part, which is non-trivial for the multivariate inverted beta-distribution. The first two columns are basically equal to column one and three in Table C5. In the CARA case, the quadratic coefficient is somewhat smaller (varying σ) and the elasticity is slightly larger (varying γ). Mathematically, in the log-utility case ($\gamma = 1$) and after applying the inversion $g(y)$, the marginal CRRA-function is only a first-order Taylor expansion of the CARA around zero: $1 + x \approx e^x$. However, the simpler CRRA-function better fits into our model assumptions and is used frequently in the literature.

Term	$bias_{\text{beta}}(\sigma)$	$\log bias_{\text{beta}}(\gamma)$	$bias_{\text{CARA}}(\sigma)$	$\log bias_{\text{CARA}}(\gamma)$
const.	−0.010 (0.007)	−0.120*** (0.002)	−0.002 (0.004)	−0.699*** (0.003)
linear	0.019** (0.008)		0.005 (0.004)	
quadr.	0.885*** (0.002)		0.499*** (0.001)	
log		1.575*** (0.002)		2.077*** (0.003)
log-quadr.		0.156*** (0.002)		0.043*** (0.003)
$\overline{R}^2$	1	1	1	1
Std.Err.	$9.91 \cdot 10^{-3}$	$5.97 \cdot 10^{-3}$	$5.16 \cdot 10^{-3}$	$8.51 \cdot 10^{-3}$
Obs.	20	20	20	20

Table C6: σ is multiplied by 100 to account for the typical step size. ***/**/* denote significance at the 1%/5%/10% level.

C.11 2^{nd}-Order Taylor Series for Eq.(3.11)

In addition to the regression analysis, we check for accuracy whether the relationship can be titled "quadratic" for a realistic range of σ, from a theoretical point of view. Approximating around $\sigma_0 = 0$ is chosen for simplicity, although this is the smallest possible value, and therefore it cannot be the optimal center point. It turns out that this approximation is sufficient for eligible values. However, as an extension, we also choose center points larger than zero to check how the formula changes and to clarify the mechanics behind the Taylor series. The following function has to be constructed from Eq.(3.11):

$$T_2(\sigma \mid \sigma_0) = bias(\sigma_0) + bias'(\sigma_0)(\sigma - \sigma_0) + \frac{1}{2} bias''(\sigma_0)(\sigma - \sigma_0)^2. \quad \text{(C23)}$$

First and second derivatives are:

$$bias'(\sigma) = 10^4 \frac{2m^3\sigma}{(m^2+\sigma^2)^2} \quad \Rightarrow \quad bias'(0) \ = 0 \quad \text{(C24.1)}$$

$$bias''(\sigma) = 10^4 \frac{2m^3(m^2-3\sigma^2)}{(m^2+\sigma^2)^3} \quad \Rightarrow \quad bias''(0) = 10^4 \cdot 2m^{-1} \quad \text{(C24.2)}$$

This ends in a simple quadratic relationship corrected by m, the centered mean:

$$T_2(\sigma \mid \sigma_0 = 0) = 10^4 \cdot \sigma^2/m \quad \text{(C25)}$$

Compared to the accurate solution, second-order Taylor expansion overestimates the bias when the standard deviation is increasing from the center point. Taking the classical log-linearizing of growth rates as a reference point, a growth rate of 5% leads to a deviation of almost 2.5%:[62]

$$\frac{5\%}{\log(1.05)} - 1 \approx 2.48\% \quad \text{(C26)}$$

Using this benchmark in terms of Eqs.(3.11) and (C25) gives

$$\frac{T_2(\sigma = 0.167)}{bias(\sigma = 0.167)} - 1 \approx 2.48\%. \quad \text{(C27)}$$

Since 0.167 is roughly four times larger than the maximum value chosen in the simulation, we can conclude that the quadratic approximation is sufficient. Nevertheless, as further extension, using a center points greater than zero, Table C7 gives an impression of how the accuracy changes over the σ's.

[62] 5% would be the approximation and log(1.05) the correct expression (in a non-linear equation). Therefore, the percentage deviation is standardized by the latter term.

Center point σ_0	$-2.5\% < \%$-difference $< 2.5\%$ min σ	max σ	Difference at $\sigma = 0.01$ Δbp
0.005	0.001	0.168	0
0.010	0.001	0.169	0
0.015	0.002	0.171	0
0.020	0.004	0.173	−0.001
0.025	0.005	0.176	−0.002
0.030	0.007	0.180	−0.007
0.035	0.009	0.184	−0.015
0.040	0.011	0.188	−0.029
0.045	0.014	0.192	−0.052
0.050	0.016	0.196	−0.085

Table C7: For σ values smaller (larger) than σ_0 the theoretical bias is underestimated (overestimated).

With a larger centre point, the maximum σ-value increases at which the approximation is sufficient. However, this gained accuracy does not compensate the simultaneously increasing minimum σ-value, with $\sigma_0 = 0.04$ already surpassing the baseline case. The latter is further examined in the last column, showing the absolute difference in basis points. This difference is still very small, not reaching a tenth of a basis point. Therefore, from a practical point of view, even when the centre point is not optimal, the approximation error is negligible.

C.12 Jensen's Inequality as Ratio

Formulating the problem as a ratio gives the advantage of a simpler formula. Initially, the factor 10^4 gets redundant, cancelling out due to the fraction. Also, in this case, the function $g(y)$ only switches denominator and numerator, resulting in a ratio > 100%:

$$bias_r(m,\sigma,\gamma) = \frac{(1+\mathrm{E}_t[growth_{t+1}])^{\gamma}}{\mathrm{E}_t[(1+growth_{t+1})^{-\gamma}]^{-1}} = \frac{m^{\gamma}}{m^{\gamma}\cdot\left(\frac{m}{\sqrt{m^2+\sigma^2}}\right)^{\gamma(\gamma+1)}} \tag{C28.1}$$

$$= \left(\frac{m}{\sqrt{m^2+\sigma^2}}\right)^{-\gamma(\gamma+1)} = \left(\frac{\sqrt{m^2+\sigma^2}}{m}\right)^{\gamma(\gamma+1)} = \left(1+\frac{\sigma^2}{m^2}\right)^{\frac{\gamma(\gamma+1)}{2}}. \tag{C28.2}$$

Additionally, the formula reduces to squared expressions for all parameters while the curvature remains in the exponent. Given positive integer values for γ, the formula can be written in a discrete form as

$$bias_r(m,\sigma,\gamma) = \left(1+(\sigma/m)^2\right)^{\sum_{i=0}^{\gamma} i} = \prod_{i=1}^{\gamma}\left(1+(\sigma/m)^2\right)^i, \tag{C29}$$

a product which factors only consist of 100% plus a squared coefficient of variation weighted by integer exponents. Figure C3 depicts Eq.(C29) for several curvature values, revealing a deviation, even for extreme scenarios, by only a few percent.

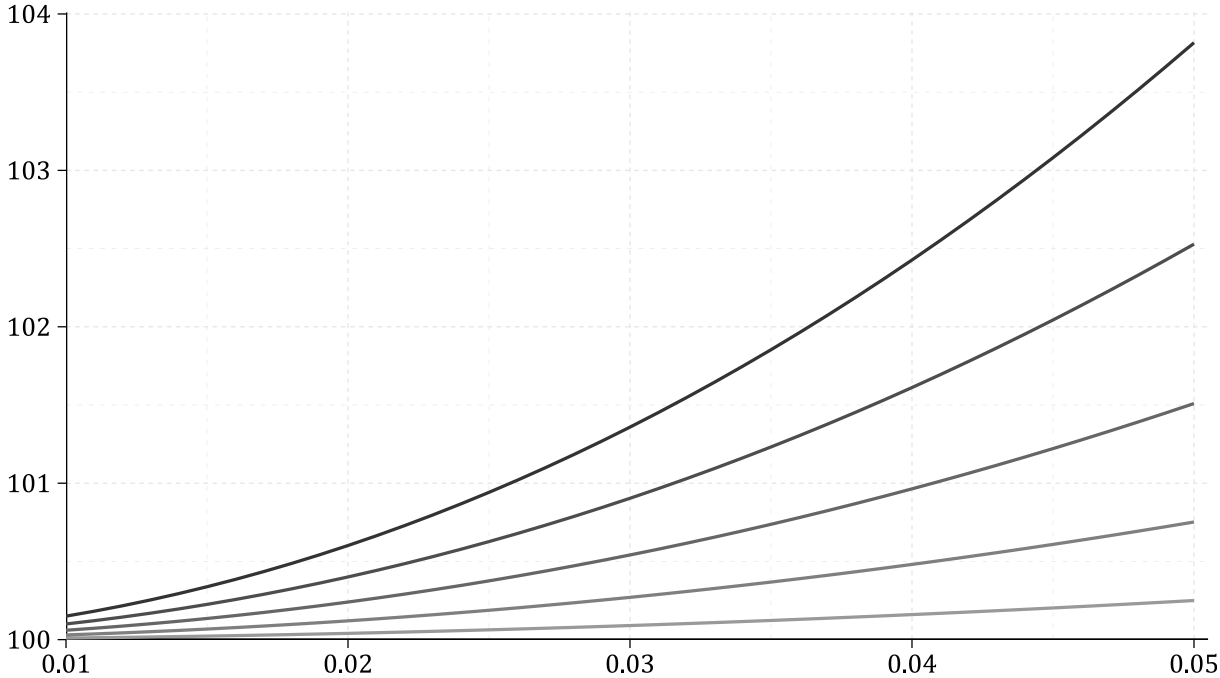

Figure C3: Comparing the Jensen ratios for $\gamma_i \in \{1,2,3,4,5\}$ and varying σ/m. Lines are ordered from lightgray ($\gamma = 1$) to darkgray ($\gamma = 5$). Horizontal axis: Coefficient of variation (σ/m). Vertical axis: Ratio of growth rates in %.

C.13 2[nd]-Order Taylor Series for Eq.(3.12)

The following function has to be constructed from Eq.(3.12):

$$T_2(\gamma \mid \gamma_0) = bias(\gamma_0) + bias'(\gamma_0)(\gamma-\gamma_0) + \frac{1}{2}bias''(\gamma_0)(\gamma-\gamma_0)^2. \tag{C30}$$

Analogous to Eq.(C25), the center point, γ_0, equals zero. The first derivative is calculated by the product rule, the chain rule, and the log-rule to switch the sign:

$$bias'(\gamma) = 10^4\Big[\log(m)m^\gamma \cdot \left(1-(m/r)^{\gamma(\gamma+1)}\right) + m^\gamma \cdot (-\log(m/r))(m/r)^{\gamma(\gamma+1)}(2\gamma+1)\Big]$$

$$
\begin{aligned}
&= 10^4 \cdot m^{\gamma}\Big[\log(m)\cdot\Big(1-(m/r)^{\gamma(\gamma+1)}\Big)+\log(r/m)(m/r)^{\gamma(\gamma+1)}(2\gamma+1)\Big] \\
&= 10^4 \cdot m^{\gamma}\Big[\log(m)+(m/r)^{\gamma(\gamma+1)}\cdot\Big(\log(r/m)(2\gamma+1)-\log(m)\Big)\Big] \qquad \text{(C31.1)}\\
\Rightarrow \quad bias'(0) &= 10^4\cdot\log(r/m) \qquad \text{(C31.2)}
\end{aligned}
$$

Applying the product rule twice, with [...] and (...) for the parts not differentiated, gives

$$
\begin{aligned}
bias''(\gamma) &= 10^4\Big[\log(m)m^{\gamma}\cdot[...]+\\
&\qquad m^{\gamma}\cdot\Big(\log(m/r)(m/r)^{\gamma(\gamma+1)}(2\gamma+1)\cdot(...)+(m/r)^{\gamma(\gamma+1)}\cdot 2\log(r/m)\Big)\Big]\\
&= 10^4\cdot m^{\gamma}\Big[\log(m)\cdot[...]+\log(r/m)(m/r)^{\gamma(\gamma+1)}(2-(2\gamma+1)(...))\Big] \qquad \text{(C32.1)}\\
\Rightarrow \quad bias''(0) &= 10^4\cdot[\log(m)\cdot\log(r/m)+\log(r/m)\cdot(2-(\log(r/m)-\log(m)))]\\
&= 10^4\cdot\log(r/m)\Big(2\log(m)+2-\log(r/m)\Big)\\
&= 10^4\cdot\log(r/m)\Big(2\log(m)+2\log(\mathrm{e})+\log(m/r)\Big)\\
&= 10^4\cdot 2\log(r/m)\log\Big(m\mathrm{e}\sqrt{m/r}\Big). \qquad \text{(C32.2)}
\end{aligned}
$$

Putting the derivatives together leads to a quadratic equation without intercept and similar coefficients for the linear and the quadratic term:

$$
T_2(\gamma \mid \gamma_0 = 0) = 10^4\cdot\log(r/m)\Big[\log\Big(\mathrm{e}\sqrt{m^3/r}\Big)\gamma^2+\gamma\Big] \qquad \text{(C33)}
$$

Compared to the accurate solution, second-order Taylor underestimates the bias when the curvature is increasing from the center point. Using the benchmark in Eq.(C26) in terms of Eqs.(3.12) and (C33) gives

$$
\left|\frac{T_2(\gamma=0.9)}{bias(\gamma=0.9)}-1\right| \approx 2.49\%. \qquad \text{(C34)}
$$

This corresponds to a curvature up to 0.9 and, thus, slightly smaller than the baseline case. In terms of the economic interpretation, the quadratic Taylor-series is accurate enough for EIS-values larger than 1.1. Following the meta-study by Havranek et al. (2015), this includes not even half of the scenarios. Hence, in contrast to the σ-version in Appendix C.11, we would not refer to this relationship as quadratic. For reasonable values, the relationship is exponential with approximately constant elasticities, $\partial\log(bias)/\partial\log(\gamma)$.

C.14 Simulation with the Elasticity of Intertemporal Substitution

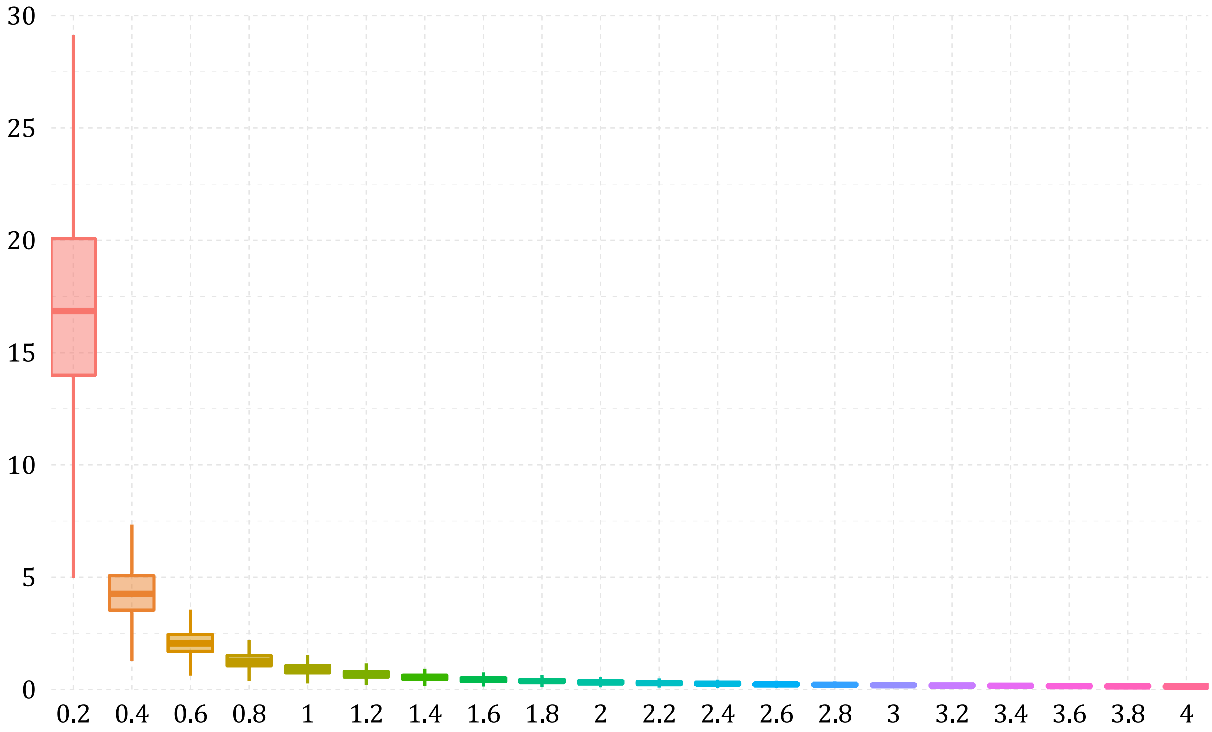

Figure C4: Horizontal axis: EIS (γ^{-1}). Vertical axis: Growth rate difference in basis points.

Appearing like the distorted mirror image of Figure 3.5, the graphic shows a significant impact for EIS = 0.2 only.

C.15 Convergence for Large Samples

We show how the empirical bias approaches the analytical bias by means of the variance formula, $\text{Var}[X] = E[X^2] - E[X]^2$, a special case of Jensen's inequality. This example is particularly traceable since the analytical bias consists of σ^2 only. Therefore, this case specifies the function f but shows the results being not dependent on the moments or the distribution, provided that the required moments are defined. The set-up is as follows:

$$\lim_{N\to\infty} \text{E}\left[\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)^2\right] = \sigma^2, \quad x_i \sim i.i.d.(\mu, \sigma^2) \tag{C35}$$

We claim that the %-difference to σ^2 for finite N is always negative and approaches this value in the form of a hyperbola:

$$\frac{\mathrm{E}\left[\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)^2\right] - \sigma^2}{\sigma^2} = -\frac{1}{N} \tag{C36}$$

The interpretation in percent is convenient since for $N = 1$ the value stays at -100% and approaches 0% for $N \to \infty$. Eq.(C36) can be simplified by using $\mathrm{E}[x_i \cdot x_j] = \mathrm{E}[x_i] \cdot \mathrm{E}[x_j] + \mathrm{Cov}[x_i, x_j]$ as the key step:

$$\Rightarrow \qquad \frac{1}{N}\sum_{i=1}^{N}\mathrm{E}[x_i^2] - \frac{1}{N^2}\mathrm{E}\left[\left(\sum_{i=1}^{N} x_i\right)^2\right] = \sigma^2 - \frac{\sigma^2}{N} \tag{C37.1}$$

$$\Leftrightarrow \qquad \mu^2 + \sigma^2 - \frac{1}{N^2}\mathrm{E}\left[(x_1 + \ldots + x_N)^2\right] = \sigma^2 - \frac{\sigma^2}{N} \tag{C37.2}$$

$$\Leftrightarrow \qquad N^2\mu^2 - \mathrm{E}\left[\sum_{i=1}^{N} x_i^2 + 2\sum_{i=1}^{N-1}\left(x_i \cdot \sum_{j=1+i}^{N} x_j\right)\right] = -N\sigma^2 \tag{C37.3}$$

$$\Leftrightarrow \quad N^2\mu^2 - N(\mu^2 + \sigma^2) - 2\mathrm{E}\Big[\underbrace{x_1x_2 + x_1x_3 + \cdots + x_1x_N}_{(N-1)\text{ terms}} + \underbrace{x_2x_3 + x_2x_4 + \cdots + x_2x_N}_{(N-2)\text{ terms}} + \cdots + \underbrace{x_{N-1}x_N}_{1\text{ term}}\Big] = -N\sigma^2 \tag{C37.4}$$

$$\Leftrightarrow \qquad N^2\mu^2 - N\mu^2 - 2\frac{N(N-1)}{2}\mu^2 = 0. \tag{C37.5}$$

From Eq.(C37.4) to Eq.(C37.5), the *i.i.d.*-property ensures that $\mathrm{Cov}[x_i, x_j] = 0$. The fraction in the last step, $N(N-1)/2 = 1 + 2 + \ldots + (N-1)$, equals the total amount of terms x_ix_j.

Interpreting N as the amount of states, in which the future economy can be situated, the analytical bias has to be corrected downwards by $(N-1)/N$. E.g., for two possible states, the bias has only half the size as analytically derived.

C.16 Overview: Figures

Figure	μ	σ	γ	ρ	n	N	~max. Δbp
3.3	−5%-20%	0.001-0.04	1.5/2/2.5	–	1	∞	70
3.4	6%	0.002-0.04	1	–	1	30	25
3.5	6%	0.01	0.25-5	–	1	30	30
3.6	6%	0.01	1	–	1	1-20	2
3.7	6%	0.002-0.04	1	–	1	2	25
3.8	6%	0.01	1	−0.95-0.95	2	30	5
3.9	6%	0.01	1	0-0.95	5	30	30
3.10	2.5%	0.01	1	0.1	1-20	30	90

Table C8: Overview for all figures using varying parameters. μ ($= m - 1$) and σ designate the growth rates' mean and standard deviation, respectively. γ reflects the non-linear function's curvature. ρ is the correlation between two or more variables. n counts the number of variables in the model. N stands for the sample size. The last column shows the maximum differences in basis points of the respective figure.

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Summary (Deutsche Zusammenfassung)

Die Dissertation "Structured Eurobonds - Optimal Construction, Impact on the Euro and the Influence of Interest Rates" ist kumulativ verfasst, bestehend aus drei einzelnen Fachartikeln. Obwohl die Dissertationsschrift aus drei in sich abgeschlossenen Kapiteln zusammengesetzt ist, sind diese, wie in der *Motivation* beschrieben, thematisch miteinander verbunden.

Der Fokus liegt bei den ersten beiden Kapiteln auf strukturierten Eurobonds, vor allem deren Sensitivität hinsichtlich verschiedener Variablen und Einfluss auf den Wechselkursmarkt. Das dritte Kapitel beschäftigt sich mit Approximationsfehlern in DSGE Modellen und der Kalibrierung von Verteilungen. Die Approximationsfehler haben einen großen Einfluss auf Eurobonds, insbesondere im Hinblick auf die von makroökonomischen Faktoren abhängige Sensitivität.

1. Sensitivität strukturierter Eurobonds

Der erste Artikel "Limited Joint Liability in Structured Eurobonds: Pricing the political costs" setzt sich mit den politischen Kosten und Gewinnmöglichkeiten einer beschränkten gemeinsamen Haftung bei strukturierten Eurobonds auseinander. Unter Eurobonds versteht man in diesem Kontext[63] die gemeinsame Verschuldung der Länder der Europäischen Währungsunion (EWU), wohingegen in der aktuellen Praxis nahezu ausschließlich auf einzelstaatlicher Ebene Anleihen emittiert werden.[64] Für die Einführung von Eurobonds gibt es verschiedene Modelle, welche nahezu allen teilnehmenden Staaten Zinsvorteile verschaffen sollen. Staaten mit einer sehr guten Bonität können gegebenenfalls keine direkten Zinsvorteile erwirtschaften, aber es wäre mit einer Stärkung der EWU sowie einer geringeren Liquiditätsprämie verbunden. Allerdings wird das moralische Risiko als sehr hoch eingeschätzt. Unter moralischem Risiko wird insbesondere das Risiko einer Querfinanzierung von Staaten, welche einen hohen Zinssatz zahlen müssen, durch Staaten mit einem besseren Rating beziehungsweise einem niedrigen Zinssatz verstanden. Da für das Funktionieren und die Akzeptanz der Eurobonds eine gemeinsame Haftung als elementar angesehen wird, besteht die Gefahr einer höheren Schuldenaufnahme durch Staaten, welche sich durch Eurobonds günstiger finanzieren können.[65]

[63] Eurobonds werden im nicht-europäischen Kontext auch als Anleihen, welche nicht in der nationalen Währung begeben werden, verstanden.

[64] Hier wird von aktuellen Praktiken wie beispielsweise den Anleihen des Europäischen Stabilitätsmechanismus (ESM) abgesehen.

[65] Zusätzlich gibt es noch verschiedene juristische Hürden, auf welche hier aber nicht weiter eingegangen werden soll.

Die Emission von Eurobonds über strukturierte Produkte – hiermit sind insbesondere forderungsbesicherte Wertpapiere gemeint – sind eine Möglichkeit, um die oben genannten Vorteile von Eurobonds zu konservieren und die Nachteile zu reduzieren. Hierzu werden die einzelstaatlichen Anleihen von einer Zweckgesellschaft aufgekauft. Im Anschluss werden von dieser Zweckgesellschaft wiederum Anleihen in verschiedenen Tranchen mit unterschiedlichem Rating emittiert, um die Kosten des Aufkaufs der einzelstaatlichen Anleihen zu decken. Der Mittelwert des Ratings der Tranchen und die damit einhergehende durchschnittliche Zinslast ist geringer als die Zinslast der bereits bestehenden Anleihen. Dies ist eine Folge von Diversifikationseffekten.

Zu dem Ansatz der Emission durch ein strukturiertes Produkt gibt es verschiedene Arbeiten, welche im Kern ein gleiches Modell vorschlagen, aber in der Ausgestaltung verschiedene Möglichkeiten aufzeigen. So variiert die Anzahl der emittierten Tranchen zwischen zwei und einer nicht fixierten Zahl. Eine weitere Modifikation ist die Etablierung eines Sicherheitenfonds, welcher mit einem gewissen Anteil des emittierten Nominals kapitalisiert wird. Dieser Fonds könnte im Falle von Staatspleiten erste Verluste auffangen. Er übernimmt die ausgefallenen Zahlungen und bildet eine Art Puffer, bevor einzelne Tranchen der strukturierten Eurobonds ausfallen. Durch diese Konstruktion wird die gemeinsame Haftung drastisch reduziert. Sie wird auf das Volumen des Sicherheitenfonds beschränkt und das übrige Risiko auf den Kapitalmarkt transferiert, da bei der Insolvenz eines Staates und der Ausnutzung des Sicherheitenfonds die Kuponzahlungen und endfällige Tilgung der Junior-Anleihen gegebenenfalls nicht bedient werden können. In diesem Fall erleidet der Investor Verluste und die übrigen Staaten müssen diese Zahlung nicht leisten. Die Emission mit einer zusätzlichen Einführung eines Sicherheitenfonds, welcher initial mit einem fixierten Anteil des Nominals – z.B. 10% – ausgestattet wird, wird in diesem Artikel simuliert.[66]

Eine Konsequenz der Konstruktion mit einem Sicherheitenfonds ist eine Verbesserung des durchschnittlichen Ratings der Struktur, allerdings mit einem abnehmenden Effekt. Die Problematik ist, dass eine höhere Kapitalisierung des Fonds eine zusätzliche Zinslast aufruft, da er im Rahmen der Emission von strukturierten Eurobonds mit kapitalisiert wird und die Staaten hierfür die Zinslast tragen müssen. Ab einem gewissen Volumen überwiegen die negativen Effekte einer zusätzlichen Kapitalisierung die positiven Effekte aus einem verbesserten durchschnittlichen Rating und dem niedrigeren durchschnittlichen Zins. Zusätzlich geht mit einem größeren Volumen eine deutlich erhöhte gemeinsame Haftung einher, was sich als politische Kosten niederschlägt. Die Untersuchungsfrage ist nun, welches Volumen beziehungsweise welche

[66]Siehe zu dem Modell insbesondere Hild et al. (2014).

initiale Kapitalisierung des Sicherheitenfonds – wie im obigen Beispiel 10% – unter verschiedenen Parametervariationen optimal ist.

Die Parameter, welche für die Analyse variiert werden, sind der risikolose Zinssatz und die Erlösquote der Staaten. Der risikolose Zinssatz ist relevant für die zu leistenden Zinszahlungen der Staaten wie auch der Zweckgesellschaft. Er schlägt sich zusätzlich im Sicherheitenfonds nieder, da das hierin enthaltene Kapital zu diesem Zinssatz verzinst wird. Die Erlösquote spiegelt den Anteil am Nominal wider, welcher nach einer Insolvenz noch an den Investor zurückgezahlt werden kann. Der Wert hat in dem Modell sowohl einen direkten Einfluss auf die Ausfallwahrscheinlichkeit, als auch auf die Kapitalzuflüsse an den Sicherheitenfonds im Falle einer Insolvenz. Der Einfluss auf die Ausfallwahrscheinlichkeit resultiert aus der Berechnungsmethodik ebenjener.

Es wird die Gleichung

$$AW = \frac{CDS_{Prämie}}{1 - EQ} \tag{1}$$

angewendet, wobei AW für die Ausfallwahrscheinlichkeit und EQ für die Erlösquote steht. Der Wert $CDS_{Prämie}$ resultiert aus den Prämien von am Kapitalmarkt gehandelten Credit Default Swaps, welche – aufgrund ihrer Wirkungsweise – eine Versicherung gegen eine Insolvenz darstellen. Eine Änderung von EQ hat somit einen inversen Einfluss auf AW. Auf die Kapitalzuflüsse des Sicherheitenfonds wirkt die Erlösquote in der Form, dass bei einer Insolvenz eines Staats sein nominaler Anteil direkt mit dieser Quote in den Sicherheitenfonds fließt. Die Veränderung der Kapitalzuflüsse aus Insolvenzen ist somit ebenfalls ein Punkt, der einen großen Einfluss auf den Erfolg von strukturierten Eurobonds haben kann.

Für die Basis-Analyse wird angenommen, dass strukturierte Eurobonds im Juli 2018 eingeführt worden wären und eine Laufzeit von 10 Jahren haben. Die Sensitivität hinsichtlich der beiden Parameter – risikoloser Zinssatz und Erlösquote – bei verschiedenen initialen Sicherheitsfondsgrößen kann in Abbildung 1 gesehen werden.

Es zeigt sich, dass die Sensitivität mit wachsender Größe des Sicherheitenfonds abnimmt und insbesondere bei niedrigeren Erlösquoten eine höhere initiale Kapitalisierung vorteilhaft ist. Bei höheren Erlösquoten kann es hingegen sinnvoll sein, mit einer niedrigeren Kapitalisierung zu starten. Eine Veränderung im risikolosen Zinssatz hat ebenfalls einen Einfluss auf den Gewinn. Dieser ist allerdings nicht eindeutig. Während bei einer niedrigen Startkapitalisierung und einer geringen Erlösquote ein negativer Zusammenhang zwischen dem risikolosen Zinssatz und dem Gewinn besteht, ändert sich dies ins Positive, wenn entweder die initiale Kapitalisierung oder die Erlösquote anwachsen.

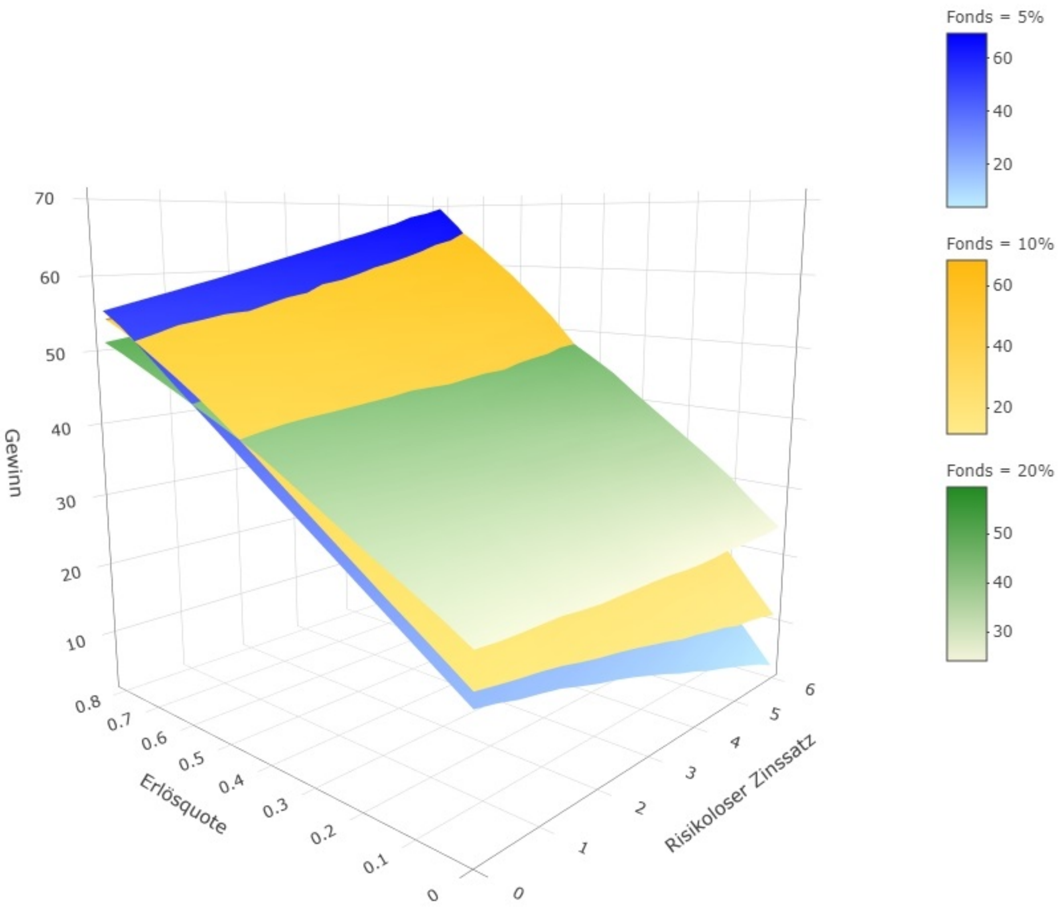

Abbildung 1: Sensitivität der Gewinne aus der Emission von strukturierten Eurobonds unter der Annahme verschiedener initialer Sicherheitenfonds-Größen.

Als nächstes stellt sich die Frage wie stark der Gewinn bei einer fixierten Erlösquote in Abhängigkeit von der Sicherheitenfondsgröße und dem risikolosen Zinssatz ist. Dies wird in Abbildung 2 visualisiert, in welcher die Quote auf 50% fixiert wird.[67]

Der Gewinn ist somit eine inverse und U-förmige Funktion in Abhängigkeit der initialen Sicherheitenfonds-Kapitalisierung. In Konsistenz zu dem vorherigen Ergebnis ist der Gewinn aufgrund der Erlösquote von 50% positiv vom risikolosen Zinssatz abhängig. Hinsichtlich der Größe des Sicherheitenfonds gibt es allerdings eine kritische Größe bis zu welcher der Gewinn steigt und im Anschluss wieder fällt. Die optimale Größe ist auch abhängig vom risikolosen Zinssatz, allerdings ist der Effekt keinen extremen Schwankungen unterworfen, wie es in Abbildung 3 zu sehen ist.

Die optimale initiale Sicherheitenfonds-Kapitalisierung bewegt sich bei einer Erlösquote von 50% zwischen 12% und 16%. Auffällig ist, dass bei einem risikolosen

[67] Die 50% orientieren sich an historisch beobachteten Erlösquoten von insolventen Staaten. Eine genauere Analyse kann bei Meyer et al. (2019) und Cruces and Trebesch (2013) gefunden werden.

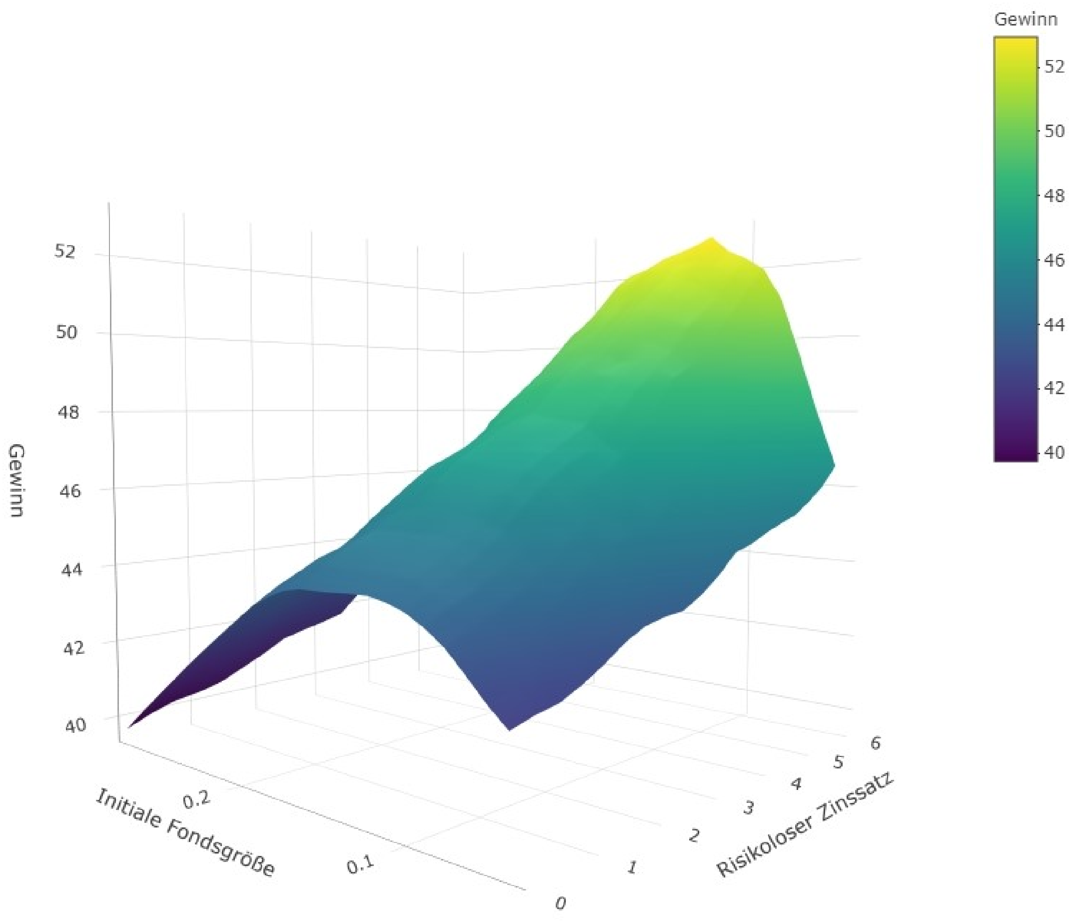

Abbildung 2: Sensitivität der Gewinne aus der Emission von strukturierten Eurobonds unter der Annahme einer fixierten Erlösquote von 50%.

Zins von 0% die optimale Kapitalisierung bei 12% liegt, danach auf 16% springt, um im Anschluss kontinuierlich – nur unterbrochen von kleinen Ausschlägen – wieder zu sinken. Bei einem risikolosen Zins von 6% ist die optimale Startkapitalisierung wieder bei 12%. Der Sprung bei niedrigen risikolosen Zinssätzen liegt daran, dass die Gewinne nur marginale Differenzen bei einer Variation des Zinssatzes in dieser Höhe aufweisen. Mit einem wachsenden risikolosen Zinssatz wird das Kapital im Sicherheitenfonds höher verzinst und dadurch entsteht ein größerer Puffer, um mögliche Staatsinsolvenzen abzufangen. Das Modell profitiert von einem Zinseszins-Effekt.

Neben der Wahl der optimalen Sicherheitenfondsgröße und deren Sensitivität hinsichtlich makroökonomischer Parameter, wird auch die Einführung zu verschiedenen historischen Zeitpunkten – 2008 und 2012 – sowie verschiedenen Subgruppen diskutiert. Für diese Analysen wird der Sicherheitenfonds bei einem Wert von 10% fixiert.[68]

[68] Für verschiedene Erlösquoten hat sich ein optimaler Wert zwischen 9% und 18% ergeben. Da die ersten Analysen von Hild et al. (2014) auf einen Wert von 10% durchgeführt wurden, soll dieser hier ebenfalls angewendet werden.

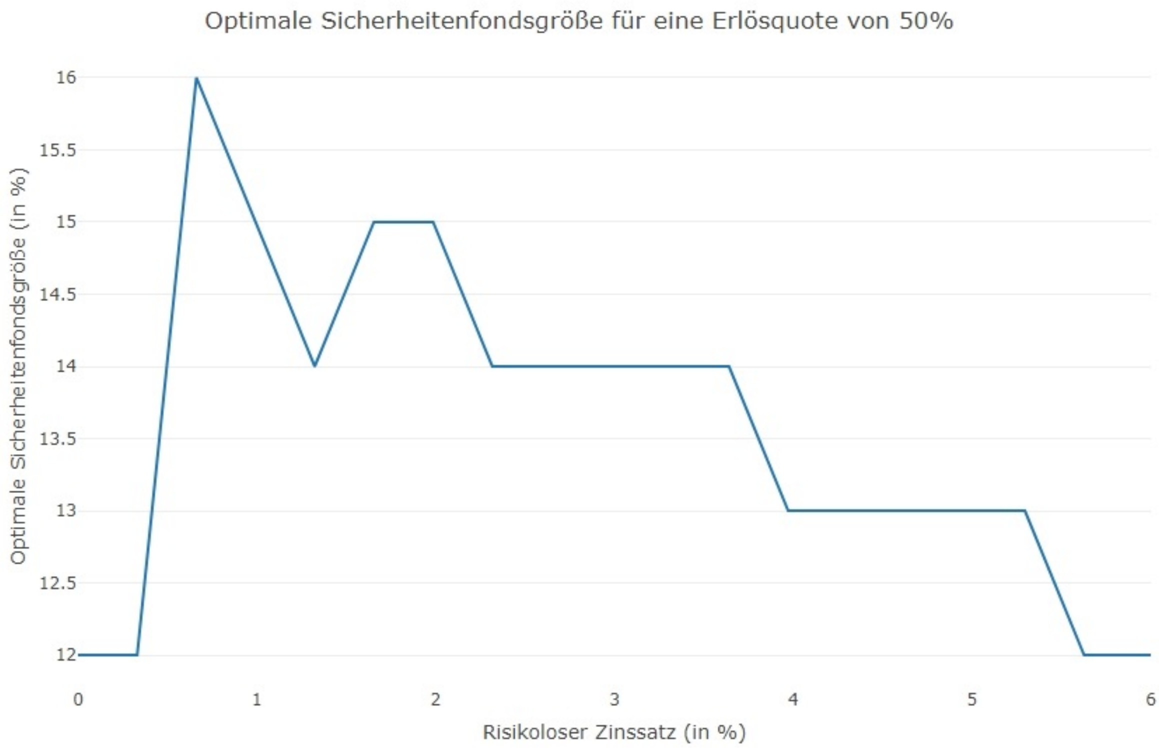

Abbildung 3: Die optimale initiale Sicherheitenfonds-Kapitalisierung in Abhängigkeit vom risikolosen Zinssatz.

Die Simulationen für 2008 und 2012 zeigen, dass das Modell stabil bleibt und unverändert Gewinne generiert.

Hervorzuheben ist insbesondere, dass es für zwei Subgruppen – EWU ohne Italien und PIIGS-Staaten – im Vergleich zum Basisszenario mit einer Einführung im Jahre 2018 zu einer starken Änderung der Gewinne kommt. In einem Szenario ohne Italien sind die Erlöse stark rückläufig. Dies ist insbesondere auf zwei Faktoren zurückzuführen. Zum einen ändert sich das durchschnittliche Rating der emittierten Tranchen, beziehungsweise des Zinssatzes, nur marginal im Vergleich zum Basisszenario, und zum anderen sind die Zinszahlungen an die Zweckgesellschaft deutlich verringert. Im Basisszenario trägt Italien ungefähr 40% der gesamten Zinszahlungen an die Zweckgesellschaft. Diese beiden Faktoren sorgen für den Rückgang der Erlöse aus der Emission von strukturierten Eurobonds. Eine Emission ausschließlich durch die PIIGS-Staaten – Portugal, Italien, Irland, Griechenland und Spanien – haben einen konträren Effekt. Hier verschlechtert sich das durchschnittliche Rating der Struktur und damit einhergehend steigt der zu zahlende Zinssatz. Dennoch bleibt ein großer Teil mit dem besten Rating, nämlich AAA, übrig, obwohl keiner der beteiligten Staaten dieses Rating hat. Die Aufwertung wird ausschließlich durch den Diversifikationseffekt erreicht. Zusätzlich bleibt das Zinsvolumen beziehungsweise der durchschnittliche Zinssatz, der von den beteiligten Staaten gezahlt wird, hoch. Durch diese Faktoren wird vergleichsweise viel Geld im Sicherheitenfonds gelagert und verzinst. Dies führt am Ende zu einer hohen Auszahlung an die fünf beteiligten Staaten, sodass sie – je nach Verteilung – zwischen 4,27% und 22,65% ihres eingesetzten Nominalvolumens an Gewinnen erzielen können. Zusätzlich wurde die Emission für kurze Laufzeiten betrachtet, wobei die Ergebnisse insbesondere bei sehr kurzen Laufzeiten für diverse Staaten auch negativ sein können. Dennoch werden weiterhin Gewinne generiert, der Verteilungsmechanismus müsste allerdings adjustiert werden.

In allen Konstellationen – Zeitpunkt der Einführung und teilnehmende Staaten – stellt sich heraus, dass das Modell der strukturierten Eurobonds hinsichtlich der positiven Resultate beständig ist. Die Höhe schwankt zwar, Gewinne werden aber unabhängig vom risikolosen Zinssatz und der Erlösquote generiert. Die einzigen Parameter, welche von der Politik oder den Entscheidungsträgern beeinflusst werden können, sind die der initialen Sicherheitenfonds-Kapitalisierung und der teilnehmenden Staaten. Die Generierung von Erlösen ist stark davon abhängig, wie viel gemeinsame Haftung die Staaten einzugehen bereit sind. Zu viel gemeinsame Haftung kann den Effekt auch umkehren. Außerdem hat sich herausgestellt, dass keine vollständige Teilnahme der gesamten EWU notwendig ist und auch Subgruppen positive Resultate erreichen. Es würde also die Möglichkeit bestehen, zuerst mit einer kleinen Gruppe zu starten, beispielsweise den Gründungsländern der Europäischen Wirtschaftsgemeinschaft, und den Teilnehmerkreis später zu erweitern.

2. Einfluss auf Wechselkurse

Der zweite Artikel "The Impact of Structured Eurobonds on Exchange Rates" setzt sich mit dem Einfluss einer Emission von strukturierten Eurobonds auf verschiedene Wechselkurse auseinander.

Zuerst wird im Artikel der Begriff der strukturierten Eurobonds ausführlich erläutert und auf die bereits bestehenden Bestrebungen einer Etablierung solcher Instrumente innerhalb der EWU verwiesen. Die Emission mithilfe eines strukturierten Produkts wird in diesem Artikel angenommen, was ebenfalls von der Europäischen Kommission angeregt wird. Zusätzlich wird die Stärkung des Euro als Reservewährung als eine Folge der Emission von Eurobonds hervorgehoben. Dies wird beispielsweise im Grünbuch über die Durchführbarkeit der Einführung von Stabilitätsanleihen der *European Commission (2011)* herausgestellt. Der Einfluss auf den Euro wird im Rahmen dieses Artikels unter Annahme einer Emission von strukturierten Eurobonds quantifiziert.

Zur Formalisierung einer Zinskurve wird das Nelson-Siegel Modell verwendet. Aus diesem können drei Parameter – Level, Steigung und Krümmung – extrahiert und verschiedene makroökonomische Effekte, beispielsweise Inflationserwartungen oder erwartetes BIP-Wachstum, erklärt werden. Zusätzlich gibt es die Möglichkeit, die Wechselkursentwicklung zwischen zwei Währungen anhand der Änderung der zugehörigen Zinskurven beziehungsweise der hieraus resultierenden Nelson-Siegel Faktoren zu bestimmen. Um dies zu bewerkstelligen, müssen die relativen Nelson-Siegel Faktoren über die Gleichung

$$y(m) - y^*(m) = L_t^R + S_t^R \left(\frac{1 - e^{-\lambda m}}{\lambda m} \right) + C_t^R \left(\frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m} \right) + \epsilon_t \tag{2}$$

und einer kleinste Quadrate Methode bestimmt werden. Hierbei beschreibt $y(m)$ die inländische Rendite (im Folgenden ist hier immer die EWU gemeint) und $y^*(m)$ die ausländische Rendite. L_t^R, S_t^R und C_t^R stehen für die relativen Nelson-Siegel Faktoren Level, Steigung und Krümmung.

Im Anschluss können die relativen Nelson-Siegel Faktoren mit den beobachteten Wechselkursänderungen in Verbindung gesetzt werden. Dies wird mithilfe einer linearen Regression, angewendet auf die Gleichung

$$\Delta s_{t+m} = \beta_{m,0} + \beta_{m,1} L_t^R + \beta_{m,2} S_t^R + \beta_{m,3} C_t^R + u_{t+m}, \tag{3}$$

bewerkstelligt, wobei Δs_{t+m} die Änderung des Wechselkurses zum Zeitpunkt t für m Monate in die Zukunft darstellt. Die Regression liefert für vier von fünf untersuchten Gegenwährungen – US-Dollar, Britischer Pfund, Schweizer Franken und Chinesische Renminbi – signifikante Ergebnisse. Für Japanischen Yen können keine signifikanten Ergebnisse beobachtet werden. Insbesondere für den einjährigen Horizont haben drei Währungspaare mindestens einen – wenn auch unterschiedlichen – signifikanten Faktor, weshalb sich die folgende Analyse auf diesen Horizont fokussiert. Bei Britischen Pfund ist im sechsmonatigen Horizont ein signifikanter Faktor vorhanden, sodass der Fokus bei dieser Währung auf dem kürzeren Horizont liegt.

Zusätzlich wird angenommen, dass im Rahmen der Einführung von strukturierten Eurobonds die einzelstaatlichen Anleihen vom Kapitalmarkt komplett verschwinden. Eine direkte Folge hiervon wäre, dass die Zinsstrukturkurven einzelner Staaten nicht mehr vorhanden sind und durch eine Zinskurve für die gesamte EWU ersetzt werden. Aktuell gibt es bereits eine Zinskurve für die EWU, welche von der Europäischen Zentralbank (EZB) berechnet wird. In diese fließen die Zinskurven der einzelnen Mitgliedsstaaten gewichtet ein. Die Gewichtung ist von der Kapitalstruktur der EZB und den Anteilen der jeweiligen Staaten an dieser abhängig. Die Anteile der Staaten berechnen sich aus deren Bevölkerung und dem BIP.

Die neue EWU-Zinskurve wird sich von der aktuellen unterscheiden und primär durch die erreichte Struktur beziehungsweise den emittierten Tranchen der strukturierten Eurobonds definiert. In diesem Artikel wird die Zinskurve unter Verwendung dreier verschiedener Indikatoren (EZB-Kapitalstruktur, BIP und Schuldenstände) berechnet. Im ersten Schritt wird für jedes einzelne Rating die Zinsstrukturkurve berechnet. Dafür wird die Gleichung

$$y_R(m) = \sum_{i=1}^{n} \frac{IND_i}{\sum_{j=1}^{n} IND_j} \cdot y_i(m), \tag{4}$$

verwendet, wobei $y_R(m)$ für die m-monatige Rendite zum Rating R, n für die Anzahl der Staaten mit dem entsprechenden Rating und IND für den gewählten der drei vorher genannten Indikatoren steht. $y_i(m)$ beschreibt in diesem Kontext die länderspezifische m-monatige Rendite. Um die finale Zinskurve zu erhalten, müssen die Zinskurven für die einzelnen Ratings nach einer festen Gewichtung kombiniert werden. Diese Gewichtung ist von den Anteilen der Tranche mit dem entsprechenden Rating an der gesamten Struktur abhängig.

In der Literatur werden verschiedene Strukturen präsentiert, der Fokus liegt hier allerdings auf zwei Extremfällen.[69] Diese zeichnen sich dadurch aus, dass die Korrelation zwischen den Staaten – einer der Hauptgründe für die Strukturierung von Anleihen – in einem Fall als sehr hoch und im anderen als geringer angesehen wird. Im ersten Fall hat dadurch die AAA-Tranche, also die Tranche mit dem besten Rating, einen Anteil von ca. 56,63% an der gesamten Emission und beim zweiten Fall steigt dieser auf 95,41% an.[70] Aus diesem Grund nimmt die entstehende Zinsstrukturkurve verschiedene Formen an. Während sie beim ersten Fall nur leicht unterhalb der aktuellen Kurve liegt, ist der Abstand beim zweiten Fall deutlich größer.

Bei der Analyse des Einflusses der Emission von strukturierten Eurobonds hat sich gezeigt, dass sich dieser nur hinsichtlich der Stärke unterscheidet. Ob der Euro auf- oder abwertet bleibt in beiden Fällen oder Strukturen identisch. Dies ist auch unabhängig vom Einführungszeitpunkt, wobei eine Emission im Januar und Februar 2018 simuliert wird. Der Euro würde gegenüber dem US-Dollar abwerten und den anderen drei Währungen (Britischer Pfund, Schweizer Franken und Chinesischer Renminbi) aufwerten. Der Effekt auf den Euro in den zwölf Monaten nach Emission von strukturierten Eurobonds bewegt sich zwischen -3,57% (vs. US-Dollar) und 5,37% (vs. Briti-

[69] Siehe hierzu insbesondere Hild et al. (2014).

[70] Im Juli 2018 haben drei Staaten in der EWU ein AAA-Rating und repräsentieren ca. 36,3% des BIP.

schen Pfund). Insgesamt ergibt sich also ein gemischtes Bild, welches den Erwartungen einer Stärkung des Euro als Reservewährung nicht genügt. Die Erwartung ist nämlich, dass eine Aufwertung in allen Währungspaaren zu finden sein wird. Es ist wichtig zu betonen, dass es sich hierbei um einen isolierten Effekt handelt und zusätzliche Effekte, welche einen Einfluss auf den Wechselkurs haben können, ausgeschlossen werden.

Um die Abwertung beim Wechselkurs des Euro gegen den US-Dollar zu analysieren, wird zusätzlich ein reduzierter Zeithorizont betrachtet. Dieser beginnt ein Jahr nach der Pleite von Lehman Brothers in den USA. Hiermit kann der unmittelbare Einfluss der Finanzkrise bzw. deren Verwerfungen exkludiert werden. Es stellt sich heraus, dass sich an der Richtung des Effekts, das heißt einer vorher betrachteten Abwertung oder Aufwertung, nichts ändert, sondern diese sogar verstärkt wird. Zusätzlich wird auch noch ein handelsgewichteter Wechselkurs betrachtet bei welchem die Einflüsse auf die vier Wechselkurse in Abhängigkeit des Handelsvolumens gewichtet werden. Bei Anwendung dieser Methode kann immer eine Aufwertung des Euro betrachtet werden.

Insgesamt lässt sich folgern, dass die Emission von strukturierten Eurobonds eine Aufwertung des Euro zur Folge hätte. Gegenüber dem US-Dollar würde sich allerdings eine Abwertung ergeben. Das Ziel einer Stärkung des Euro als Reservewährung wäre zumindest hinsichtlich des Wechselkurses teilweise erfolgreich.

3. Unsicherheiten als Resultat von Jensens Ungleichung

Im dritten Artikel "Evaluating the Approximation Bias in Forward-Looking DSGE Models" wird der Approximationsfehler, welcher durch Anwendung von Jensens Ungleichung in um Erwartungen erweiterten DSGE Modellen entsteht, untersucht. Das Feld der strukturierten Eurobonds aus den ersten beiden Artikeln wird somit verlassen. Dennoch haben die Ergebnisse einen direkten Einfluss auf die vorherigen Resultate.

In einem ersten Schritt werden DSGE Modelle und ihre Relevanz für die Analyse makroökonomischer Veränderungen dargestellt. Der Einfluss von zweiten (oder höheren) Momenten einer Verteilung, welche insbesondere während einer Finanzkrise große Auswirkungen haben können, werden bei der Log-Linearisierung der Modelle unterdrückt. Durch eine Approximation höheren Grades kann diese Problematik umgangen werden, aber hier stellt sich die Frage der Darstellung im Modell beziehungsweise der Lösung. Eine Möglichkeit besteht in der Anwendung von Jensens Ungleichung. Der Fehler, welcher hierdurch entsteht, soll in diesem Artikel quantifiziert werden.

Von der Euler-Gleichung, welche in DSGE Modellen wichtig ist, wird der nicht-lineare und in die Zukunft schauende Teil isoliert betrachtet. Der nicht-lineare Teil der Euler-Gleichung kann in Form einer isoelastischen Nutzenfunktion behandelt werden. Für die makroökonomischen Variablen, hier im speziellen Inflationsraten, und deren späteren Kalibrierung werden Daten von Consensus Forecasts (CF) genutzt. Für diese Variablen, welche im Kontext des Artikels als Zufallsvariablen angesehen werden, wird eine Log-Normalverteilung angenommen. Durch weitere Berechnungen stellt sich die Gleichung

$$bias(X) = 10^4 \cdot \left((\mathrm{E}[1+X]^{\gamma}) - \mathrm{E}[(1+X)^{-\gamma}]^{-1} \right), \quad (1+X) \sim \log \mathcal{N}\left(\mu, \sigma^2\right), \tag{5}$$

wobei X eine log-normalverteilte Zufallsvariable und γ die Krümmung der isoelastischen Nutzenfunktion ist, als die zentrale Gleichung zur Berechnung des Fehlers (*bias*) dar. Die Auswertung des Fehlers wird mit zwei Varianten, analytisch und mithilfe von Monte Carlo Simulationen, durchgeführt.

Zur Herleitung des analytischen Ergebnisses wird Gl.(5) zu

$$bias(\mu, \sigma, \gamma) = \mathrm{E}[10^4 \cdot \left((\mathrm{E}[1 + growth_{t+1}]^{\gamma}) - \mathrm{E}[(1 + growth_{t+1})^{-\gamma}]^{-1} \right)] \tag{6}$$

umgeschrieben und gelöst. Hier entspricht $growth_{t+1}$ der log-normalverteilten Zufallsvariablen aus dem ursprünglichen Problem. Da Zufallsvariablen enthalten sind, muss zum Ermitteln der Lösung der Erwartungswert gebildet werden. Durch Anwendung der momenterzeugenden Funktion für die Normalverteilung kann die stochastische Komponente aus der Gleichung entfernt werden, sodass ausschließlich ein deterministischer Zusammenhang zurückbleibt. Dieser wird durch

$$bias(\mu, \sigma, \gamma) = 10^4 \cdot (1+\mu)^{\gamma} \cdot \left(1 - \left(\frac{1+\mu}{\sqrt{(1+\mu)^2 + \sigma^2}} \right)^{\gamma(\gamma+1)} \right) \tag{7}$$

beschrieben. Dabei entspricht μ dem Mittelwert und σ der Standardabweichung der Verteilung. Bei ausreichend guten empirischen Datensätzen können die hieraus abgeleiteten Parameter bereits zur Bestimmung des Fehlers eingesetzt werden. Wenn diese allerdings nicht ausreichend sind, muss eine Monte Carlo Simulation durchgeführt werden. Für die Simulation müssen Parameter vorliegen, welche anhand einer Kalibrierung erzielt werden können. Dies gibt eine Bandbreite möglicher Parameter vor.

Bevor zur Monte Carlo Simulation übergegangen wird, wird mit einer Datenanalyse und der Kalibrierung begonnen. Die Kalibrierung wird zuerst anhand der USA durchgeführt. Damit sich die Variablen unabhängig vom betrachteten Monat vergleichen lassen können, müssen diese mithilfe der Gleichung

$$\mathrm{E}_t[x_{t+1}|m] = \frac{13-m}{12} \cdot \mathrm{E}_{t,m}[x_t] + \frac{m-1}{12} \cdot \mathrm{E}_{t,m}[x_{t+1}] \tag{8}$$

harmonisiert werden. Hierdurch wird die Variable immer im einjährigen Horizont betrachtet. Damit die nächsten Schritte durchgeführt werden können, muss die Anforderung hinsichtlich der Log-Normalverteilung der Zufallsvariablen aus dem analytischen Teil überprüft werden. Die harmonisierten Zufallsvariablen aus Gl.(8) werden adjustiert und anschließend mithilfe verschiedener Tests hinsichtlich einer Normalverteilung in jedem Monat überprüft. Es zeigt sich, dass die Annahme der Normalverteilung der adjustierten und somit die Log-Normalverteilung der harmonisierten Zufallsvariablen nicht abgelehnt werden kann.

Aufgrund der Ergebnisse kann nun eine Kalibrierung der Zufallsvariablen an der Log-Normalverteilung vorgenommen werden. Für diese wird auf die kleinste Quadrate Methode zurückgegriffen, wobei die Differenz zwischen den Variablen aus dem CF Datensatz und den Gezogenen aus der Log-Normalverteilung gebildet und minimiert wird. Die Parameter für die Log-Normalverteilung werden aus einem Gitter gewählt, welches pro Iterationsschritt feiner wird und schlussendlich eine optimale Parameterkombination liefert. Für eine Robustheitsanalyse wird die Kalibrierung auch für die inverse Betaverteilung durchgeführt. Es zeigt sich, dass der Fehler aus der zweiten Verteilung größer ist als derjenige aus der Log-Normalverteilung. Die gleichen Schritte wie für die USA werden ebenfalls für Ägypten, Nigeria und Südafrika durchgeführt. Diese drei Länder sind die einzigen im CF Datensatz, welche den Schwellenländern oder dem *frontier*-Markt zugeordnet werden können. Der *frontier*-Markt enthält Länder, die auf dem Weg zum Schwellenland sind. Für Institute solcher Staaten könnte die Analyse von zusätzlicher Relevanz sein, da die Datenlage hier häufig noch nicht so gut wie in westlichen Industrienationen ist. Dies zeigt sich bereits im CF Datensatz, in welchem die monatlichen Beobachtungen nicht mehr für eine Analyse und Kalibrierung ausreichen und deshalb auf eine quartalsweise Betrachtung umgestellt werden muss. Im Schnitt gibt es hier 22 Beobachtungen pro Quartal und die Prüfung, ob die Zufallsvariablen log-normalverteilt sind, liefert ähnliche Ergebnisse wie bei den USA. Bei der Kalibrierung zeigt sich wieder, dass die Log-Normalverteilung besser als die inverse Betaverteilung ist. Während sich der Parameter für den Mittelwert bei den USA zwischen -0,0032 und 0,042 und die Standardabweichung zwischen 0,0001 und 0,006 bewegt hat, ist bei diesen Nationen die Bandbreite größer. Hier liegt der Mittelwert zwischen 0,0464 und 0,2033 und die Standardabweichung zwischen

0,0047 und 0,0341. Um diesen unterschiedlichen Bandbreiten und dem größeren Interesse der Schwellenländer Rechnung zu tragen, wird in der folgenden Monte Carlo Simulation der Parameter μ auf 0,06 und die Standardabweichung σ auf 0,01 in einem Basisszenario fixiert. Aufgrund der Beobachtungen pro Monat respektive Quartal wird die Anzahl der pro Monte Carlo Schritt gezogenen Zufallsvariablen auf 30 fixiert.

Es werden nacheinander verschiedene Fälle simuliert, wobei immer die gleiche Vorgehensweise genutzt wird. Zuerst werden die verwendeten Parameter spezifiziert, anschließend die Zufallsvariablen – standardmäßig 30 Stück – 100.000 Mal gezogen und in Gl.(5) eingesetzt. Daraufhin wird der zu untersuchende Parameter variiert und wieder die Monte Carlo Simulation durchgeführt. Die Ergebnisse werden anschließend anhand einer grafischen Darstellung interpretiert.

Den Anfang bildet eine Variation der Standardabweichung. Der Fehler beziehungsweise *bias* bewegt sich zwischen einem Wert nahe bei Null für eine Standardabweichung von 0,002 und 25 Basispunkte (bp) für einen Wert von 0,04. Zur Bestimmung des Zusammenhangs wurde eine Regression durchgeführt, welche einen quadratischen Zusammenhang liefert. Als nächstes wird die Krümmung – gemessen durch den Parameter γ – in einem Intervall von 0,25 bis 5 variiert. Die Ergebnisse sind sehr nahe an den vorherigen mit dem Unterschied, dass der maximale *bias* bei circa 30 bp liegt. Die Regression zeigt wieder, dass ein nahezu quadratischer Zusammenhang vorliegt. Eine weitere Analyse behandelt die Anzahl der gezogenen Zufallsvariablen pro Monte Carlo Simulation, wobei 1 bis 20 Variablen simuliert werden. Es zeigt sich, dass mit einer steigenden Zahl von Zufallsvariablen das numerische Ergebnis schnell an das analytische aus Gl.(7) konvergiert.

Eine weitere wichtige Analyse behandelt multivariate Funktionen, welche die zusätzliche Betrachtung der Korrelation notwendig machen. Dies ist vor allem im Hinblick auf große DSGE Modelle wie dem ECB-Global oder dem IMF Global Projection Model von Relevanz.[71] Hierdurch ändert sich die ursprüngliche Ausgangsproblematik (5) zu

$$bias(X,Y) = 10^4 \cdot \left(\mathrm{E}[1+X] \cdot \mathrm{E}[1+Y] - \mathrm{E}\left[(1+X)^{-1}(1+Y)^{-1}\right]^{-1} \right), \tag{9}$$

im zweidimensionalen Fall, beziehungsweise zu

$$bias(X_1,...,X_n) = 10^4 \cdot \left(\prod_{i=1}^{n} \mathrm{E}[1+X_i] - \mathrm{E}\left[\prod_{i=1}^{n}(1+X_i)^{-1}\right]^{-1} \right) \tag{10}$$

[71] Siehe hierzu insbesondere Dieppe et al. (2018) und Carabenciov et al. (2013).

im mehrdimensionalen Problem. Im zweidimensionalen Fall ist die Differenz relativ klein mit Werten zwischen 1 und knapp 3 bp für Korrelationen zwischen -0,95 und 0,95. Der Zusammenhang ist in diesem Fall linear.

Das Bild ändert sich drastisch, wenn die Anzahl der verschiedenen Zufallsvariablen erhöht wird und es sich somit um den multivariaten Fall handelt. Für den Fall, dass es fünf verschiedene Zufallsvariablen gibt, deren Korrelation von 0 bis 0,95 variiert wird, ergibt sich wieder ein linearer Zusammenhang. Allerdings bewegt sich der Fehler nun in einem Bereich von 5 bis 15 bp, mit Ausreißern bis 25 bp. Bei Fixierung der Korrelation auf einen Wert von 0,1 und ausschließlicher Variation der Anzahl der verschiedenen Zufallsvariablen bis zu einer Anzahl von 20 werden die Differenzen noch größer. Hier werden Ergebnisse von über 50 bp mit Ausreißern bei 90 bp erreicht, was in Abbildung 4 zu sehen ist. Bei dieser Konstellation liegt wieder ein quadratischer Zusammenhang vor.

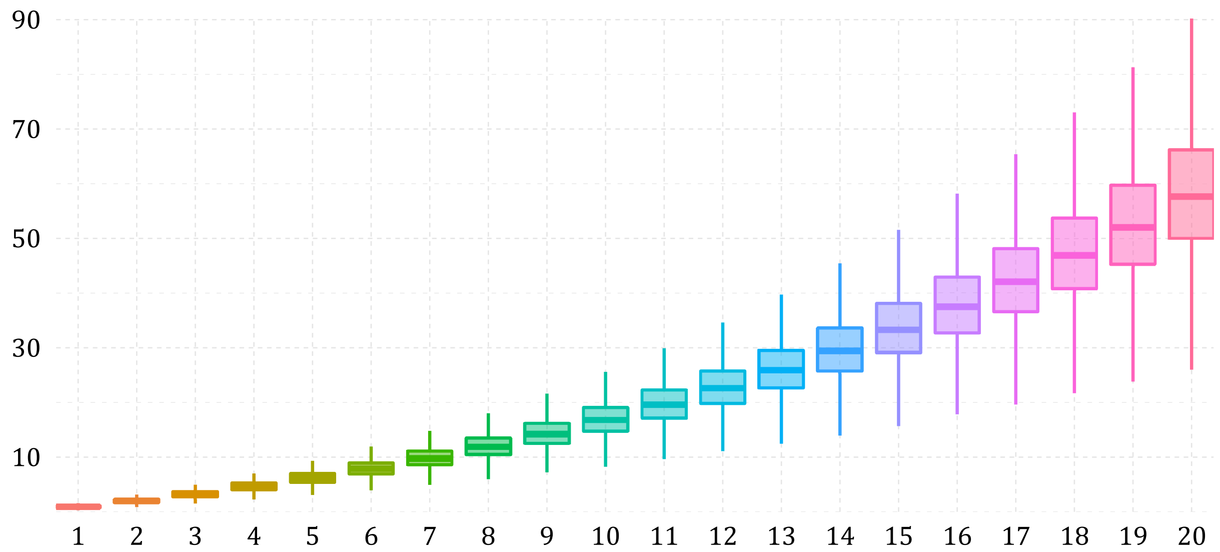

Abbildung 4: Simulation im multivariaten Fall mit einer Korrelation von $\rho = 0{,}1$. Horizontale Achse: Anzahl der Variablen. Vertikale Achse: Differenz der Wachstumsraten in Basispunkten.

Insgesamt zeigt sich, dass der Fehler von einigen Parametern, z.B. der Standardabweichung oder der Größe des Modells, beeinflusst werden kann. Insbesondere in einer Situation mit einer hohen Ungewissheit, beispielsweise einer Finanzkrise, kann dieser Einfluss nicht vernachlässigt werden. Durch die Anwendung der Ungleichung kann insgesamt eine Überschätzung der Wachstumsrate – hier der Inflationsrate – von 25 bp erreicht werden. Wenn dies beispielsweise von der EZB, welche Modelle mit vielen Variablen betreibt, zusätzlich in die Berechnung mit einbezogen wird, dann müssten die entsprechenden Zinssätze angepasst werden.

Hier lässt sich der eingangs genannte Zusammenhang zu den anderen beiden Artikeln deutlich erkennen. Eine Adjustierung der Zinssätze der EZB würde zu zwei zusammengehörenden Szenarien führen. Hierbei handelt es sich (1) um eine Verringerung des risikolosen Zinssatzes und somit ebenfalls um eine (2) Veränderung der Zinsstrukturkurve. Die Zinsstrukturkurven der Mitgliedsstaaten hängen vom risikolosen Zins ab, welcher von EZB über ihren Hauptrefinanzierungssatz gesteuert werden kann. Die Änderung von (1) hätte einen direkten Einfluss auf den Gewinn, welcher durch strukturierte Eurobonds generiert werden könnte, wie auch auf die gemeinsame Haftung, welche eingegangen werden sollte. Eine Änderung der Zinskurve (2) kann zusätzlich dafür sorgen, dass der Einfluss auf die Wechselkurse eine andere Größenordnung annimmt. Dies hängt allerdings stark davon ab inwieweit andere Staaten, z.B. die USA, beziehungsweise die darin ansässigen Zentralbanken, ebenfalls eine Änderung des Leitzinses vornehmen.

Schlussendlich liegt es an der Europäischen Währungsunion ob sie eine gemeinsame Haltung zu Eurobonds und insbesondere strukturierten Eurobonds finden können. Falls es hierzu kommen sollte, können anhand dieser Arbeit einige Effekte wie die Wirkung des risikolosen Zinssatzes auf die optimale gemeinsame Haftung analysiert werden, die vor der Einführung und vor den finalen Ausgestaltungen bewertet werden müssen. Zusätzlich werden die Folgen auf die Wechselkurse dargestellt, welchen bei der Eurobond-Konstruktion Rechnung getragen werden sollte. Des Weiteren bietet sich die Möglichkeit, eine Änderung des Zinssatzes der Zentralbanken aufgrund der Ergebnisse aus dem dritten Artikel in die Evaluation mit einfließen zu lassen.