Vom Fachbereich IV (Wirtschafts- und Sozialwissenschaften, Mathematik und Informatikwissenschaften) der Universität Trier zur Verleihung des akademischen Grades Dr. rer. pol. genehmigte Dissertation

Issues in Price Measurement

vorgelegt von

Alena Shumskikh, M.Sc.

shumskikhalena@gmail.com

Disputation: 8. Dezember 2022

Gutachter:

Univ.-Prof. Dr. Ludwig von Auer
Univ.-Prof. Dr. Normann Lorenz

Trier, 2022

Preface

The present doctoral thesis is a cumulative work, which consists of three independent papers. Two of the papers are connected to each other and are written in co-authorship with Prof. Dr. Ludwig von Auer, while the third one is an individual research paper. The article "Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction" was published in the *Journal of Official Statistics* in March, 2022.

I would like to warmly thank my first supervisor Prof. Dr. Ludwig von Auer for the opportunity to write my doctoral thesis at his chair and to work with him. I am very grateful for his constant support, professional guidance, insightful comments and detailed feedback to every question I asked him. I would like to thank also Prof. Dr. Normann Lorenz, my second supervisor, for his kind readiness to evaluate and review my dissertation. In addition, this journey would not have been possible without the support of Prof. Dr. Matthias Neuenkirch, who has accompanied me since my master's studies and always supports me with advice if necessary.

Furthermore, I would like to thank my friends and colleagues who have supported me in the beginning and/or throughout the entire doctoral phase. I would like to mention in particular: Karsten Schultheiß und Heike Einhorn, Jan Specovius, Dr. Tu Anh Pham, Arina Wischnewsky, Mehrzad B. Baktash, Dr. Andranik Stepanyan, Dr. Artem Dyachenko, Kerstin Müller.

Finally, my very special thanks and gratitude go to my most important people in life, my beloved parents Sviatlana and Uladzimir Shumskikh. Thank you for guiding me on my personal and professional journey, for all your advice, inspiration and support every step of the way. This dissertation is dedicated to you. There are not enough words to express all my love and gratitude to you. I am very proud to be a daughter of the best parents in the world and I love you very much.

Overview of Academic Career

Education:

2008-2013: Economic Theory - Belarusian State University, graduation with honours, Minsk, Belarus

2013-2015: Economist, Statistical Reporting and Economics Division, Finance Department - Telco, ICT and content provider velcom (currently A1 Belarus, Die Telekom Austria Group), Minsk, Belarus

2015 - 2018: MSc. Economics, DAAD/OSF Scholarship Holder - University of Trier - Trier, Germany

2018-2022: Doctoral Student in Economics/Research assistant - University of Trier - Trier, Germany

Publications:

von Auer, L. and Shumskikh, A. (2022). Retrospective Computations of Price Index Numbers: Theory and Application. Review of Income and Wealth/04.12.2022. DOI: https://doi.org/10.1111/roiw.12627.

von Auer, L. and Shumskikh, A. (2022). Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction. Journal of Official Statistics, Vol. 38, No. 1, 2022, pp. 107–126. DOI: https://doi.org/10.2478/jos-2022-0006.

Shumskikh, A. (2022). Economic Valuation of Free Digital Products. Unpublished Manuscript.

Lemeschenko, P. and Shumskikh, A. (2013). Information Economy of the Republic of Belarus in the Context of Global Development Trends (in Russian). Minsk: Misanta, 2013. - 96 p.

Contents

List of Tables i

List of Figures ii

List of Abbreviations iii

Summary 1

Extended Summary 3

1 Retrospective Computations of Price Index Numbers: Theory and Application 7

1.1 Introduction 7

1.2 Four Types of Symmetric Price Indices 9

1.3 Retrospectively Computed Price Indices 12

1.4 Retrospective Correction of the German CPI 15

1.5 Decomposition of the Bias 18

1.6 Decomposition of the Bias in the German CPI 19

1.7 Concluding Remarks 21

Appendix A 22

Appendix B 25

Appendix C 26

Appendix D 27

2 Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction 29

2.1 Introduction 29

2.2 Calculation of the Terms of Trade 31

2.3 Retrospective Correction of Substitution Bias 33

2.4 Application to German Foreign Trade Data 39

2.5 Concluding Remarks 46

Appendix E: Retroactive Correction in Practice 47

3 Economic Valuation of Free Digital Products **53**

3.1 Introduction 53

3.2 Free Digital Products: Definition and Features 54

3.3 Economic Valuation of Digital Products 58

3.3.1 Methods for Economic Valuation of Non-market Products . . . 58

3.3.2 A Review of the Results of Estimating the Economic Value of Free Digital Products from the Literature 62

3.4 Value, Price and Cost: Differences and Relationship 66

3.5 Usage Cost Model 73

3.5.1 A Short Overview of the Travel Cost Method 73

3.5.2 Introduction to the Usage Cost Model 74

3.5.3 Specification of the Usage Cost Model 77

3.5.3.1 Description of Necessary Data 80

3.5.3.2 Adjusted Subscription Fee for Internet Access 81

3.5.3.3 Opportunity costs of time 84

3.5.3.4 Value of personal data and user's attention 87

3.6 Usage Cost Model in Practice 90

3.6.1 A General Estimation Plan of the Model 90

3.6.2 Empirical Application: Results 97

3.7 Concluding Remarks and Further Considerations 103

Appendix F 105

Appendix G 106

Appendix H 114

References **115**

List of Tables

1 Laspeyres index numbers and Walsh index numbers for December 2014 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2010 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI. 20

2 Laspeyres index numbers and Walsh index numbers for December 2004 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2000 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI. 27

3 Laspeyres index numbers and Walsh index numbers for December 2009 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2005 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI. 28

4 Methods of valuing non-market goods/services. 58

5 Development of the definitions of “value”, “price” and “cost” over time. 67

6 Facebook results. 98

7 Instagram results. 99

8 WhatsApp results. 100

9 Test results for overdispersion. 102

10 Groups of price definitions. 105

11 List of variables defined 114

List of Figures

1 German consumer price index (January 2000 = 100) for January 2000 to December 2014. 15

2 Terms of Trade of the G7 countries (1995 = 100) from 1995 to 2018. . . 33

3 German import price index (1995 = 100) for January 1995 to May 2019. 41

4 German export and import price indices (Jan. 1980 = 100), real oil prices (Brent oil, in dollars of 1980) and exchange rate (€/$) for January 1980 to May 2019. 43

5 German export price index (1995 = 100) from January 1995 to May 2019. 44

6 German terms of trade (1995 = 100) from January 1995 to May 2019. . 45

7 An illustrative expected demand function for a free digital product. . . 76

8 Zonal usage cost model. 78

9 Subscription fee transformation and conversion mechanism to be applied to the UCM. 82

List of Abbreviations

CA - Conjoint analysis

CM - Choice Modelling

CPI - Consumer Price Index

CV - Contingent valuation

Destatis - The Federal Statistical Office of Germany ("Statistisches Bundesamt")

GB - Gigabyte

GDP - Gross Domestic Product

ILO - International Labour Organization

ITCM - Individual Travel Cost Model

IUCM - Individual Usage Cost Model

MB - Megabyte

NSO - National Statistical Offices

SF - Subscription fee

TCM - Travel Cost Model

UCM - Usage Cost Model

WTA - Willingness-to-accept

WTP - Willingness-to-pay

ZTCM - Zonal Travel Cost Model

ZUCM - Zonal Usage Cost Model

Summary

This dissertation is devoted to the issues of price measurement. It comprises three chapters each of which replicates a separate paper. The first two papers are concerned with the theory and application of retroactive revisions of price indices. Both papers are co-authored with Ludwig von Auer. The third paper studies the monetary evaluation of free digital products, that is, products with zero prices. The empirical applications for all three papers are conducted using the programming language R.

Price indices play an important role in the economy as they monitor changes in the price level and measure price movements. Price level measures are used as a conversion mechanism for various nominal economic indicators (e.g. gross domestic product) when they are converted into real indicators. They are also used for indexing pensions, rents, alimony, and other contractual payments. Furthermore, they are the reference for the monetary policy of central banks. Therefore, providing accurate and consistent price data is one of the most important objectives of National Statistical Offices (NSOs).

Chapter 1 develops different options for the retrospective computations of price index numbers. Most NSOs compute their Consumer Price Index (CPI) as Laspeyres type indices. Such indices use outdated weighting information. It is well known that outdated weighting information causes accumulating upward bias. This bias is usually denoted as substitution bias. The lable refers to the fact that Laspeyres type indices ignore that consumers attempt to adjust their consumption behaviour such that they get the best price-performance ratio. The greater the individual price changes, the larger is the substitution bias of the Laspeyres type indices. Therefore, the present study introduces and examines simple and transparent revision approaches that retrospectively address the source of the bias. They provide a consistent long-run time series of the CPI and they require no additional information. Furthermore, a coherent decomposition of the bias into the contributions of individual product groups is developed. In a case study with German price and expenditure data, the approaches are applied to a Laspeyres-based CPI. The empirical results confirm the theoretical predictions. The proposed revision approaches are not only adoptable to most national CPIs, but

also to other price level measures such as the producer price index or the import and export price indices.

Chapter 2 is dedicated to the measurement of import and export price indices. Such indices are complicated by the impact of exchange rates. Like the CPI, the import and export price indices of an economy are usually compiled by some Laspeyres type index. Therefore, substitution bias is an issue. Since the terms of trade of a country are defined as the ratio of the export and import price index, they too are likely to be distorted. The underlying substitution bias accumulates over time. Hence, the second paper introduces a simple and transparent retroactive correction approach that addresses the source of the substitution bias and produces meaningful long-run time series of import and export price levels and, therefore, of the terms of trade. In addition, an empirical case study based on German data is conducted to demonstrate the effectiveness and versatility of the corrective approach.

Chapter 3 leaves the field of index revision and studies another issue in price measurement, namely, the monetary evaluation of digital products that have zero market prices. Such products differ from products with positive market prices. The latter are constantly monitored by the NSOs. Prices and quantities consumed are evaluated and included in the calculation of price indices. In addition, their contribution to other economic indicators (e.g., gross domestic product) is calculated. Products with zero market price are largely ignored, even though they have economic value and their production requires economic resources. The present analysis argues that each consumer of a free digital product pays an individual implicit price (or shadow price). Thus, in theory, the value of free digital products can be derived from the consumers' implicit prices. In the literature, some methods for assessing the monetary value of free digital products has been discussed. The economic valuation of free digital products is usually conducted with the help of direct and/or indirect methods (e.g., contingent valuation, discrete choice experiments). As an alternative, the present study proposes a usage cost model (UCM) that is based on revealed preferences. The approach adapts the well known Travel Cost Model (TCM) to the case of free digital products. The analysis starts out by describing the theoretical model and the underlying assumptions of the UCM. The theoretical part is followed by an empirical case study that applies the UCM to various free digital products.

Extended Summary

National Statistical Offices (NSOs) are entrusted with the provision of an accurate measurement of the economies' price levels and price level changes. To this end, they collect price data and transform them into price index numbers. These index numbers are used for various purposes. On the macroeconomic level they serve as a reference for monetary policy and as a conversion mechanism for the transformation of nominal indicators of the economy into real indicators (e.g., gross domestic product). On the microeconomic level, they are used for the indexation of pensions, rents, social benefits, and other contractual payments. The goal of the NSOs is to provide consistent and reliable measures of price levels. Therefore, a lot of research has been devoted to this task.

This cumulative dissertation contributes to this research. It comprises three chapters each of which replicates a separate paper. The first two papers are concerned with the theory and application of retroactive revisions of price indices. Both papers are co-authored with Ludwig von Auer. The third paper studies the monetary evaluation of free digital products, that is, products with zero prices. The empirical applications for all three papers are conducted using the programming language R.

In price level measurement, a trade-off exists between timeliness and accuracy. One way to mitigate this trade-off is to publish preliminary index numbers without much delay and, at a later point of time, to complement these preliminary numbers by a retrospectively revised price index number that uses more reliable data. A comparison between the preliminary index numbers and the revised index numbers helps to evaluate the reliability of the preliminary index numbers.

Most NSOs compute their Consumer Price Index (CPI) as Laspeyres type indices. To fulfill the requirement of timeliness, such indices use outdated weighting information. It is well known that outdated weighting information causes accumulating upward bias. This bias is usually denoted as substitution bias (ILO et al., 2004, p. 4). The label refers to the fact that Laspeyres type indices ignore that consumers attempt to adjust their consumption behaviour such that they get the best price-performance ratio. The greater the individual price changes, the larger is the substitution bias of the Laspeyres type indices. Various empirical studies compute retrospectively revised price index numbers. They confirm that substitution bias is still a problem in official price measurement; e.g., Hansen (2007), de Haan et al. (2010), Greenlees and Williams (2010), Huang et al. (2017), Klick (2018), and Herzberg et al. (2021).

Chapter 1 consolidates former proposals for computing retrospectively revised index numbers (de Haan et al., 2010) into two basic approaches and introduces various alternative options for each of these approaches. The two approaches are denoted as *correction approach* and *imputation approach*. Both of them provide a consistent long-run time series of the CPI and they require no additional information. Furthermore, a coherent decomposition of the bias into the contributions of individual product groups is developed. In a case study with German price and expenditure data, the approaches are applied to a Laspeyres-based CPI.

The empirical results confirm the theoretical predictions. Without any retrospective revision, the long-run upward substitution bias during the time interval January 2000 and December 2014 would have amounted to almost 0.22 percentage points per year. The official retrospective revisions of the German CPI did not eliminate the yearly bias, but reduced it to 0.087 percentage points. This is a conservative estimate, since it does not include the upward bias attributable to the lower level of aggregation. It is also shown that half of the bias in the German CPI can be attributed to a single group of items even though the expenditure share of that group is below 12 percent.

Theoretically, the proposed revision approaches are not only adoptable to most national CPIs, but also to other price level measures such as the producer price index or the import and export price indices. However, each of these measures has its own specific issues. Therefore, the adoption of the revision approach to these indices requires additional considerations.

Chapter 2 is dedicated to the retrospective revision measurement of import and export price indices. The compilation of import and export price indices is complicated by the impact of exchange rates. The "Export and Import Price Index Manual" published by the IMF (2009) provides recommendations for the compilation of import and export price indices. Like the CPI, the import and export price indices of an economy are usually compiled by some Laspeyres type index. Therefore, substitution bias is again an issue (e.g., Dridi and Zieschang, 2004, p. 169; IMF, 2009, pp. 413-439). Since the terms of trade of a country are defined as the ratio of the export and import price index, they too are likely to be distorted. The underlying substitution bias accumulates over time. Hence, Chapter 2 introduces a simple and transparent retroactive correction approach that addresses the source of the substitution bias and produces meaningful long-run time series of import and export price levels and, therefore, of the terms of trade. In addition, an empirical case study based on German data is conducted to demonstrate the effectiveness and versatility of the corrective approach.

For the time interval January 1995 to May 2019 the accumulated upward substitution bias of the import price index is more than five percent, while the upward bias of the export price index is slightly above one percent. Therefore, the upward bias in the terms of trade is roughly four percent. The empirical case study verifies that the accumulating substitution bias can be easily avoided by a three-stage retroactive revision process. This process requires no additional data and is adaptable to the specific index compilation procedures of the various national statistical offices.

Chapter 3 leaves the field of index revision and studies another issue in price measurement, namely, the monetary evaluation of digital products that have zero market prices. Such products differ from products with positive market prices. The latter are constantly monitored by the NSOs. Prices and quantities consumed are evaluated and included in the calculation of price indices. In addition, their contribution to other economic indicators (e.g., gross domestic product) is calculated. By contrast, products with zero market price are largely ignored, even though they have economic value and their production requires economic resources. The present analysis argues that each consumer of a free digital product pays an individual implicit price (or shadow price). Thus, in theory, the value of free digital products can be derived from the consumers' implicit prices.

In the literature, various methods for assessing the monetary value of free digital products have been discussed. They include direct and indirect methods (e.g. contingent valuation mathods, discrete choice experiments, experimental auctions of accessing WTA/WTP category etc.). Studies that apply these methods to the case of digital products include Brynjolfsson (1996), Rappoport et al. (2003), Goolsbee and Petrin (2004), Zhang (2016), and Byrne et al. (2018). However, these studies focus on digital products that are not entirely costless. Recently, more attention has been paid to free digital products and the measurement of their invisible "shadow" price. Contributions to this field include Post (2009), Brynjolfsson and Oh (2012), Brynjolfsson et al. (2019b), Herzog (2018), Mosquera et al. (2019), and Corrigan et al. (2018).

The present study introduces a new approach that is based on revealed preferences. It proposes the "usage cost model" (UCM). The approach adapts the well known travel cost model (Hotelling, H., 1947, Smith, 1989) to the case of free digital products. The analysis starts out by describing the theoretical model and the underlying assumptions of the UCM. The theoretical part is followed by an empirical case study that applies the UCM to the three social media platforms Facebook, Instagram, and WhatsApp.

Only parts of the empirical results are plausible. Two alternative conclusions can be drawn from this observation. Possibly, the UCM does not represent the expected improvement over the existing methods for measuring the shadow price of free digital goods. Alternatively, the size and structure of the sample was unsuitable for this method. Applying the UCM to a much broader sample may provide more plausible results.

1 Retrospective Computations of Price Index Numbers: Theory and Application[1]

1.1 Introduction

Various important fields of economic analysis rely on indicators such as real gross domestic product, real wages, real interest rates, and real public debt. To obtain a long-run time series of such an indicator, the time series of the corresponding nominal indicator is deflated by some appropriate price level measure. National statistical offices (NSOs) are entrusted to provide these price level measures in an accurate and timely manner.

The most important national price level measure is the Consumer Price Index (CPI). In their CPI compilations, NSOs relate the *comparison period*'s (or current period's) price level of final consumption to the price level of some former *price reference period* (or base period). In the applied price index formulas, the weighting of the various consumption items reflects the consumer expenditures on these items. Most NSOs apply an index derived from the Laspeyres index; in other words, a "Laspeyres type index". Usually this is a Lowe index (e.g., Australia, Canada, Switzerland, U.S.) or a Young index (e.g., Denmark, Georgia, South Africa). Only few NSOs use a proper Laspeyres index (e.g., Germany, Japan). While the weighting of a *proper* Laspeyres index exclusively depends on expenditure information of the price reference period, the weighting of a Laspeyres *type* index may draw on expenditure information from periods other than (and usually preceding) the price reference period.

Unfortunately, the Laspeyres index as well as Laspeyres type indices are known to suffer from upward substitution bias because in their item weights they fail to incorporate the substitution behavior reflected in the consumed quantities of the comparison period (e.g., ILO et al., 2004, p. 4). Among the more recent studies that provide empirical evidence of this bias are Hansen (2007), de Haan et al. (2010), Greenlees and Williams (2010), Huang et al. (2017), Klick (2018), and Herzberg et al. (2021). The more outdated the expenditure information – and, consequently, the applied item

[1] Helpful comments from Olivia Ståhl, Can Tongur, and Sebastian Weinand are gratefully acknowledged. The present paper was published as a research paper in the *Research Papers in Economics, Universität Trier*: See please von Auer and Shumskikh (2022a).

weights – the larger the risk of substitution bias and the associated accumulating upward bias in the long-run time series of monthly price levels. Such findings are worrying for institutions and analysts that rely on an unbiased long-run time series of the CPI because upward biased CPI numbers would, for example, result in downward biased real growth rates.

To reduce the risk of biased CPI numbers, many NSOs attempt to shorten the time interval between the comparison period and the period to which the item weights relate. Another strategy is to wait until sufficiently up-to-date information on expenditures and, thus, on weighting is available and then to publish revised index numbers. However, in many countries such revisions raise legal issues. Furthermore, NSOs may worry that revisions undermine the credibility of their published numbers. Therefore, many NSOs are reluctant to retrospectively revise their initial results.

Generally, it is acknowledged that retrospectively revised numbers tend to be more accurate than original ones. Therefore, revisions are extremely valuable for scientific purposes and, thus, for designing sound economic and social policies. More NSOs might be willing to consider the provision of such revised numbers if the extra work load appears manageable. Therefore, the compilation of the revised numbers should require only information that the NSOs collect and process anyway.

The first contribution of the present paper is to present and elaborate two general approaches of such work-saving retrospective revision methods. The two approaches are denoted as the "correction approach" and the "imputation approach". For the implementation of both approaches a wide range of options is available. Therefore, they are general enough to be applied by most NSOs. The approaches effectively address the source of the accumulating substitution bias and they generate a consistent long-run time series of monthly price levels. The revised series also indicates the upward substitution bias inherent in the non-revised series.

In addition, it is shown that the imputation approach can decompose the upward substitution bias into the contributions of individual groups of items. This decomposition method is the paper's second contribution. In the future, this method may help to mitigate the bias already from the outset.

The paper's third contribution is empirical. To illustrate the relevance of the problem and the efficacy of the proposed revision approach, it is applied to the upper level aggregation of the German CPI. Even though the study relates to the CPI, its insights carry over to other price level measures such as the producer price index or the import and export price indices.

The rest of the paper is organized as follows. Section 1.2 recaptures the relevant concepts from index theory. The correction approach and the imputation approach are introduced in Section 1.3. In Section 1.4, an empirical application of the new revision concept is presented. The decomposition of the upward bias into the contributions of individual groups of items is developed in Section 1.5 and applied in Section 1.6. Section 1.7 concludes.

1.2 Four Types of Symmetric Price Indices

Let p_i^t and x_i^t denote the price and the quantity of item $i \in A = [1,\ldots,N]$ at time period $t \in [0,\ldots,T]$. The expenditures on item i are denoted by $v_i^t = p_i^t x_i^t$. The symbols $\sum$ and $\prod$ are the shorthand notation for the sum and the product over all N items in set A. Only for periods 0 and T the expenditure shares of the N items are known:

$$s_i^0 = \frac{v_i^0}{\sum v_j^0} \qquad \text{and} \qquad s_i^T = \frac{v_i^T}{\sum v_j^T}\,.$$

For each comparison period $t \in [0,\ldots,T]$, a Laspeyres index can be compiled that compares the average price level of the comparison period t to the average price level of the price reference period 0. The resulting sequence of Laspeyres indices, $P_{\mathrm{L}}^{0\to t}$, is

$$P_{\mathrm{L}}^{0\to 0} = \sum s_i^0 \frac{p_i^0}{p_i^0} = 1, \quad P_{\mathrm{L}}^{0\to 1} = \sum s_i^0 \frac{p_i^1}{p_i^0}, \quad \ldots \quad , \quad P_{\mathrm{L}}^{0\to T} = \sum s_i^0 \frac{p_i^T}{p_i^0}\,. \tag{1}$$

In this study, the period that provides the information for the calculation of the expenditure weights is denoted as the *expenditure reference period*. In the sequence of Laspeyres indices (1), period 0 is both, the expenditure reference period and the price reference period.[2]

[2]The expenditure reference period should not be confused with the *weight reference period* as defined in ILO et al. (2004, p. 3). The latter is the period whose *quantities* are actually used in the index. In a Laspeyres index of the form (1) the weight reference period and the expenditure reference period coincide. Another type of reference period is the *index reference period*. This is the period for which the index is set equal to 100.

Once the expenditure shares of the new expenditure reference period T become available, a retrospective price index can be compiled for each comparison period $t \in [0,\ldots,T]$. Possible candidates for such retrospective price indices are appropriately modified versions of so-called *symmetric* standard price index formulas. ILO et al. (2004, pp. 5-6) attach the label "symmetric" to price index formulas that give equal importance to the information of the first expenditure reference period 0 and the second expenditure reference period T. However, the phrase "equal importance" turns out to be rather ambiguous because it can come in different forms. Therefore, we propose to distinguish between four types of symmetry:

A: Symmetric Treatment of Quantities

The quantities, x_i, represent some combination of $x_i^0 = v_i^0/p_i^0$ and $x_i^T = v_i^T/p_i^T$. Examples are

$$\begin{aligned}
\text{Marshall-Edgeworth: } P_{\text{ME}}^{0 \to T} &= \frac{\sum p_i^T (x_i^0 + x_i^T)}{\sum p_i^0 (x_i^0 + x_i^T)} \\
\text{Walsh: } P_{\text{W}}^{0 \to T} &= \frac{\sum p_i^T \sqrt{x_i^0 x_i^T}}{\sum p_i^0 \sqrt{x_i^0 x_i^T}} \,.
\end{aligned} \tag{2}$$

B: Symmetric Treatment of Expenditures

The expenditures, v_i, represent some combination of v_i^0 and v_i^T. Well known examples are

$$\begin{aligned}
\text{Walsh-2: } \ln P_{\text{W2}}^{0 \to T} &= \sum \frac{\sqrt{v_i^0 v_i^T}}{\sum \sqrt{v_j^0 v_j^T}} \ln \frac{p_i^T}{p_i^0} \\
\text{Theil: } \ln P_{\text{Th}}^{0 \to T} &= \sum \left[\frac{\sqrt[3]{\frac{1}{2}\left(v_i^0 + v_i^T\right) v_i^0 v_i^T}}{\sum \sqrt[3]{\frac{1}{2}\left(v_j^0 + v_j^T\right) v_j^0 v_j^T}} \right] \ln \frac{p_i^T}{p_i^0} \\
\text{Vartia: } \ln P_{\text{Va}}^{0 \to T} &= \sum \frac{L\left(v_i^0, v_i^T\right)}{L\left(\sum v_j^0, \sum v_j^T\right)} \ln \frac{p_i^T}{p_i^0} \,, \\
&\text{where } L(a,b) = \frac{a-b}{\ln a - \ln b} \text{ for } a \neq b \text{ and } L(a,b) = a \text{ for } a = b.
\end{aligned}$$

C: Symmetric Treatment of Expenditure Shares

The expenditure shares, s_i, represent some combination of s_i^0 and s_i^T. This type of symmetry is represented by

$$\begin{aligned}
\text{Törnqvist}: \ \ln P_{\text{Tö}}^{0\to T} &= \sum \frac{1}{2}\left(s_i^0 + s_i^T\right) \ln \frac{p_i^T}{p_i^0} \\
\text{Walsh-Vartia}: \ \ln P_{\text{WV}}^{0\to T} &= \sum \sqrt{s_i^0 s_i^T} \ln \frac{p_i^T}{p_i^0} \\
\text{Sato-Vartia}: \ \ln P_{\text{SV}}^{0\to T} &= \sum \frac{L\left(s_i^0, s_i^T\right)}{\sum L\left(s_j^0, s_j^T\right)} \ln \frac{p_i^T}{p_i^0} \,.
\end{aligned} \tag{3}$$

D: Symmetric Treatment of Indices

The indices, $P^{0\to T}$, represent some combination of an index with expenditure reference period 0 and some index with expenditure reference period T. For this purpose, the Laspeyres index, $P_L^{0\to T}$, and the Paasche index,

$$P_{\text{P}}^{0\to T} = \frac{\sum p_i^T x_i^T}{\sum p_i^0 x_i^T} = \left[\sum s_i^T \left(\frac{p_i^T}{p_i^0}\right)^{-1}\right]^{-1} ,$$

are particularly popular. They are used in price index formulas such as

$$\begin{aligned}
\text{Fisher}: \ P_{\text{F}}^{0\to T} &= \sqrt{P_{\text{L}}^{0\to T} P_{\text{P}}^{0\to T}} \\
\text{Drobisch}: \ P_{\text{D}}^{0\to T} &= \frac{1}{2}\left(P_{\text{L}}^{0\to T} + P_{\text{P}}^{0\to T}\right) .
\end{aligned} \tag{4}$$

Some price index formulas can be assigned to more than one type of symmetry. For example, the Walsh index can also be written in the forms

$$P_{\text{W}}^{0\to T} = \sum \frac{\sqrt{v_i^0 v_i^T \left(p_i^T/p_i^0\right)^{-1}}}{\sum \sqrt{v_j^0 v_j^T \left(p_j^T/p_j^0\right)^{-1}}} \frac{p_i^T}{p_i^0} \tag{5}$$

$$= \sum \frac{\sqrt{s_i^0 s_i^T \left(p_i^T/p_i^0\right)^{-1}}}{\sum \sqrt{s_j^0 s_j^T \left(p_j^T/p_j^0\right)^{-1}}} \frac{p_i^T}{p_i^0} \,. \tag{6}$$

This is a weighted arithmetic mean of the price ratios (p_i^T/p_i^0) where the weights depend not only on the expenditures (or expenditure shares) of the price reference period, but also on the expenditures (or expenditure shares) of the comparison period T deflated to the price reference period. Therefore, the Walsh index can be interpreted as a representative of symmetry types A, B, and C.

The listed symmetric price indices compare the prices of the comparison period T to those of the price reference period 0. In ILO et al. (2004, p. 173), the retrospective compilation of a price index such as (2), (3), or (4) is advocated. However, we need a price index formula that can relate not only period T but also all intermediate comparison periods $t \in [1,\ldots,T-1]$ to the price reference period 0. What can be achieved if neither the quantities, nor the expenditures, nor the expenditure shares of these intermediate comparison periods are known? The objective is a price index formula that provides a more reliable series of retrospective index numbers than the Laspeyres-based series (1).

1.3 Retrospectively Computed Price Indices

To construct a reliable series of retrospective price index numbers we recommend to use some symmetric price index formula. Unfortunately, the quantity and expenditure information of the intermediate periods $(t = 1,\ldots,T-1)$ is missing. Therefore, we must make use of the available quantity or expenditure information of the two expenditure reference periods, $t = 0$ and $t = T$, and we must invoke some form of interpolation for the intermediate periods.

The construction of a retrospective price index formula can follow at least two alternative principles. The index can be defined as

- the product of the original Laspeyres index (or Laspeyres type index) and some correction factor that accounts for the gradually increasing deviation between the Laspeyres index and a symmetric price index as t progresses from 0 to T, or as

- a symmetric price index formula that imputes the missing information on x_i^t, v_i^t, or s_i^t $(t = 1,\ldots,T-1)$ where the imputeted values are weighted averages of the values of periods 0 and T, and where the weight of the value of period 0 gradually decreases from 1 to 0 as t progresses from 0 to T.

We denote the two principles as *correction approach* and *imputation approach*. In both approaches, the variable $\lambda_t = t/T$ plays a crucial role. As time progresses, λ_t increases from $\lambda_0 = 0$ to $\lambda_T = 1$ and reflects in the retrospective price index formula the gradually diminishing relevance of the first expenditure reference period ($t = 0$) and the increasing relevance of the second expenditure reference period ($t = T$). The two approaches can be implemented in various ways.

Correction Approach: Using the Walsh index (2), the series of Laspeyres indices (1) can be revised by the following retrospective price index formula:

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right) \cdot \left(\frac{P_{\mathrm{W}}^{0\to \mathrm{T}}}{P_{\mathrm{L}}^{0\to \mathrm{T}}}\right)^{\lambda_t}, \qquad t = 0\ldots,T\,. \tag{7}$$

For $t = 0$, we have $\lambda_0 = 0$. Therefore, the correction factor $(P_{\mathrm{W}}^{0\to\mathrm{T}}/P_{\mathrm{L}}^{0\to\mathrm{T}})^{\lambda_t}$ is equal to 1 (that is, no correction), and the retrospective price index formula gives $P^{0\to 0} = P_{\mathrm{L}}^{0\to 0} = 1$. As time progresses towards period $t = T$, λ_t increases from 0 to 1, the correction factor reaches its maximum deviation from 1, and the retrospective price index gradually turns into the Walsh index: $P^{0\to T} = P_{\mathrm{W}}^{0\to\mathrm{T}}$.

Instead of the Walsh index, other symmetric price index formulas could be used to define the correction factor. For example, von Auer and Shumskikh (2022b) apply the Törnqvist index (3). Using the Fisher index (4), gives

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right)\left(\frac{P_{\mathrm{P}}^{0\to \mathrm{T}}}{P_{\mathrm{L}}^{0\to \mathrm{T}}}\right)^{\lambda_t/2}, \qquad t = 0\ldots,T\,, \tag{8}$$

and, therefore, $P^{0\to 0} = 1$ and $P^{0\to T} = P_{\mathrm{F}}^{0\to\mathrm{T}}$. The same two values are generated by a retrospective price index formula that is proposed in de Haan et al. (2010). They replace in the correction factor of formula (8) the Laspeyres index, $P_{\mathrm{L}}^{0\to\mathrm{T}}$, by the chained index $P_{\mathrm{L}}^{0\to\mathrm{t}}P_{\mathrm{P}}^{\mathrm{t}\to\mathrm{T}}$.

Instead of the multiplicative correction of the Laspeyres index, $P_{\mathrm{L}}^{0\to t}$, applied in (7), one could use an additive one:

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right) + \lambda_t\left(P_{\mathrm{W}}^{0\to \mathrm{T}} - P_{\mathrm{L}}^{0\to \mathrm{T}}\right), \qquad t = 0\ldots,T\,. \tag{9}$$

Again, one obtains $P^{0 \to 0} = 1$ and $P^{0 \to T} = P_{\mathrm{W}}^{0 \to \mathrm{T}}$.

Imputation Approach: When the Walsh index (2) is applied, the unknown quantities, x_i^t ($t = 1, \ldots, T-1$), must be substituted by imputed values, $\hat{x}_i^t$. If we assume that the rate of change of x_i^t is constant between periods 0 and T, the imputed values are given by the weighted geometric average

$$\hat{x}_i^t = \left(x_i^0\right)^{1-\lambda_t} \left(x_i^T\right)^{\lambda_t} , \tag{10}$$

and we obtain $P^{0 \to 0} = 1$ and $P^{0 \to T} = P_{\mathrm{W}}^{0 \to \mathrm{T}}$. Alternatively, other symmetric price indices can be used. Furthermore, the imputed values, $\hat{x}_i^t$, can be obtained from a weighted arithmetic average:

$$\hat{x}_i^t = (1 - \lambda_t) x_i^0 + \lambda_t x_i^T . \tag{11}$$

The same options are available, if the missing values of the expenditures, v_i^t, or the expenditure shares, s_i^t, have to be imputed.

An early application of the imputation approach can be found in de Haan et al. (2010). They propose to use the Törnqvist index (3) or the Fisher index (4) and to impute the unknown expenditure shares, s_i^t ($t = 1, \ldots, T-1$), by $\hat{s}_i^t = (1 - \lambda_t) s_i^0 + \lambda_t s_i^T$.

In this section we have presented two different construction principles for a retrospective price index, the correction approach and the imputation approach. For both approaches, a large set of symmetric price indices is available. In the correction approach, a multiplicative or an additive form of correction can be applied. In the imputation approach, the imputed values can be obtained from weighted arithmetic or geometric averages. Since $\lambda_T = 1$, the index number relating to the comparison period $t = T$ (the second expenditure reference period) is independent from the choice between the correction and the imputation approach.

Both, the correction approach and the imputation approach have intuitive appeal. The correction approach appears somewhat less intrusive than the imputation approach. Therefore, in the following section, we demonstrate how the correction approach can be implemented in the German CPI. On the other hand, the imputation approach allows for an additive decomposition of the overall price change. This property is useful in various ways. For example, it helps to identify those groups of items that cause the upward substitution bias of the Laspeyres index. This identification is a prerequisite for reducing the bias already from the outset. Therefore, in Section 1.5 the imputation approach is applied to identify the critical items in the German CPI.

1.4 Retrospective Correction of the German CPI

Destatis (shortened form of "Statistisches Bundesamt") compiles proper Laspeyres indices. The most recent price and expenditure reference period of Destatis is the year 2015. The previous ones were the years 2010, 2005, and so on. For example, the Laspeyres index of the comparison period January 2000 was published in February 2000 and used the year 1995 as price and expenditure reference period. The same price and expenditure reference period was used for the subsequent months.

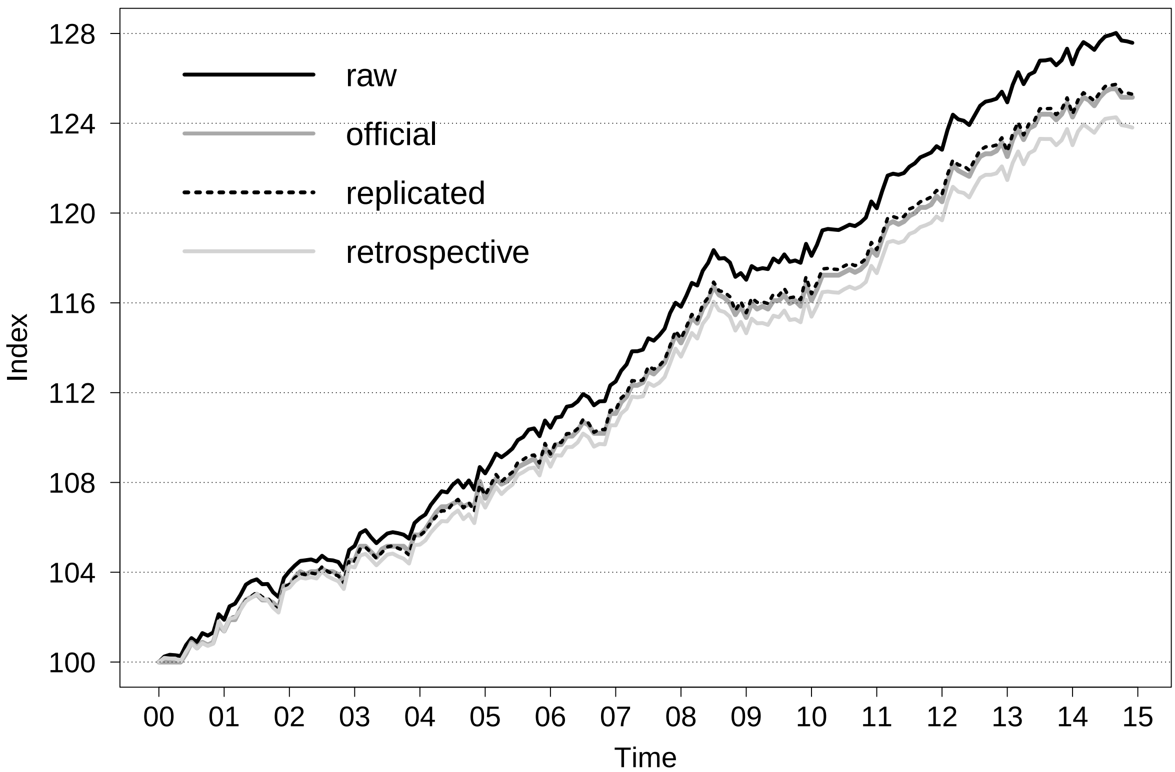

Figure 1: German consumer price index (January 2000 = 100) for January 2000 to December 2014.

When in 2003 the expenditures of the year 2000 became available, Destatis recalculated the index numbers of the comparison periods starting in January 2000. The revised Laspeyres indices use the year 2000 as the price and expenditure reference period. These index calculations were continued until in 2008 the expenditure weights of the year 2005 became available. This new information prompted a retrospective revision of the Laspeyres index numbers starting in January 2005. Analogous retrospective revisions occurred in 2013 when the expenditures of the year 2010 became available.

In total, these compilations generate a sequence of three consecutive time series of monthly Laspeyres indices, each covering a five-year period. These three series are chained, such that a consistent time series starting in January 2000 is obtained. The details of the official CPI compilation procedure are documented in Appendix A. The official source is Statistisches Bundesamt (2018, pp. 6-7, 15-19).[3] The official time series of monthly price levels is depicted in Figure 1 (dark gray line labeled as "official").

We cannot use the official time series as a benchmark because we do not have access to the same data set. Using the data set available to us, we replicate the compilation procedure of Destatis. The resulting time series will serve as a benchmark for the time series derived from the retrospective index formula. The benchmark time series is depicted in Figure 1 by the black dotted line labeled as "replicated". The differences between the officially published Laspeyres CPI index and our replicated Laspeyres index are hardly visible. In the final comparison period (December 2014), the deviation between the official and the replicated index number are merely 0.14 percentage points. This is a remarkable result because the information accessible for our research purposes is not as granular as the information processed by Destatis. In the accessible data set, the consumption basket is decomposed into 102 classes of products (third level of Classification of Individual Consumption according to Purpose, COICOP). The original decomposition of Destatis is much finer and some of the original classes are missing in the accessible data set.

Now, if Destatis had ruled out retrospective revisions, in 2003 it would have continued with the old price and expenditure reference period (1995) until January 2005, the month scheduled for the introduction of regular methodological modifications and other updates of the index. Thus, this would have been the time to switch from the old

[3]Additional descriptions are provided by Egner (2003), Elbel and Egner (2008), Egner (2013), and Egner (2019).

(1995) to the new (2000) price and expenditure reference period. Subsequently, an analogous changeover would have been conducted in January 2010.

Based on the data set available to us, we simulate this non-revisionary compilation procedure. The details are documented in Appendix A. In Figure 1, the black line labeled as “raw” depicts the resulting time series of unrevised monthly price levels. It covers the time interval January 2000 to December 2014 (the index numbers of earlier months would require the expenditure information of the year 1990 which we do not have). This time series is obtained from Laspeyres index numbers based on weights that are outdated by five to ten years. As a result, it is prone to substantial upward substitution bias.

The comparison of the two time series “raw” and “replicated” reveals that the retrospective revisions of Destatis successfully curbed the long-run upward substitution bias. Did these revisions even eliminate the bias? To answer this question, a time series of index numbers is required that can be considered as free of long-run substitution bias.

Therefore, we compile a time series of retrospective Walsh indices and compare it to the time series of replicated Laspeyres indices. The Walsh-based series applies the correction approach. The comparison between the Laspeyres- and Walsh-based series gives us clues about the existence and the extent of the remaining distortion in the long-run time series of the official CPI. The retrospective Walsh index numbers of the time interval 2015 to 2020 would require the expenditure weights of the year 2020. These will become available not before 2023. Therefore, we decided to restrict the analysis to the interval January 2000 to December 2014. The formal details of the compilation procedure of the retrospective Walsh index are explained in Appendix B.

The result of this procedure is the light gray line in Figure 1 labeled as “retrospective”. The graph confirms the theoretical predictions. The retrospective Walsh index runs below the replicated Laspeyres index, indicating that the official index still suffers from upward substitution bias. The deviation accumulates over time and in December 2014 it reaches almost 1.5 percentage points. This deviation represents only the distortion that can be attributed to the upper level aggregation, that is, the aggregation of the 102 classes into the overall CPI. Since the subclasses at the lower level aggregation exhibit a larger substitutability than the classes at the upper level aggregation, it is quite likely that the actual upward bias is larger than the 1.5 percentage points observed here.

1.5 Decomposition of the Bias

It would be valuable to know which groups of items are responsible for the upward substitution bias of the Laspeyres index. Usually the CPI is formed by major expenditure categories where each category comprises several items. More formally, the set of items, A, can be partitioned into the subsets A_k where $k \in K$ and K is the set of (major expenditure) categories.

We know from formulas (1) and (6) that both, the Laspeyres index and the Walsh index can be expressed in the additive form

$$P = \sum_{i \in A} z_i \frac{p_i^T}{p_i^0} , \tag{12}$$

where p_i^T / p_i^0 is the price ratio of item i and, therefore, the *primary attribute* of the index and z_i is the *secondary attribute*. It can be interpreted as the weight of the primary attribute. In this section, we drop the superscript "0 → T" at the index P. Since $\sum_{i \in A} = \sum_{k \in K} \sum_{i \in A_k}$, index formula (12) can also be written in the form

$$P = \sum_{k \in K} Z_k P_k , \tag{13}$$

where $P_k = \sum_{i \in A_k} (z_i / Z_k)(p_i^T / (p_i^0)$ is the price index computed for category k and $Z_k = \sum_{i \in A_k} z_i$ is the weight of category k, with

$$z_i = z_{i,\mathrm{L}} = s_i^0 \qquad \text{(Laspeyres index)} \tag{14}$$

$$z_i = z_{i,\mathrm{W}} = \frac{\sqrt{s_i^0 s_i^T \left(p_i^T / p_i^0\right)^{-1}}}{\sum \sqrt{s_j^0 s_j^T \left(p_j^T / p_j^0\right)^{-1}}} \qquad \text{(Walsh index)} . \tag{15}$$

Equations (12) and (13) imply that the same index number is obtained, regardless of whether the index is computed in a single stage over all items in set A or in two stages, where, on the first stage, for each subset A_k the price index P_k and the aggregated weight Z_k are computed and, on the second stage, these values are used to compute the overall result P. In that second stage, the same index formula is used as in the first stage and the single stage computation. Z_k and P_k simply replace z_i and p_i^t/p_i^0, respectively. Therefore, von Auer and Wengenroth (2021) denote the Laspeyres and the Walsh index as *consistent in aggregation with respect to the secondary attributes* (14) and (15), respectively. The authors also show that all other indices listed in Section 1.2 of the present study are consistent in aggregation with respect to some appropriately defined secondary attribute (e.g., Törnqvist index, Fisher index).

Usually, the property of consistency in aggregation would be used to decompose the overall price change into the contributions of the individual categories. However, this property also allows for the decomposition of the deviation between two different index formulas into the contributions of the individual categories. Since we interpret the deviation between the Laspeyres index and the Walsh index as the upward substitution bias of the Laspeyres index, we obtain a decomposition of that bias.

The categories' Z_k-values can be considered as the weight of the category in the overall index computation. Definitions (14) and (15) imply that the notation must distinguish between the Z_k-values of the Laspeyres index, $Z_{k,\mathrm{L}} = \sum_{i \in A_k} z_{i,\mathrm{L}}$, and the Z_k-values of the Walsh index, $Z_{k,\mathrm{W}} = \sum_{i \in A_k} z_{i,\mathrm{W}}$. Noting that $\sum_{k \in K} Z_{k,\mathrm{L}} = \sum_{k \in K} Z_{k,\mathrm{W}} = 1$ and that the Laspeyres index and Walsh index are consistent in aggregation, we propose the following decomposition of the overall bias:

$$P_{\mathrm{L}} - P_{\mathrm{W}} = \sum_{k \in K} \left[Z_{k,\mathrm{L}} \left(P_{k,\mathrm{L}} - 1 \right) - Z_{k,\mathrm{W}} \left(P_{k,\mathrm{W}} - 1 \right) \right] , \tag{16}$$

where

$$P_{k,\mathrm{L}} = \sum_{i \in A_k} z_{i,\mathrm{L}} \frac{p_i^T}{p_i^0} \qquad \text{and} \qquad P_{k,\mathrm{W}} = \sum_{i \in A_k} z_{i,\mathrm{W}} \frac{p_i^T}{p_i^0} .$$

1.6 Decomposition of the Bias in the German CPI

The German CPI is formed by twelve major expenditure categories, $K = [1, \ldots, 12]$. The names of these twelve categories are listed in the first column of Table 1. The decomposition (16) allows us to examine whether the bias in the Laspeyres index is concentrated in only a few of these categories. We start with the comparison period December 2014 ($t = 12/14$) and the price reference period January 2010 ($t = 1/10$). In the notation, we drop the superscript "$1/10 \to 12/14$" at the price index. For the retrospective computation of the Walsh index, the imputation approach is applied. The computational details are described in Appendix C. The same type of analysis is repeated for the time intervals January 2000 to December 2004 as well as January 2005 to December 2009.

Table 1: Laspeyres index numbers and Walsh index numbers for December 2014 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2010 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

Expenditure Category	Laspeyres $P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	Walsh $P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	Contrib. to Bias
Food, non-alcoholic beverages	111.87	10.45	111.74	10.09	0.056
Alcoholic beverages, tobacco	111.78	3.81	111.70	3.78	0.007
Clothing, footwear	109.89	4.49	109.84	4.51	0.000
Housing, water, electricity, gas, other fuels	109.04	32.28	108.88	31.92	0.085
Furniture, other household equipment	102.83	5.06	102.80	5.24	−0.003
Health	102.58	4.52	101.61	4.80	0.039
Transport	105.91	13.71	105.78	13.65	0.021
Communication	90.67	3.06	90.04	3.19	0.032
Recreation, entertainment, culture	110.88	11.69	109.36	11.61	0.186
Education	70.29	0.40	70.29	0.37	−0.008
Restaurant, accommodation services	109.62	4.54	109.63	4.66	−0.012
Miscellaneous goods and services	105.42	5.97	105.13	6.17	0.007
Total (Jan. 2010 - Dec. 2014)	107.75	100.00	107.34	100.00	0.410

The bottom line of Table 1 shows the all-items index numbers for the price reference period January 2010 and the comparison period December 2014. The all-items Laspeyres index (1) gives the number $P_{\mathrm{L}} = 107.75$, whereas the retrospectively computed Walsh index yields $P_{\mathrm{W}} = 107.34$. Thus, the upward bias of the all-items Laspeyres index is 0.410. The index numbers of each category are also listed in Table 1. The last column decomposes the all-items bias (0.410) into the contributions of the twelve individual categories. To this end, we use the decomposition approach presented in formula (16), where

$$P_{k,\mathrm{L}} = \sum_{i \in A} s_i^{10} \frac{p_i^{12/14}}{p_i^{1/10}} \qquad \text{and} \qquad P_{k,\mathrm{W}} = \sum_{i \in A} \frac{\sqrt{s_i^{10} s_i^{15} \left(p_i^{15}/p_i^{10}\right)^{-1}}}{\sum \sqrt{s_j^{10} s_j^{15} \left(p_j^{15}/p_j^{10}\right)^{-1}}} \frac{p_i^{12/14}}{p_i^{1/10}}$$

$$Z_{k,\mathrm{L}} = \sum_{i \in A_k} s_i^{10} \qquad \text{and} \qquad Z_{k,\mathrm{W}} = \sum_{i \in A_k} \frac{\sqrt{s_i^{10} s_i^{15} \left(p_i^{15}/p_i^{10}\right)^{-1}}}{\sum \sqrt{s_j^{10} s_j^{15} \left(p_j^{15}/p_j^{10}\right)^{-1}}}\,.$$

For example, the contribution of the category "Education" is negative (−0.008), even though the price indices of that category are identical ($P_{10,\mathrm{L}} = P_{10,\mathrm{W}} = 70.29$). The negative contribution says that the category "Education" *reduces* the upward bias inherent in the overall Laspeyres index. The cause of that reduction is the difference in the weights. In the Laspeyres index, the price decline receives the weight $Z_{10,\mathrm{L}} = 0.40$, while in the Walsh index the weight is only $Z_{10,\text{W-im}} = 0.37$. The category "Education" reveals that a comparison of the Laspeyres index and Walsh index of some given category is not sufficient to evaluate that category's contribution to the overall bias. Also the weights must be considered. Besides "Education", two other categories mitigate the bias. The strongest positive contribution to the bias comes from the category "Recreation, entertainment and culture".

The decomposition of the bias can be conducted also for the intervals January 2000 to December 2004 as well as January 2005 to December 2009. The results are documented in Appendix D. The overall bias for January 2000 to December 2004 and for January 2005 to December 2009 are 0.683 and 0.573, respectively. Looking over all three time intervals, one can say that roughly half of the bias can be attributed to the category "Recreation, entertainment, culture". For the compilers of the German CPI, this is a valuable insight. It may prompt a closer inspection of the computational procedures applied in this category. Even though the category "Housing, water, electricity, gas, other fuels" has by far the highest weight, its contribution to the bias is small. The contributions of the categories "Clothing, footwear", "Education", "Restaurant, accomodation services", and "Miscellaneous goods and services" are negligible.

1.7 Concluding Remarks

The expenditure information for the weighting of a national Consumer Price Index (CPI) is usually outdated by 13 to 60 months. The longer this implementation lag, the larger the risk of upward substitution bias in the CPI. The national statistical offices (NSOs) have addressed this issue in different ways. To keep the implementation lag short, some NSOs collect and process the expenditure information every year. Some NSOs allow for retrospective revisions of their headline CPI or some supplementary CPI. However, even such revisions may not completely solve the problem.

The German CPI is a case in point. Without any retrospective revision, the long-run upward substitution bias during the time interval January 2000 and December 2014 would have amounted to almost 0.22 percentage points per year. The official retrospective revisions of the German CPI reduced that yearly bias to 0.09 percentage points. This is a conservative estimate, since it does not include the upward bias attributable to the lower level of aggregation.

This study has introduced a procedure that can effectively address the source of the bias without requiring any additional information. This procedure replaces the Laspeyres or Laspeyres type index (typically a Lowe or a Young index) by a retrospective price index. Such a retrospective price index can be computed in different ways. Here we discussed the correction approach and the imputation approach and presented various options for implementing these approaches. Both approaches rely on some symmetric price index formula. Suitable candidates include the Walsh, Törnqvist, Fisher, Sato-Vartia, or Marshall-Edgeworth index.

The imputation approach allows for a decomposition of the bias into the contributions of individual groups of items. It was shown that half of the bias in the German CPI can be attributed to a single group of items even though the expenditure share of that group is below 12 percent.

The proposed revision process is not only adoptable to most national CPIs, but also to other price level measures such as the producer price index or the import and export price indices.

Appendix A

First, this appendix describes the compilation process of the officially revised and published German CPI. The resulting index numbers were depicted in Figure 1 by the dark gray line labeled as “official” and the index numbers derived by the replication of that process by the dotted black line labeled as “replicated”. Only afterwards, the compilation process that abstains from any retrospective revisions is presented. The resulting index numbers were represented in Figure 1 by the black line labeled as "raw".

When in 2003 the yearly expenditure weights of the year 2000 become available, Destatis calculates a series of monthly Laspeyres CPIs. The price and expenditure reference period of this series is the complete year 2000 (not January 2000!) and the first comparison period is January 2000:

$$P_{\mathrm{L}}^{00\to t} = \sum_{i=1}^{N} s_i^{00} \frac{p_i^t}{p_i^{00}} , \qquad t = 1/00, 2/00, \ldots , \tag{17}$$

where $s_i^{00} = v_i^{00} / \sum_{j=1}^{N} v_j^{00}$ is the expenditure share of item i in 2000, with $v_i^{00} = p_i^{00} x_i^{00}$ denoting total expenditure on item i during the year 2000, x_i^{00} being the item's total consumed quantity, and p_i^{00} representing the item's average price.

Destatis compiles the yearly expenditure weights for every fifth year. When the expenditure information of the year 2005 becomes available, Destatis compiles a time series of Laspeyres indices with the year 2005 serving as price and expenditure reference period:

$$P_{\mathrm{L}}^{05\to t} = \sum_{i=1}^{N} s_i^{05} \frac{p_i^t}{p_i^{05}} , \qquad t = 1/05, 2/05, \ldots . \tag{18}$$

The same procedure is applied when the expenditure weights of the year 2010 become available.

How are the three time series merged into one consistent time series of monthly price levels covering the interval January 2000 to December 2014? Consider the two time series compiled by formulas (17) and (18). Because in the year 2005 the expenditure weights of the expenditure reference period 2000 are outdated, the index number $P_{\mathrm{L}}^{00\to 1/05}$ and all subsequent index numbers produced by formula (17) are likely to suffer from considerable substitution bias. Therefore, as soon as the expenditure shares of the year 2005 become available, Destatis replaces them by results obtained from formula (18).

Obviously, simple chaining of $P_{\mathrm{L}}^{00\to 1/05}$ and $P_{\mathrm{L}}^{05\to t}$ is not feasible because the comparison period of the first index differs from the price reference period of the second index. Therefore, chaining requires an additional chain link that compares the price level of the year 2005 to the price level of January 2005. To this end, Destatis uses a simple Paasche index with the year 2005 as comparison period and January 2005 as price reference period:

$$P_{\mathrm{P}}^{1/05 \to 05} = \left(\sum_{i=1}^{N} s_i^{05} \left(\frac{p_i^{05}}{p_i^{1/05}} \right)^{-1} \right)^{-1} = \frac{1}{P_{\mathrm{L}}^{05 \to 1/05}} \,. \tag{19}$$

Therefore, the chain index is

$$\tilde{P}^{00 \to t} = P_{\mathrm{L}}^{00 \to 1/05} \cdot P_{\mathrm{P}}^{1/05 \to 05} \cdot P_{\mathrm{L}}^{05 \to t} \,.$$

Applying this chaining principle to the comparison period January 2005 and exploiting the second equality of (19), yields

$$\tilde{P}^{00 \to 1/05} = P_{\mathrm{L}}^{00 \to 1/05} \cdot P_{\mathrm{P}}^{1/05 \to 05} \cdot P_{\mathrm{L}}^{05 \to 1/05} = P_{\mathrm{L}}^{00 \to 1/05} \,.$$

This coincidence between the Laspeyres index $P_{\mathrm{L}}^{00 \to 1/05}$ and the chain index $\tilde{P}^{00 \to 1/05}$ confirms that the index numbers before and after January 2005 are consistently connected.

Once the expenditure weights of the year 2010 become available to Destatis, the new Laspeyres index numbers (that is, $P_{\mathrm{L}}^{10 \to 1/05}, P_{\mathrm{L}}^{10 \to 2/05}, \ldots$) must be linked to the index number $\tilde{P}^{00 \to 1/10}$. This is achieved by the same approach as before:

$$\tilde{P}^{00 \to t} = \tilde{P}^{00 \to 1/10} \cdot P_{\mathrm{P}}^{1/10 \to 10} \cdot P_{\mathrm{L}}^{10 \to t} \,.$$

In sum, Destatis calculates the long-run time series for the time interval January 2000 to December 2014 in the following way:

$$t = 1/00, \ldots, 1/05: \qquad \tilde{P}^{00 \to t} = P_{\mathrm{L}}^{00 \to t} \,, \tag{20}$$

$$t = 1/05, \ldots, 1/10: \qquad \tilde{P}^{00 \to t} = \tilde{P}^{00 \to 1/05} \cdot P_{\mathrm{P}}^{1/05 \to 05} \cdot P_{\mathrm{L}}^{05 \to t} \,, \tag{21}$$

$$t = 1/10, \ldots, 12/14: \qquad \tilde{P}^{00 \to t} = \tilde{P}^{00 \to 1/10} \cdot P_{\mathrm{P}}^{1/10 \to 10} \cdot P_{\mathrm{L}}^{10 \to t} \,. \tag{22}$$

Expression (20) defines an index that covers a time span of 61 months (a five-year period plus the first month of the following five-year period). The same applies to expression (21), while expression (22), being the last five-year period, covers only 60 months. For the replication of the official CPI numbers of Destatis the same system of formulas is used.

Now, to estimate the price trend that would arise if no retrospective revisions were admissible (depicted in Figure 1 by the black line labeled as "raw"), the expenditure reference period precedes the price reference period by five years. Therefore, the system of equations (20) to (22) must be slightly adjusted:

$$\begin{aligned}
t &= 1/00,\ldots,1/05: &\quad \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/00}\cdot P_{\mathrm{P}}^{1/00\to 95}\cdot P_{\mathrm{L}}^{95\to t}\,, \\
t &= 1/05,\ldots,1/10: &\quad \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/05}\cdot P_{\mathrm{P}}^{1/05\to 00}\cdot P_{\mathrm{L}}^{00\to t}\,, \\
t &= 1/10,\ldots,12/14: &\quad \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/10}\cdot P_{\mathrm{P}}^{1/10\to 05}\cdot P_{\mathrm{L}}^{05\to t}\,,
\end{aligned} \tag{23}$$

where $\tilde{P}^{00\to 1/00} = P_{\mathrm{L}}^{95\to 1/00}/[(1/12)\sum_{r=1}^{12} P_{\mathrm{L}}^{95\to r/00}]$. Therefore, formula (23) simplifies to $\tilde{P}^{00\to t} = P_{\mathrm{L}}^{95\to t}/[(1/12)\sum_{r=1}^{12} P_{\mathrm{L}}^{95\to r/00}]$.

Appendix B

In Section 1.4, the correction approach is applied to compute the long-run time series of retrospective Walsh indices. The expenditure reference periods are not single months but complete years. Therefore, the weighting parameter λ_t in formula (28) must be modified. Instead of $\lambda_t = t/T$, one has to set $\lambda_t = 0$ for the first twelve months of the five-year period. These twelve months are indexed by $m = 1,\ldots,12$. For the subsequent months of that five-year period up to and including the first month of the next five-year period ($m = 13,\ldots,61$), the value of λ_t is defined by $\lambda_t = (m-12)/(61-12) = (m-12)/49$. For example, consider the first five-year period (2000-2005). January to December 2000 ($t = 1/00,\ldots,12/00$) give $m = 1,\ldots,12$ and $\lambda_{1/00} = \lambda_{2/00} = \ldots = \lambda_{12/00} = 0$. In January 2001 ($t = 1/01$) one gets $m = 13$ and $\lambda_{1/01} = 1/49$. Finally, in January 2005 ($t = 1/05$) the values are $m = 61$ and $\lambda_{1/05} = 49/49 = 1$.

The long-run time series of retrospective Walsh indices uses this weighting parameter, λ_t, and a modified version of the system of formulas (20) to (22). In this modified system, the Laspeyres indices $P_{\mathrm{L}}^{00\to t}$, $P_{\mathrm{L}}^{05\to t}$, and $P_{\mathrm{L}}^{10\to t}$ are replaced by the retrospective

Walsh indices $P_{\text{W-co}}^{00 \to t}$, $P_{\text{W-co}}^{05 \to t}$, and $P_{\text{W-co}}^{10 \to t}$, where "co" stands for "correction approach". The index $P_{\text{W-co}}^{00 \to t}$ is defined by

$$P_{\text{W-co}}^{00 \to t} = \left(P_{\text{L}}^{00 \to t}\right) \cdot \left(\frac{P_{\text{W}}^{00 \to 05}}{P_{\text{L}}^{00 \to 1/05} P_{\text{P}}^{1/05 \to 05}}\right)^{\lambda_t}, \qquad t = 1/00, \ldots, 1/05\,,$$

with

$$P_{\text{W}}^{00 \to 05} = \sum \frac{\sqrt{s_i^{00} s_i^{05} \left(p_i^{05}/p_i^{00}\right)^{-1}}}{\sum \sqrt{s_j^{00} s_j^{05} \left(p_j^{05}/p_j^{00}\right)^{-1}}} \frac{p_i^{05}}{p_i^{00}}\,.$$

For $t = 1/05$, this index gives $\lambda_{1/05} = 1$ and $P_{\text{W-co}}^{00 \to 1/05} = P_{\text{W}}^{00 \to 05}/P_{\text{P}}^{1/05 \to 05}$. The indices $P_{\text{W-co}}^{05 \to t}$ and $P_{\text{W-co}}^{10 \to t}$ are defined analogously.

Then, the system for the time series of retrospective Walsh indices can be written in the following form:

$$t = 1/00, \ldots, 1/05: \qquad \tilde{P}_{\text{W-co}}^{00 \to t} = P_{\text{W-co}}^{00 \to t}\,, \tag{24}$$

$$t = 1/05, \ldots, 1/10: \qquad \tilde{P}_{\text{W-co}}^{00 \to t} = \tilde{P}_{\text{W-co}}^{00 \to 1/05} \cdot P_{\text{P}}^{1/05 \to 05} \cdot P_{\text{W-co}}^{05 \to t} = P_{\text{W}}^{00 \to 05} \cdot P_{\text{W-co}}^{05 \to t}\,, \tag{25}$$

$$t = 1/10, \ldots, 12/14: \qquad \tilde{P}_{\text{W-co}}^{00 \to t} = \tilde{P}_{\text{W-co}}^{00 \to 1/10} \cdot P_{\text{P}}^{1/05 \to 05} \cdot P_{\text{W-co}}^{10 \to t} = P_{\text{W}}^{00 \to 05} \cdot P_{\text{W}}^{05 \to 10} \cdot P_{\text{W-co}}^{10 \to t}\,. \tag{26}$$

This system of formulas is consistent in the sense that, for January 2005 ($t = 1/05$), formulas (24) and (25) generate the same index number and, for January 2010 ($t = 1/10$), formulas (25) and (26) generate the same index number.

Appendix C

In Section 1.6, the imputation approach is applied to compute a series of retrospective Walsh indices with the price reference period January 2010 ($t = 1/10$) and comparison periods that begin in January 2010 and end in December 2014. For this purpose, the Walsh formula (5) is generalized in the following way:

$$P_{\text{W-im}}^{1/10 \to t} = \sum \frac{\left(v_i^{10}\right)^{1-\lambda_t} \left(v_i^{15} (p_i^{10}/p_i^{15})\right)^{\lambda_t}}{\sum \left(v_j^{10}\right)^{1-\lambda_t} \left(v_j^{15} (p_j^{10}/p_j^{15})\right)^{\lambda_t}} \frac{p_i^t}{p_i^{1/10}}\,, \tag{27}$$

where "im" stands for "imputation approach" and λ_t is defined as in $P_{\text{W-co}}^{1/10 \to t}$. Since the accessible data set contains the expenditure shares s_i^{10} and s_i^{15} instead of the expenditures v_i^{10} and v_i^{15}, the numerator and denominator of the first quotient in formula (27) is divided by the factor $\left[\left(\sum v_j^{10}\right)^{(1-\lambda_t)}\left(\sum v_j^{15}\right)^{\lambda_t}\right]$. As a result,

$$P_{\text{W-im}}^{1/10 \to t} = \sum \frac{\left(s_i^{10}\right)^{1-\lambda_t}\left(s_i^{15}(p_i^{10}/p_i^{15})\right)^{\lambda_t}}{\sum\left(s_j^{10}\right)^{1-\lambda_t}\left(s_j^{15}(p_j^{10}/p_j^{15})\right)^{\lambda_t}} \frac{p_i^t}{p_i^{1/10}} . \tag{28}$$

The retrospective Walsh indices $P_{\text{W-im}}^{1/00 \to t}$ and $P_{\text{W-im}}^{1/05 \to t}$ are defined analogously.

Appendix D

Table 2: Laspeyres index numbers and Walsh index numbers for December 2004 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2000 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

Expenditure Category	Laspeyres $P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	Walsh $P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	Contrib. to Bias
Food, non-alcoholic beverages	104.46	10.5	104.37	10.68	0.001
Alcoholic beverages, tobacco	127.82	3.73	126.76	3.55	0.088
Clothing, footwear	99.96	5.58	99.92	5.47	0.002
Housing, water, electricity, gas, other fuels	108.99	30.74	109.06	30.83	−0.030
Furniture, other household equipment	101.87	6.96	101.79	6.47	0.014
Health	124.02	3.60	123.52	3.58	0.024
Transport	110.72	14.08	110.52	13.53	0.086
Communication	85.33	2.56	84.81	3.19	0.109
Recreation, entertainment, culture	106.29	11.26	102.88	11.86	0.366
Education	120.75	0.17	120.75	0.18	−0.001
Restaurant, accommodation services	113.5	4.73	113.75	4.47	0.024
Miscellaneous goods and services	107.77	6.09	107.65	6.19	0.000
Total (Jan. 2000 - Dec. 2004)	108.25	100.00	107.57	100.00	0.683

Table 3: Laspeyres index numbers and Walsh index numbers for December 2009 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2005 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

Expenditure Category	Laspeyres $P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	Walsh $P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	Contrib. to Bias
Food, non-alcoholic beverages	110.3	10.54	110.28	10.43	0.014
Alcoholic beverages, tobacco	114.3	3.96	114.07	3.80	0.032
Clothing, footwear	105.63	4.97	105.63	4.80	0.009
Housing, water, electricity, gas, other fuels	110.37	31.34	110.16	31.59	0.041
Furniture, other household equipment	104.37	5.69	104.23	5.45	0.018
Health	104.90	4.10	104.78	4.38	−0.008
Transport	113.47	13.42	113.54	13.26	0.013
Communication	87.99	3.15	86.69	3.50	0.087
Recreation, entertainment, culture	106.25	11.77	103.67	12.06	0.293
Education	227.00	0.20	227.00	0.20	0.010
Restaurant, accommodation services	114.73	4.48	114.21	4.40	0.034
Miscellaneous goods and services	109.12	6.38	108.99	6.14	0.030
Total (Jan. 2005 - Dec. 2009)	109.30	100.00	108.72	100.00	0.573

2 Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction[4]

2.1 Introduction

Besides the consumer price index and the producer price index, the national statistical offices (NSOs) usually publish a monthly or quarterly export price index and import price index. The latter two indices are used in the indexation of various types of international contracts and they are also required in the national accounts as deflators of nominal values of exports and imports. These are necessary to derive volume estimates of GDP by the expenditure approach. In the assessment of an economy's inflationary trends, special attention is paid to the import price index, because it is considered as an early indicator of increasing or weakening inflationary pressure. Correspondingly, the export price index is an early indicator of the inflationary pressure in the destination countries of the exports.

The terms of trade index of an economy is usually defined as the ratio of the economy's export price index and import price index. Changes in the terms of trade translate into changes of the real income of the economy's population.

Bias in the measurement of the import and export indices could weaken the reliability of economic statistics used for public and private economic decision making. Therefore, the "Export and Import Price Index Manual" published by the IMF (2009) provides recommendations for the measurement of the export and import price indices. In practice, the NSOs must also ensure a cost efficient and timely publication of the newest index numbers. Therefore, most NSOs rely on some type of Laspeyres index, although these indices are suspected of (upper-level) substitution bias (e.g., Dridi and Zieschang, 2004, p. 169; IMF, 2009, pp. 413-439). In chained Laspeyres type indices the substitution bias would accumulate over time.

Therefore, the central contribution of the present paper is a fully worked out retroactive correction approach that, with some delay, provides more reliable index numbers than those produced by a chained Laspeyres type index. Our correction approach produces meaningful long-run time series of import and export prices and, therefore, of

[4]We gratefully acknowledge the extremely helpful advice of two anonymous referees. The present paper was published in the *Journal of Official Statistics*: See please von Auer and Shumskikh (2022b)

the terms of trade. The approach is simple and transparent. It can be applied not only to an import or export price index, but also to a consumer or producer price index.

The first pillar of our correction approach is a retroactively computed price index that compares the prices of the two latest expenditure reference periods, that is, the two latest periods for which detailed information about the relative importance of the various products is available. Because of its symmetric treatment of the two expenditure reference periods, we use the Törnqvist formula, though other index formulas that treat the two periods in a symmetric fashion would be equally appropriate (e.g., Walsh, Marshall-Edgeworth, Fisher index). Chaining consecutive Törnqvist indices instead of Laspeyres indices removes the long-run substitution bias. However, it does not correct the index numbers of the periods between the expenditure reference periods. To revise also these index numbers, we construct from the ratio of the Laspeyres and Törnqvist index a correction factor the impact of which gradually increases between the two expenditure reference periods. This correction factor is the second pillar of our approach.

The second contribution of the present paper is an empirical case study that not only estimates a lower bound of the long-run substitution bias in officially published import and export price indices, but also demonstrates how the correction approach can be implemented to mitigate both the short-run bias arising between the expenditure reference periods and the long-run bias. We have opted for the trade data of the Federal Statistical Office of Germany (Statistisches Bundesamt, usually referred to as Destatis). These data are publicly available, they comprise price data collected from producers and wholesalers (instead of the less reliable unit values compiled from customs sources), and the official compilation procedure of Destatis is documented in accessible publications (Statistisches Bundesamt, 2019, pp. 6-7).

Theoretically, if the substitution bias in the export and import price indices was of equal magnitude, the terms of trade index would remain unbiased. However, we show that there is hardly any substitution bias in the German export price index, while the upward substitution bias in the German import price index is substantial.

The explanation of this finding is the third contribution of this paper. Our analysis reveals that the difference between the bias in the export and import price index is mostly driven by the volatility of the prices of oil and gas and by the fluctuations in the exchange rates.

Our findings are relevant not only for Destatis, but for all NSOs that use infrequently chained Laspeyres type indices for their measurement of import and export prices. Without an appropriate retroactive correction of these indices, the substitution bias in the officially published index numbers accumulates over time.

The paper proceeds as follows. Section 2.2 presents in stylized form the interpretation and official computation of the import and export price indices. Section 2.3 explains why they are likely to be biased. Furthermore, a retroactive correction approach is introduced. Its application to the German foreign trade data is presented in Section 2.4. Section 2.5 concludes.

2.2 Calculation of the Terms of Trade

Suppose that during a reference period $t = 0$ a country imports only one single good i and exports only one other good j. Let p_i^0 be the euro price of the imported good and z_j^0 the euro price of the exported good. Then, the terms of trade of the reference period (ToT^0) are defined by the ratio of these two prices: $\text{ToT}^0 = z_j^0/p_i^0$. Correspondingly, the terms of trade of some comparison period $t = 1$ are $\text{ToT}^1 = z_j^1/p_i^1$. The change in the terms of trade between the reference and the comparison period can be expressed by the following ratio:

$$\frac{\text{ToT}^1}{\text{ToT}^0} = \frac{z_j^1/p_i^1}{z_j^0/p_i^0} = \frac{z_j^1/z_j^0}{p_i^1/p_i^0}\,. \tag{29}$$

The right-hand side equality is the change in the export price in relation to the change in the import price.

In the case of many goods, average price changes can be measured by price index formulas. Applying the Laspeyres formula, the change in the export prices is given by

$$E_{\text{L}}^{0\to 1} = \frac{\sum_{j=1}^{N} z_j^1 x_j^0}{\sum_{j=1}^{N} z_j^0 x_j^0}\,, \tag{30}$$

where x_j^0 is the quantity of good j exported during the reference period and N is the number of exported goods. The subscript “L” stands for “Laspeyres” and the superscript "0→1" indicates that the index measures the average price change between the reference period 0 and the comparison period 1. Analogously, the price index of the import prices is

$$I_{\mathrm{L}}^{0\to1} = \frac{\sum_{i=1}^{M} p_i^1 m_i^0}{\sum_{i=1}^{M} p_i^0 m_i^0}\,, \tag{31}$$

where m_i^0 is the quantity of good i imported during the reference period and M is the number of imported goods.

The change in the terms of trade between the reference and the comparison period is derived from the quotient of the price indices (30) and (31):

$$\mathrm{ToT}^{0\to1} = \frac{\mathrm{ToT}^1}{\mathrm{ToT}^0} = \frac{E_{\mathrm{L}}^{0\to1}}{I_{\mathrm{L}}^{0\to1}}\,. \tag{32}$$

This ratio is the terms of trade index. It can be interpreted as a generalisation of the right-hand side equality of Equation (29) to the case of many goods. Even though $\mathrm{ToT}^{0\to1}$ defined by (32) measures "the *change* in the terms of trade between the reference and the comparison period", it is conventionally denoted as "*the* terms of trade of the comparison period". We follow this convention.

The terms of trade index (32) can also be written in the form that corresponds to the left-hand side equality in Equation (29):

$$\mathrm{ToT}^{0\to1} = \frac{\left(\sum_{j=1}^{N} z_j^1 x_j^0\right) \Big/ \left(\sum_{i=1}^{M} p_i^1 m_i^0\right)}{\left(\sum_{j=1}^{N} z_j^0 x_j^0\right) \Big/ \left(\sum_{i=1}^{M} p_i^0 m_i^0\right)}\,. \tag{33}$$

Interpreting the exported quantities of the reference period, $(x_1^0,\ldots,x_N^0)$, as the "reference period export basket" and the imported quantities of the reference period, $(m_i^0,\ldots,m_M^0)$, as the "reference period import basket", the denominator of Equation (33) measures the purchasing power of a reference period export basket measured in units of reference period import baskets. The numerator indicates the purchasing power of the reference period export basket during the comparison period. Again, this purchasing power is measured in units of reference period import baskets. Therefore, the terms of trade index (33) indicates the change in the purchasing power of the reference period export basket, measured in units of reference period import baskets.

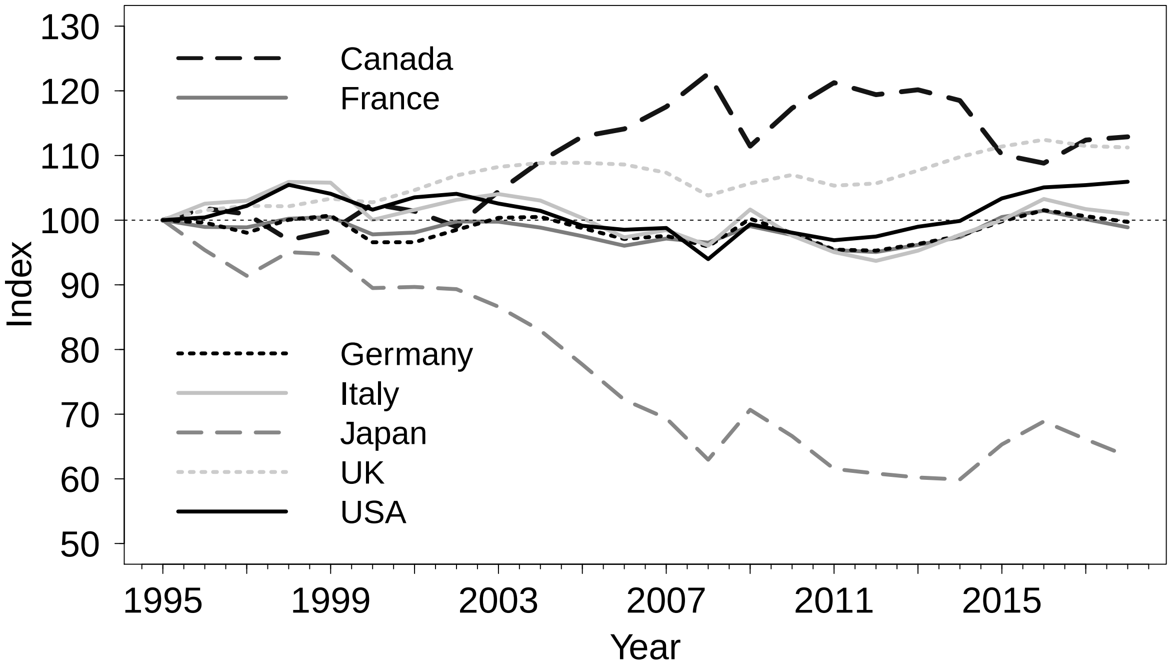

Figure 2: Terms of Trade of the G7 countries (1995 = 100) from 1995 to 2018.

Figure 2 shows the officially published terms of trade of the G7-countries from 1995 to 2018 (OECD, 2021).[5] The Canadian terms of trade are largely driven by the price of oil and gas. The terms of trade of all other G7-countries are negatively correlated with the Canadian ones. Between 2000 and 2007, there was a strong devaluation of the Japanese yen against the euro, though not against the dollar. Despite the yen's subsequent appreciation until 2012, the Japanese terms of trade remained at their lower level.

The terms of trade depicted in Figure 2 are derived from the ratio of the price index of export prices and the price index of import prices. The next section explains why such indices may exhibit substitution bias and how the problem can be mitigated.

2.3 Retrospective Correction of Substitution Bias

Bias in the terms of trade arises, when the export price index, E_{L}, or the import price index, I_{L}, or both price indices are distorted – provided that the two distortions do not

[5]The statistical methodologies of the NSOs of the G7-countries (Canada, France, Germany, Italy, Japan, UK, and US) are described in Statistics Canada (2019), INSEE (2019), Peter (2009, 2014, 2019), Statistisches Bundesamt (2004), Istat (2019), Bank of Japan (2019), ONS (2017), and Bureau of Labor Statistics (2018a,b, 2020).

cancel. Of course, such distortions can occur already during data collection and processing. These problems are well known and extensively discussed in IMF (2009, pp. 287-297). Therefore, it is assumed here that the available price and quantity data are accurate and that the only remaining source of bias is the choice of the index formula.

In IMF (2009, pp. 413-439) it is argued that imports and exports can be viewed from a resident's perspective or a non-resident's perspective. For each perspective, economic theory makes predictions about the direction of the measurement bias arising from the Laspeyres index formula.

When the resident's perspective is applied, the import quantities and prices collected by NSOs reflect the residents' cost minimizing consumer behavior (including firms purchasing their inputs). The consumer side increases the purchases of products that become relatively less expensive and they reduce the purchases of products that become relatively more expensive. This consumer behavior would result in a negative correlation between intertemporal price and quantity changes and, therefore, in upward substitution bias of the Laspeyres index.

Furthermore, the observed export quantities and prices reflect the revenue maximization of the residents' producer side. This side increases the output of products that become relatively more expensive and they reduce the output of products that become relatively less expensive. Therefore, the intertemporal price and quantity changes are positively correlated. In a Laspeyres index, this would lead to downward substitution bias.

In sum, in the resident's perspective the numerator in the terms of trade index (32) would understate the average change in export prices, while the denominator would overstate the average change in import prices. Therefore, the measured terms of trade would exhibit downward bias.

When a non-resident's perspective were applied, the direction of the bias would be reversed. However, most NSOs use the resident's perspective.

These considerations are conjectures that are based on economic theory. An empirical examination of these conjectures requires a measurement approach that can be expected to produce unbiased index numbers. A deviation between these unbiased index numbers and the Laspeyres numbers is an indication of the direction and extent of the actual substitution bias of the Laspeyres index.

The derivation of such unbiased index numbers starts with the Laspeyres index (31). It is often expressed in the following equivalent form:

$$I_{\mathrm{L}}^{0 \to 1} = \sum_{i=1}^{M} s_i^0 \frac{p_i^1}{p_i^0}\,, \tag{34}$$

with

$$s_i^0 = \frac{p_i^0 m_i^0}{\sum_{j=1}^{M} p_j^0 m_j^0}\,.$$

Therefore, the Laspeyres index can be interpreted as a weighted arithmetic mean of price ratios where the weights are the expenditure shares of the reference period.

The Laspeyres index is not the only index formula that is prone to substitution bias. The Paasche index,

$$I_{\mathrm{Pa}}^{0 \to 1} = \frac{\sum_{i=1}^{M} p_i^1 m_i^1}{\sum_{i=1}^{M} p_i^0 m_i^1} = \left(\sum_{i=1}^{M} s_i^1 \left(\frac{p_i^1}{p_i^0} \right)^{-1} \right)^{-1}\,, \tag{35}$$

has a similar problem, though its substitution bias would point in the opposite direction. The right-hand side of (35) expresses the Paasche index as the weighted harmonic mean of the price ratios where the weights are the expenditure shares of the comparison period.

A large number of price indices avoid the substitution bias of the Laspeyres index. Examples are the Walsh, Marshall-Edgeworth, Fisher and Törnqvist index. These index formulas utilize not only the set of reference period quantities or the set of comparison period quantities, but both sets of quantities. Usually, these four formulas generate very similar index numbers. Therefore, we confine our analysis to the Törnqvist index. It is defined by

$$I_{\mathrm{Tö}}^{0 \to 1} = \exp\left(\sum_{i=1}^{M} \frac{1}{2} \left(s_i^0 + s_i^1 \right) \ln\left(\frac{p_i^1}{p_i^0} \right) \right)\,. \tag{36}$$

The compilation of this price index requires not only the expenditures $p_i^0 m_i^0$ and the price ratios p_i^1/p_i^0, but also the expenditures $p_i^1 m_i^1$. However, the collection and compilation of the latter expenditures is a labor intensive process that would significantly delay the publication of the indices (IMF, 2009, p. 58). Therefore, NSOs do not have the capacity to timely calculate and publish the results of such an index formula.

Nevertheless, the IMF (2009, p. 56) points out that a retroactive revision of the index numbers would be feasible when the requisite data on updated expenditure weights become available. The method presented here retroactively applies the updated expenditure weights to gradually transform the Laspeyres indices to Törnqvist indices, thus addressing the substitution bias with a historical revision.

For example, suppose that in January 2010 (shorthand notation 1/10) a survey was conducted providing us with the import expenditure weights for that month, $s_i^{1/10}$. Therefore, this month is our first *expenditure reference* period. For February 2010 (2/10) and all subsequent months we calculate a monthly Laspeyres import price index that measures the average price change between the *price reference* (and first expenditure reference) period January 2010 and some comparison period t:

$$I_{\mathrm{L}}^{1/10 \to t} = \sum_{i=1}^{M} s_i^{1/10} \frac{p_i^t}{p_i^{1/10}}, \qquad t = 1/10, 2/10, \ldots . \tag{37}$$

For the comparison period $t = 1/10$, this index yields $I_{\mathrm{L}}^{1/10 \to 1/10} = 1$, that is, the reference price level of the index series is Jan. 2010 = 1. Therefore, January 2010 is not only the price and weight reference period, but also the *index reference* period.

Suppose that January 2015 is the next expenditure reference period, but that the expenditure weights relating to that month become available only in July 2018. Therefore, January 2010 remains the price (and index) reference period also for the Laspeyres indices compiled between January 2015 and July 2018. In July 2018, when the expenditure weights of January 2015 become available, we can conduct a retroactive revision of past index numbers. We propose to conduct this revision in three stages. The first and second stage revise the index numbers of January 2015 to July 2018, while the third stage revises the index numbers of January 2010 to December 2014. The three stages yield a consistent time series of price levels that stretches from January 2010 to July 2018. It can be easily continued without harming its consistency.

Stage 1: We begin the revision by computing a new series of Laspeyres index numbers for January 2015 to July 2018. This new series uses as price, expenditure and index reference period January 2015 instead of January 2010:

$$I_{\mathrm{L}}^{1/15 \to t} = \sum_{i=1}^{M} s_i^{1/15} \frac{p_i^t}{p_i^{1/15}}, \qquad t = 1/15, 2/15, \ldots, 7/18. \tag{38}$$

This new series of Laspeyres index numbers can be expected to exhibit considerably less substitution bias than the original series, because the quantity information is more up to date (January 2015 instead of January 2010). However, some substitution bias remains, because we still apply the Laspeyres index formula. Only when the results of the next expenditure reference period will become available, this bias can be addressed.

Stage 2: To rebase the new series to the index reference period January 2010, advocates of the Laspeyres index would multiply the index numbers compiled by (38) by the Laspeyres index $I_{\mathrm{L}}^{1/10 \to 1/15}$. However, we know that this Laspeyres index exhibits substantial substitution bias. Therefore, we recommend to apply the Törnqvist index (36) instead:

$$I_{\mathrm{Tö}}^{1/10 \to 1/15} = \exp\left(\sum_{i=1}^{M} \frac{1}{2} \left(s_i^{1/10} + s_i^{1/15} \right) \ln\left(\frac{p_i^{1/15}}{p_i^{1/10}} \right) \right). \tag{39}$$

In view of the upward substitution bias of the original Laspeyres index, we expect that $I_{\mathrm{Tö}}^{1/10 \to 1/15} < I_{\mathrm{La}}^{1/10 \to 1/15}$. The rebased series is obtained from

$$I^{1/10 \to t} = I_{\mathrm{Tö}}^{1/10 \to 1/15} \cdot I_{\mathrm{L}}^{1/15 \to t}, \qquad t = 1/15, 2/15, \ldots, 7/18. \tag{40}$$

Therefore, $I^{1/10 \to 1/15} = I_{\mathrm{Tö}}^{1/10 \to 1/15}$. The rebased series of price index numbers relates each of the price levels between January 2015 and July 2018 to Jan. 2010 = 1. Overall, we expect a significant downward revision of the original price levels compiled by (37).

The first two stages of revision affect only the time series of import price levels from January 2015 to July 2018. The price levels of the previous months have not yet been revised. Quite likely, the original price level of December 2014, $I_{\mathrm{L}}^{1/10 \to 12/14}$, is larger than the downwards revised price level of January 2015, $I_{\mathrm{Tö}}^{1/10 \to 1/15}$. To obtain a consistent time series stretching from January 2010 to July 2018, the price level of the index reference period January 2010 should remain at 1, but the price levels of February 2010 to December 2014 must be revised. This is the third and most challenging stage of the retroactive revision.

Stage 3: In the course of Stage 2, we replaced the Laspeyres index $I_{\mathrm{L}}^{1/10 \rightarrow 1/15}$ by the Törnqvist index $I_{\text{Tö}}^{1/10 \rightarrow 1/15}$. Obviously, we cannot make the same replacement for the previous months ($t = 2/10$ to $t = 12/14$), because the quantities of these months are unknown. Quantity information is available only for the two expenditure reference periods January 2010 and January 2015. However, we can utilize in our retroactive revision of these earlier index numbers the ratio $\left(I_{\text{Tö}}^{1/10 \rightarrow 1/15} \big/ I_{\mathrm{L}}^{1/10 \rightarrow 1/15}\right)$ as a correction factor:

$$I^{1/10 \rightarrow t} = I_{\mathrm{L}}^{1/10 \rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{1/10 \rightarrow 1/15}}{I_{\mathrm{L}}^{1/10 \rightarrow 1/15}} \right)^{\lambda_t} , \qquad t = 1/10, 2/10, \ldots, 1/15 . \tag{41}$$

The parameter λ_t represents the impact that we concede to the correction factor. For the comparison period $t = 1/15$, the correction factor should exert its full impact ($\lambda_{1/15} = 1$) such that formula (41) yields $I^{1/10 \rightarrow 1/15} = I_{\text{Tö}}^{1/10 \rightarrow 1/15}$. However, for the initial months ($t = 2/10,\ 3/10, \ldots$) the situation is different. During these comparison periods, the import basket of month $t = 1/10$ is far less outdated than in month $t = 1/15$ and, therefore, the substitution bias of $I_{\mathrm{L}}^{1/10 \rightarrow 2/10}$, say, tends to be much smaller than that of $I_{\mathrm{L}}^{1/10 \rightarrow 1/15}$. Accordingly, during the initial months, the impact λ_t should be smaller than 1. In fact, for the comparison period $t = 1/10$, the correction factor should have no impact ($\lambda_{1/10} = 0$). Otherwise, we would get $I^{1/10 \rightarrow 1/10} \neq 1$. As the comparison period t moves away from the price reference period 1/10, the impact λ_t should gradually increase above 0 until, in $t = 1/15$, it finally reaches its maximum value 1.

For a formal definition of the impact λ_t we introduce the counter variable s. In the first month, $t = 1/10$, this integer has the value $s = 1$, in month $t = 2/10$ the value $s = 2$, and so on. In month $t = 1/15$ the counter reaches its maximum value $s = 61$. Denoting this maximum value by S, we can define the impact λ_t by

$$\lambda_t = \frac{s-1}{S-1} . \tag{42}$$

Formulas (41) and (42) yield the desired series of revised price index numbers for the months January 2010 to January 2015. Combining this series with the revised index numbers that were compiled during Stages 1 and 2 of the revision process, yields a consistent series of revised index numbers stretching from January 2010 to July 2018. It is consistent in the sense, that, for month $t = 1/15$, formulas (40) and (41) yield the same index number, namely $I_{\text{Tö}}^{1/10 \to 1/15}$.

Formulas (41) and (42) are not the only conceivable method for Stage 3 of the revision process. Some alternative options are explored in von Auer and Shumskikh (2022a).[6]

2.4 Application to German Foreign Trade Data

Different NSOs apply different compilation methods for their export and import price indices. Our correction approach is adaptable to a wide range of such methods. To verify this claim and to get an impression of the magnitude of the bias inherent in official compilation procedures, we adapt our approach to the method and the trade data of Destatis, the Federal Statistical Office of Germany.

An important difference between the calculation outlined in Section 2.2 and the method of Destatis is the choice of the period lengths. In Section 2.2, all periods had a uniform length, namely one month. By contrast, the price, expenditure and index reference periods of Destatis are years, while the comparison periods are months. The resulting complications arising in the Destatis method are described in the Appendix E. There we also demonstrate how our correction approach can be adapted to these complications.

For January 1995 to May 2019, we have monthly price levels of 30 categories of German imports and 28 categories of German exports. In addition, we know the categories' expenditure weights for the years 1995, 2000, 2005, 2010 and 2015.

As documented in Pötzsch (2004) and Peter (2009, 2014, 2019), the officially published long-run import price index of Destatis (Statistisches Bundesamt, 2019, pp. 8-9) incorporates Stage 1 of our revision process but not Stages 2 and 3. For example, in September 2018 Destatis published the index numbers for the comparison months $t = 1/10$ to $t = 7/18$. They were compiled by the Laspeyres index with expenditure and

[6]Instead of a correction factor, these alternative options would replace the Laspeyres index $I_{\text{L}}^{1/10 \to 1/15}$ by a modified version of the Fisher, Marshall-Edgeworth, Törnqvist, or Walsh index, or some other index formula that incorporates the quantities of both, the price reference and the comparison period (e.g., Theil index).

index reference year 2010, $I_{\mathrm{L}}^{2010\to t}$. However, the index numbers for the comparison months $t = 1/15$ to $t = 7/18$ were only preliminary. As soon as the survey results of the year 2015 became available, Destatis replaced these index numbers by revised index numbers that were compiled by the Laspeyres index with price, expenditure and index reference year 2015, $I_{\mathrm{L}}^{2015\to t}$ (Peter, 2019, p. 37). This revision is equivalent to Stage 1 of our three-stage revision process outlined in Section 2.3.

Our first task is to replicate the compilation process of Destatis and to compile a consistent time series of price levels relating to the index reference year 1995. The replicated index is denoted by $I_{\mathrm{repl}}^{95\to t}$. Following the exposition in the Appendix E, we use the following formulas:

$$
\begin{aligned}
t &= 1/95,\dots,1/00: & I_{\mathrm{repl}}^{95\to \mathrm{t}} &= I_{\mathrm{L}}^{95\to t}\\
t &= 1/00,\dots,1/05: & I_{\mathrm{repl}}^{95\to \mathrm{t}} &= I_{\mathrm{repl}}^{95\to 1/00}\cdot I_{\mathrm{Pa}}^{1/00\to 00}\cdot I_{\mathrm{L}}^{00\to t}\\
t &= 1/05,\dots,1/10: & I_{\mathrm{repl}}^{95\to \mathrm{t}} &= I_{\mathrm{repl}}^{95\to 1/05}\cdot I_{\mathrm{Pa}}^{1/05\to 05}\cdot I_{\mathrm{L}}^{05\to t}\\
t &= 1/10,\dots,1/15: & I_{\mathrm{repl}}^{95\to \mathrm{t}} &= I_{\mathrm{repl}}^{95\to 1/10}\cdot I_{\mathrm{Pa}}^{1/10\to 10}\cdot I_{\mathrm{L}}^{10\to t}\\
t &= 1/15,\dots,5/19: & I_{\mathrm{repl}}^{95\to \mathrm{t}} &= I_{\mathrm{repl}}^{95\to 1/15}\cdot I_{\mathrm{Pa}}^{1/15\to 15}\cdot I_{\mathrm{L}}^{15\to t}\,.
\end{aligned}
$$

Our results are depicted in Figure 3[7] and compared to the (rebased) official import price index published by Destatis. Even though our data only relate to rather broad categories and do not include all subcategories of the official import price index of Destatis, our replicated import price index (labelled as "repl"), is very close to the official one (labelled as "official").

Our second task is to apply our correction approach and to compute the revised import price index, $I_{\mathrm{rev}}^{95\to t}$. Since the replicated index includes Stage 1 of the correction approach, any deviations between the revised index $I_{\mathrm{rev}}^{95\to t}$ and the replicated index $I_{\mathrm{repl}}^{95\to t}$ can be attributed to Stages 2 and 3.

[7] Calculations based on data of database "Genesis" of Destatis (Statistisches Bundesamt, 2021)

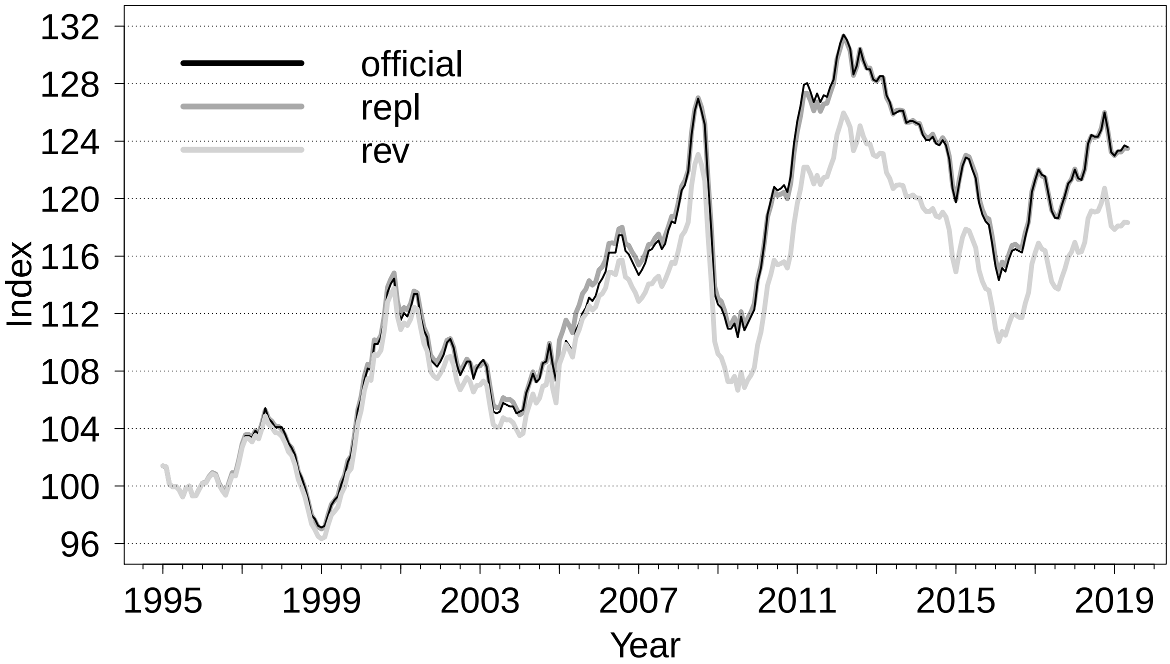

Figure 3: German import price index (1995 = 100) for January 1995 to May 2019.

Our compilations follow the process outlined in the Appendix E:

$$
\begin{aligned}
t = 1/95,\ldots,1/00: \qquad & I_{\text{rev}}^{95\rightarrow t} = I_{\text{L}}^{95\rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{95\rightarrow 00}}{I_{\text{L}}^{95\rightarrow 1/00} \cdot I_{\text{Pa}}^{1/00\rightarrow 00}} \right)^{\lambda_t} \\
t = 1/00,\ldots,1/05: \qquad & I_{\text{rev}}^{95\rightarrow t} = I_{\text{Tö}}^{95\rightarrow 00} \cdot I_{\text{L}}^{00\rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{00\rightarrow 05}}{I_{\text{L}}^{00\rightarrow 1/05} \cdot I_{\text{Pa}}^{1/05\rightarrow 05}} \right)^{\lambda_t} \\
t = 1/05,\ldots,1/10: \qquad & I_{\text{rev}}^{95\rightarrow t} = I_{\text{Tö}}^{95\rightarrow 00} \cdot I_{\text{Tö}}^{00\rightarrow 05} \cdot I_{\text{L}}^{05\rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{05\rightarrow 10}}{I_{\text{L}}^{05\rightarrow 1/10} \cdot I_{\text{Pa}}^{1/10\rightarrow 10}} \right)^{\lambda_t} \\
t = 1/10,\ldots,1/15: \qquad & I_{\text{rev}}^{95\rightarrow t} = I_{\text{Tö}}^{95\rightarrow 00} \cdot I_{\text{Tö}}^{00\rightarrow 05} \cdot I_{\text{Tö}}^{05\rightarrow 10} \cdot I_{\text{L}}^{10\rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{10\rightarrow 15}}{I_{\text{L}}^{10\rightarrow 1/15} \cdot I_{\text{Pa}}^{1/15\rightarrow 15}} \right)^{\lambda_t} \\
t = 1/15,\ldots,5/19: \qquad & I_{\text{rev}}^{95\rightarrow t} = I_{\text{Tö}}^{95\rightarrow 00} \cdot I_{\text{Tö}}^{00\rightarrow 05} \cdot I_{\text{Tö}}^{05\rightarrow 10} \cdot I_{\text{Tö}}^{10\rightarrow 15} \cdot I_{\text{L}}^{15\rightarrow t},
\end{aligned}
$$

with

$$
\lambda_t = \begin{cases} 0 & \text{for} \quad s = 1, 2, \ldots, 12 \\ \dfrac{s-12}{61-12} & \text{for} \quad s = 13, 14, \ldots, 60\,. \end{cases}
$$

In month $t = 1/95$, the value of the counter s is equal to 1. In the subsequent months it increases until in month $t = 12/99$ it reaches the value 60. In the following month, $t = 1/00$, the value of s is reset to 1. The same reset happens in months 1/05 and 1/10.

Price changes within an expenditure reference year ($s = 1, 2, \ldots, 12$) are not revised ($\lambda_t = 0$).

The formulas generate the revised import price index $I_{\text{rev}}^{95 \to t}$ depicted in Figure 3 (labelled as "rev"). The index deviates from the replicated index $I_{\text{repl}}^{95 \to t}$ ("repl") and, therefore, from the official index of Destatis ("official"). The deviation increases over time and, in 2019, reaches more than five percentage points. This value can be considered as a lower bound of the accumulated (upper-level) long-run substitution bias in the official price index.

This reinforces the case for a revision that does not stop at Stage 1, but includes also Stages 2 and 3. Only this comprehensive revision can avoid the accumulating long-run substitution bias inherent in chained Laspeyres indices. The revision requires no additional data. It exclusively draws on information that is used in the original price index compilation.

To this point, we were exclusively concerned with the import price index. If the export price index exhibited the same bias, the terms of trade index (ratio of the export price index and import price index) would remain unbiased. Unfortunately, such a compensatory effect is unlikely. The reason is illustrated in the upper part of Figure 4.[8] It shows the export and import price indices of Germany as compiled by Destatis. Both indices are normalized to Jan. 1980 = 100.

The export price index rises very evenly, while the import price index rises much more erratically.[9] In the following, we explain why a more erratic index is more vulnerable to substitution bias. The line of reasoning starts with an empirical observation. Our data reveal that for a given pair of consecutive months the change in the import index (measured in percent where the sign is eliminated) is positively correlated with the coefficient of variation of the intertemporal price relatives of the 30 categories of German imports. For the time span for which we have price data on the 30 import categories (January 1995 to May 2019), the (Pearson) correlation coefficient is almost 0.54.

[8]Calculations are based on the data from Statistisches Bundesamt (2021), Deutsche Bundesbank (2021) and World Bank (2021).

[9]Also in France, Italy, and Japan the import price index is much more volatile than the export price index.

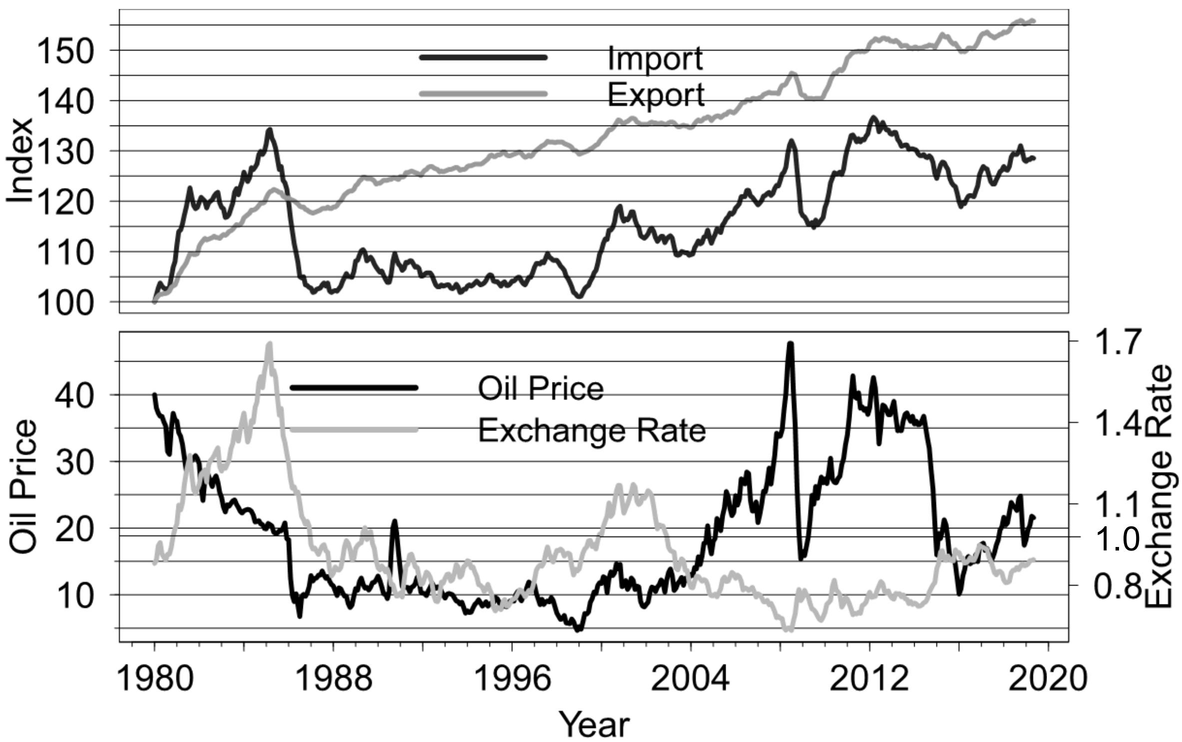

Figure 4: German export and import price indices (Jan. 1980 = 100), real oil prices (Brent oil, in dollars of 1980) and exchange rate (€/\$) for January 1980 to May 2019.

From the work of Bortkiewicz (1923, pp. 374-376) we know that the relative divergence between the Laspeyres and Paasche index depends on three factors: the coefficient of variation of the intertemporal quantity relatives, the coefficient of variation of the intertemporal price relatives, and the coefficient of linear correlation between the price and quantity relatives. Thus, for a given correlation, the divergence between the Laspeyres and Paasche index tends to increase with the volatility of the prices and quantities. An increasing Paasche-Laspeyres spread translates into an increasing Törnqvist-Laspeyres spread, because the Törnqvist index closely approximates the geometric average of the Laspeyres and Paasche index (known as the Fisher index).

Since the Törnqvist-Laspeyres spread is interpreted as an indication of substitution bias, the previous considerations can be condensed to a simple conjecture: The larger the volatility of a Laspeyres index, the larger the risk of substitution bias. Therefore, the rather steadily evolving export price index is less likely to suffer from substitution bias than the much more volatile import price index.

To verify this conjecture, we first replicate the official export price index of Destatis, $E_{\mathrm{repl}}^{95\to t}$, then compile the retroactively revised export price index, $E_{\mathrm{rev}}^{95\to t}$, and finally compare the difference of the two indices to the difference that we computed in the context of the import price index. We follow the same procedures that we used for the import price index. The results are depicted in Figure 5.[10]

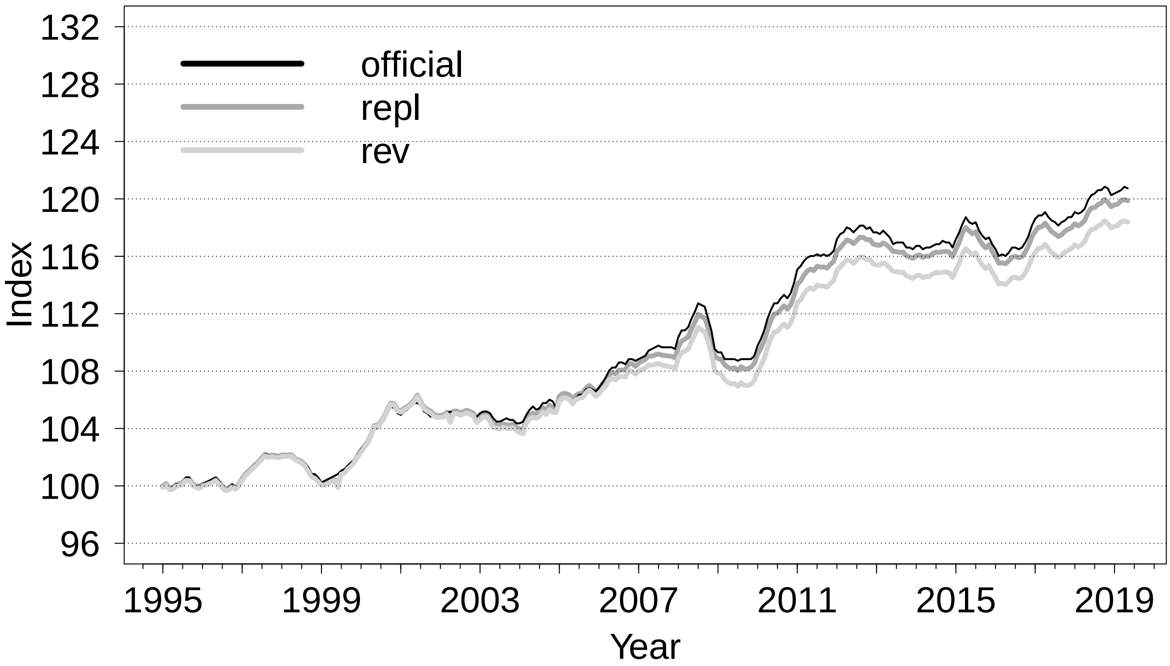

Figure 5: German export price index (1995 = 100) from January 1995 to May 2019.

We find our claim confirmed. The revision is smaller than in the import price index, indicating that also the accumulated long-run substitution bias in the official export price index is smaller. The direction of the bias is upwards, hinting at a negative correlation between price and quantity changes. This negative correlation is more in line with the behavior of purchasers than with the behavior of producers or sellers. In other words, our empirical findings related to the German export prices corresponds to the non-resident's perspective of economic theory (IMF, 2009, pp. 414).

The preceding results have important implications for the German terms of trade index, that is, for the ratio of the export and import price index. Since the export price index exhibits only a minor upward bias, the substantial upward bias in the import price index translates into a substantial downward bias in the German terms of trade index. This is depicted in Figure 6.[11] The terms of trade index compiled from

[10]Calculations are based on data of database "Genesis" of Destatis (Statistisches Bundesamt, 2021).
[11]Calculations are based on data of database "Genesis" of Destatis (Statistisches Bundesamt, 2021).

the revised indices, $E_{\text{rev}}^{95\to t}$ and $I_{\text{rev}}^{95\to t}$, deviates from the terms of trade index compiled from the replicated indices, $E_{\text{repl}}^{95\to t}$ and $I_{\text{repl}}^{95\to t}$. The deviation suggests that in 2019 the accumulated bias in the official terms of trade reaches roughly four percentage points.

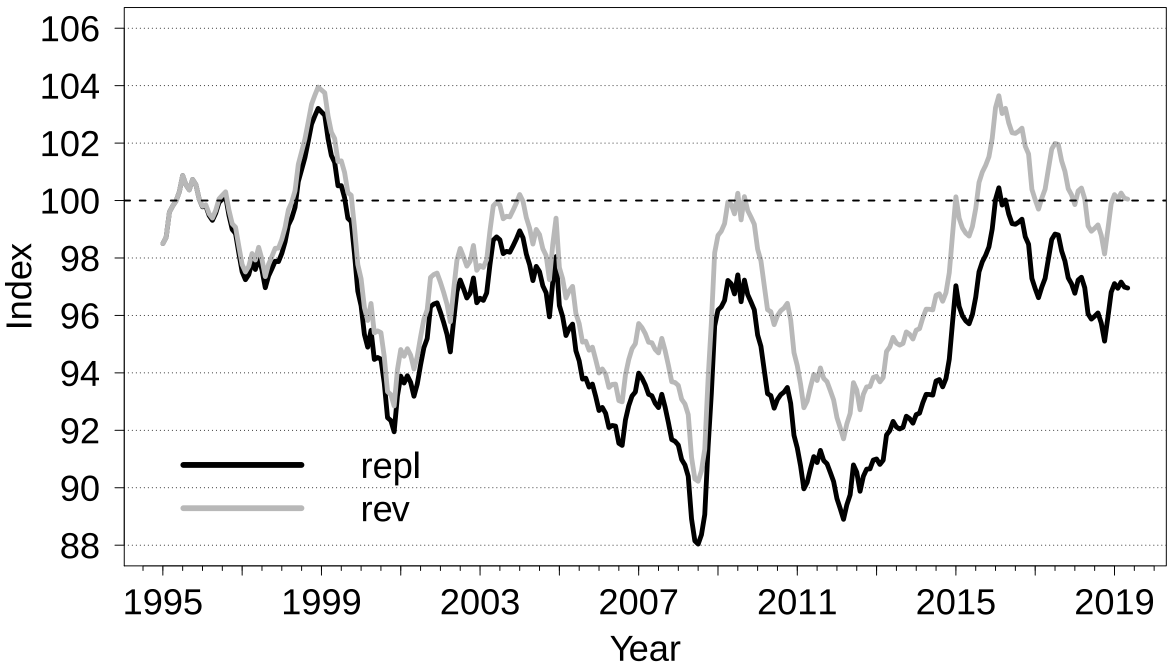

Figure 6: German terms of trade (1995 = 100) from January 1995 to May 2019.

The bias in the terms of trade can be attributed to the difference in the volatility of the import and export price index. What causes the difference in volatility? Until the introduction of the euro in January 1999, German imports had to be converted into Deutsche Mark before they entered the import price index. As a consequence, the German import price index depended not only on the price changes in the countries of origin, but also on the exchange rate of the Deutsche Mark against the foreign currencies, most importantly against the dollar. Therefore, until December 1998 we expect a strong positive correlation between the import price index and the nominal exchange rate in price notation, that is the price of one dollar.

Figure 4 confirms this expectation. The gray line in the lower part of the graph (reference is the right axis) shows the euro-dollar exchange rate in price notation, that is, the price of one dollar expressed in euros (or in units of 1.95583 Deutsche Mark before 1999). Between January 1980 and December 1998, the correlation coefficient between the first differences of the import price index and the first differences of the exchange rate is almost 0.74.

Since January 1999 parts of the imports and exports are invoiced in euro.[12] This is likely to dampen the impact of the foreign exchange rate on the import price index. Figure 4 also confirms this second conjecture. For the time interval January 1999 to May 2019, the correlation between the first differences of the import price index and the first differences of the exchange rate is below 0.26. This indicates that also other factors must be responsible for the larger volatility of the import prices as compared to the export prices.

Obvious suspects are the prices of oil and gas. These two products represent almost ten percent of the German imports as compared to an export share of less than one percent (Peter, 2019, pp. 39-40). The black line in the lower part of Figure 4 depicts the evolution of the real oil prices (reference is the left axis). The strong volatility of the real oil prices and their positive correlation with the import price index are clearly visible. The correlation coefficient of the first differences of the import index and the first differences of the real oil price for the time interval January 1999 to May 2019 is almost 0.72 (before 1999 it is below 0.29).

2.5 Concluding Remarks

For a given pair of months, the extent of the substitution bias of Laspeyres type indices is positively correlated with the variation of the intertemporal price relatives, the variation of the intertemporal quantity relatives, and the linear correlation between the two. Chaining of such distorted indices leads to accumulated bias. In most countries, the official import and export price indices are compiled as chained Laspeyres type indices. Therefore, these indices are likely to suffer from substitution bias.

In the present study, we examined this conjecture. In a case study of the German trade statistics we compiled a lower bound for (upper-level) substitution bias in the German import price index and export price index. For the time interval January 1995 to May 2019 the accumulated upward substitution bias of the import price index is

[12] Eurostat (2017) shows that in 2016 almost 50 percent of German imports from non-EU countries are invoiced in euro, while almost 60 percent of the exports in non-EU countries are invoiced in euro.

more than five percent, while the upward bias of the export price index is slightly above one percent.

Furthermore, we demonstrated that the accumulating substitution bias can be easily avoided by a three-stage retroactive revision process. This process requires no additional data and is adaptable to the specific index compilation procedures of the various national statistical offices.

The longer the intervals between the surveys providing the quantity data of the import and export price indices, the larger the expected substitution bias. Several countries rely on five-year-intervals (e.g., Canada, Germany, Italy, Japan, UK). Such countries are most likely to benefit from the implementation of a retroactive revision process. However, also in countries with shorter intervals (e.g., France, US) the proposed revision process may help to improve the reliability of the published long-run indices.

In countries like France, Germany, Italy and Japan the import price index is considerably more volatile than the export price index. For Germany, we showed that a larger volatility of the index translates into a larger variation of the price and quantity relatives and this results in a larger substitution bias. In other words, the substitution bias of the import price index is likely to exceed the bias of the export price index. As a consequence, also the terms of trade index should be biased. Our empirical analysis confirmed this conjecture for the German case.

What causes the difference in the volatility of the import and export prices? One likely reason are the strong fluctuations in the prices of oil and gas. These two products are all but absent from the exports of France, Germany, Italy and Japan, but they represent a substantial share of these countries' imports. In some cases, exchange rate fluctuations may also play a role (e.g., Germany before the introduction of the euro).

Our retroactive correction approach can also be applied to other important areas of price measurement such as the consumer price index or the producer price index.

Appendix E: Retroactive Correction in Practice

In this appendix we describe how our three-stage revision process can be applied to the trade statistics of Destatis. The monthly import and export price indices of Destatis are compiled in two variants. One is based on price data collected from exporting producers and wholesale traders, while the other is compiled from customs sources. Gehle (2013, p. 932) and Lippe and Mehrhoff (2010) show that the two variants generate different results. Following the general recommendation of the IMF (2009, p. xiv), we study the variant based on price data. Until December 2004 we had to excerpt the price levels from printed publications of Destatis (Statistisches Bundesamt, 2004). Later price levels we could retrieve from the online data base "Genesis" provided by Destatis.[13] In addition, we could compute from the online data base the categories' expenditure weights for the years 1995, 2000, 2005, 2010 and 2015.

We begin with the monthly German import price index from January 2010 until July 2018. The official compilation method of Destatis (a monthly Laspeyres index) is sketched out in Statistisches Bundesamt (2019, pp. 6-7). Destatis knows the yearly expenditure weights of the year 2010 (shorthand notation: 10). Therefore, the price, expenditure and index reference period is not a month, but the year 2010, that is, 2010 = 1. The corresponding Laspeyres index is

$$I_{\mathrm{L}}^{10 \to t} = \sum_{i=1}^{M} \bar{s}_i^{10} \frac{p_i^t}{\bar{p}_i^{10}}, \qquad t = 1/10, 2/10, \ldots, 7/18, \tag{43}$$

with

$$\bar{s}_i^{10} = \frac{\bar{p}_i^{10} \bar{m}_i^{10}}{\sum_{j=1}^{M} \bar{p}_j^{10} \bar{m}_j^{10}},$$

where $\bar{p}_i^{10}$ denotes the average euro price of good i in 2010 and $\bar{m}_i^{10}$ denotes the total imported quantity of good i in 2010. Therefore, $\bar{s}_i^{10}$ is the import share of good i in 2010.

In July 2018, the expenditure weights of the year 2015 become available. Therefore, Destatis revises the Laspeyres indices calculated for January 2015 to July 2018. The revised series uses the year 2015 as price, expenditure and index reference period:

$$I_{\mathrm{L}}^{15 \to t} = \sum_{i=1}^{M} \bar{s}_i^{15} \frac{p_i^t}{\bar{p}_i^{15}}, \qquad t = 1/15, 2/15, \ldots, 7/18. \tag{44}$$

[13] Genesis database (Available at: https://www-genesis.destatis.de/genesis/online (accessed June 2021)).

This new series must be connected to the index numbers computed by the Laspeyres index (43). Direct chaining does not work here, because the comparison period of the Laspeyres index (43) is a month (for chaining it would be January 2015, that is, $t = 1/15$), while the index reference period of the Laspeyres index (44) is the year 2015. Therefore, Destatis introduces the following Paasche index that binds the price level of January 2015 to the price level of the year 2015:

$$I_{\mathrm{Pa}}^{1/15 \to 15} = \frac{\sum_{i=1}^{M} \bar{p}_i^{15} \bar{m}_i^{15}}{\sum_{j=1}^{M} p_j^{1/15} \bar{m}_j^{15}} = \left(\sum_{i=1}^{M} \bar{s}_i^{15} \left(\frac{\bar{p}_i^{15}}{p_i^{1/15}} \right)^{-1} \right)^{-1} = \frac{1}{I_{\mathrm{L}}^{15 \to 1/15}} \, . \tag{45}$$

This Paasche index is more convenient than the Laspeyres index $I_{\mathrm{L}}^{1/15 \to 15}$, because the Paasche index requires the readily available expenditure weights of the year 2015, while the Laspeyres index would require the unknown expenditure weights of January 2015.

Based on the three formulas (43), (44) and (45), Destatis calculates a consistent series of monthly index numbers covering the time span January 2010 to July 2018. To this end, Destatis multiplies the price index numbers of January 2010 to December 2014 compiled by formula (43) by the so-called "Verkettungsfaktor" $[I_{\mathrm{L}}^{10 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15}]^{-1}$ (e.g., Statistisches Bundesamt, 2017, p. 6; Statistisches Bundesamt, 2019, p. 6). Note that the first factor of the "Verkettungsfaktor" is index (43) with comparison period $t = 1/15$ and the second factor is index (45). The Verkettungsfaktor is the reciprocal of the price change between the years 2010 and 2015. The index numbers of January 2015 to July 2018 are directly computed by formula (44). The index reference period of the resulting series is the year 2015, that is, $2015 = 1$.

To rebase this series to the index reference year 2010, we multiply the whole series by $[I_{\mathrm{L}}^{10 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15}]$, that is, by the inverse of the "Verkettungsfaktor". As a result, the index numbers of January 2010 to December 2014 are compiled by formula (43), while the index numbers of January 2015 to July 2018 are compiled by:

$$I^{10 \to t} = I_{\mathrm{L}}^{10 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15} \cdot I_{\mathrm{L}}^{15 \to t} \, , \qquad t = 1/15, 2/15, \ldots, 7/18 \, . \tag{46}$$

For the comparison month $t = 1/15$, the Laspeyres index (43) and formula (46) give $I_{\mathrm{L}}^{10 \to 1/15}$ – see the last equality in (45). Therefore, formulas (43) and (46) generate a consistent time series with the reference price level 2010 = 1. This series differs from the official time series only by a constant factor, namely the "Verkettungsfaktor".

Formula (43) is a Laspeyres index and, therefore, prone to upward substitution bias. Formula (46) is a chain index comprising two upwardly biased Laspeyres indices and a Paasche index. The Paasche index is prone to downward bias. However, the bias is probably much smaller than the combined upward bias of the two Laspeyres indices, because the time distance between January 2015 and the full year 2015 (Paasche index) is much smaller than that between the year 2010 and January 2015 or later months (Laspeyres indices). Therefore, not only formula (43), but also formula (46) is likely to exhibit severe upward bias (this was empirically confirmed in Section 2.4).

When the expenditure weights of the year 2015 have become available to Destatis, it is possible to reduce the substitution bias. To this end, we conduct the retroactive three-stage revision process outlined in Section 2.3. To adapt this process to the methodology of Destatis, only one modification is necessary. It relates to the denominator of the correction factor in Stage 3.

Stage 1: Using formula (44), a new series of Laspeyres indices for January 2015 to July 2018 can be compiled. This part of the revision process is already implemented in the official index compilations of Destatis.

Stage 2: To rebase the new series of Laspeyres index numbers compiled in Stage 1 to the price level 2010 = 1, we compute the following Törnqvist index:

$$I_{\text{Tö}}^{10 \to 15} = \exp\left(\sum_{i=1}^{M} \frac{1}{2} \left(\bar{s}_i^{10} + \bar{s}_i^{15} \right) \ln\left(\frac{\bar{p}_i^{15}}{\bar{p}_i^{10}} \right) \right) . \tag{47}$$

Next, we replace in the chain index (46) the first two links by the Törnqvist index (47) and obtain the following chain index:

$$I^{10 \to t} = I_{\text{Tö}}^{10 \to 15} \cdot I_{\text{L}}^{15 \to t} , \qquad t = 1/15, 2/15, \ldots, 7/18 . \tag{48}$$

For January 2015 to July 2018, this chain index yields more reliable results than the chain index (46).

Stage 3: We want to revise the index numbers of January 2010 to December 2014 compiled by the Laspeyres index (43). The new series of index numbers must be consistent with the revised index number of January 2015 compiled by the chain index (48). Therefore, we multiply the Laspeyres index (43) by a correction factor that is constructed in analogy to the correction factor in formula (41). The result is the following index formula:

$$I^{10 \to t} = I_{\mathrm{L}}^{10 \to t} \cdot \left(\frac{I_{\mathrm{Tö}}^{10 \to 15}}{I_{\mathrm{L}}^{10 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15}} \right)^{\lambda_t} , \qquad t = 1/10,\ 2/10, \dots,\ 12/14 , \tag{49}$$

with

$$\lambda_t = \begin{cases} 0 & \text{for} \quad s = 1, 2, \dots, 12 \\ \dfrac{s-12}{61-12} & \text{for} \quad s = 13, 14, \dots, 61 . \end{cases} \tag{50}$$

For the twelve months of the year 2010 ($s = 1, 2, \dots, 12$) the impact λ_t is 0. Therefore, the correction factor has the value 1, that is, no correction of the Laspeyres index $I_{\mathrm{L}}^{10 \to \mathrm{t}}$ occurs until December 2010. This is intended, because only the expenditure weights of the year 2010 and not the expenditure weights of the year 2015 should be included in the calculation of price changes *within* the year 2010. In January 2011 ($s = 13$) the impact λ_t is equal to 1/49. The correction factor then deviates minimally from 1, leading to a small correction of the Laspeyres index $I_{\mathrm{L}}^{10 \to 1/11}$. As t increases, the counter s and the impact λ_t also gradually increase. Only in January 2015, the counter s would reach its maximum value 61 and, therefore, the impact λ_t its maximum value 1. In this last month, formula (49) would simplify to

$$I^{10 \to 1/15} = \frac{I_{\mathrm{Tö}}^{10 \to 15}}{I_{\mathrm{Pa}}^{1/15 \to 15}}$$

which, in view of the last equality in (45), is identical to formula (48). Therefore, using formula (49) for the comparison months January 2010 to December 2014, and using formula (48) for all subsequent months, generates a consistent time series of price levels.

Quite likely, in 2023 the expenditure weights for the year 2020 will become available. Since the expenditure weights of the year 2020 add no relevant information for measuring the price changes between 2010 and 2015, there is no need for a second revision of the index numbers of January 2010 to December 2014.

The new import price index numbers of January 2020 and all subsequent months (Stages 1 and 2) are compiled by the chain index

$$I^{10 \to t} = I_{\text{Tö}}^{10 \to 15} \cdot I_{\text{Tö}}^{15 \to 20} \cdot I_{\text{L}}^{20 \to t}\,, \qquad t = 1/20, 2/20, \dots\,. \tag{51}$$

The revised import price index numbers of January 2015 to December 2019 (Stage 3) are calculated by the chain index

$$I^{10 \to t} = I_{\text{Tö}}^{10 \to 15} \cdot I_{\text{L}}^{15 \to t} \cdot \left(\frac{I_{\text{Tö}}^{15 \to 20}}{I_{\text{L}}^{15 \to 1/20} \cdot I_{\text{Pa}}^{1/20 \to 20}} \right)^{\lambda_t}, \qquad t = 1/15, 2/15, \dots, 12/19\,,$$

where the impact λ_t is defined as in (50) with $s = 1$ in January 2015. These computations generate a consistent time series of index numbers with the index reference period 2010. The series covers the time interval from January 2010 to 2023 and beyond.

The compilation and revision of the export price index can be conducted in a perfectly analogous manner. The ratio of the revised export price index and import price index of some month t yields the revised terms of trade index of that month.

3 Economic Valuation of Free Digital Products [14]

3.1 Introduction

Over the last two decades a key trend in the development of modern civilization is the increasing role of information and communication technology (further - ICT). One of the consequences of ICT development is the spread and expansion of free digital products.

The number of active users of free digital products increases every year. For example, the number of active monthly Facebook users worldwide nearly doubled in 2021 compared to 2016 (2.91 million versus 1.7 million).[15] These trends pose new challenges for statisticians measuring the contribution of different goods and services to the economy. The main difficulty here is to determine the monetary value and price of the goods and services that can be consumed without monetary payment. Therefore, the present paper addresses the measurement problems, arising in the economic valuation of free digital products.

Such products have some particular characteristics (near-public goods, non-market nature, heterogeneity, networking effect and shareability) that affect the methods used for their economic valuation. We analyse the main existing methods of valuing free digital products. Current research on the economic value of free digital products is mostly based on the use of stated direct and/or indirect methods. We discuss and compare these methods. A common drawback of these methods is their hypothetical nature.

The present paper proposes and explores another possible option for determining the economic value of free digital products in monetary terms, the so-called usage cost model. It is based on realistically observed market behaviour of users. The necessary theoretical framework is presented along with a discussion and analysis of its strength and weaknesses.

The second part of the paper consists of an empirical part. A specific focus group in Germany was interviewed. Based on free digital products such as Facebook, Instagram

[14] I would like to thank Prof. Dr. Ludwig von Auer for all his helpful comments.
[15] Facebook (2021).

and WhatsApp, an empirical application is conducted. The results are analysed and possible problems are discussed.

Overall, the purpose of this paper is to investigate the proposed model and test its applicability for measuring the shadow price and value of free digital products.

The paper is structured as follows. Section 3.2 provides a definition of free digital products and its peculiarities relevant for the research framework. Different theoretical and empirical findings in the literature about the measurement of (free) digital products are discussed in Section 3.3. Section 3.4 provides definitions of value, price and cost. The relationship between them is also shown. Section 3.5 proposes and analyses a novel UCM for evaluating the value of free digital products. An empirical application of this theoretical model to a specific focus group in Germany is provided in Section 3.6. Concluding remarks and some considerations for future research are contained in Section 3.7.

3.2 Free Digital Products: Definition and Features

Digital products represent a relatively new phenomenon that has emerged through the development of ICT and consequently the expansion of the digital economy. There are many definitions of digital products or digital goods/services.[16] According to Hui and Chau (2002, p. 73), digital products "are any goods and services that can be digitized (converted into a binary format)"."Digital products are conceptualized as bit-based objects distributed through electronic channels, i.e. via wired and wireless networks" (Koiso-Kanttila, 2004, p. 46). We follow these two definitions in our paper.

There are also different classifications within digital products. Kling and Lamb (1999, p. 18) distinguish between "highly digital goods and services, that are mostly delivered digitally" and "mixed digital goods and services, which retail tangible goods". Most of the researchers also distinguish traditional goods and services from digital ones. The following characteristics are usually used: physical form/intangibility, heterogeneity, inseparability and perishability (or IHIP framework) (Zeithaml et al. (1985,

[16]In our paper, as well as in many papers in the current literature (Micken et al., 2020, p. 100), the definitions "digital products" and "digital goods/services" are used interchangeably. Other synonyms are "digital content" and "digital offerings" (e.g. Shapiro and Varian (1998, pp. 23;93); Koiso-Kanttila (2004, p. 46); Micken et al. (2020, p. 99)).

p. 34); Koiso-Kanttila (2004, p. 47); Micken et al. (2020, p. 99 - 101)). Near-zero marginal costs are another important peculiarity of digital products.

Some authors argue that digital products are fully intangible, i.e. it is not possible for the digital product to be physically touched. However, digital products can be heard and/or viewed (Koiso-Kanttila, 2004, p. 46). Others also suggest that intangibility does not fully reflect the characteristics of digital products. Because tangible tools and utilities are required to create, store, distribute and use digital products (Micken et al., 2020, p. 99).

Heterogeneity usually means difference. In the case of digital products, one cannot fully speak of heterogeneity, because they are mainly based on some standardized computer code that can be easily reproduced and repeated (Micken et al., 2020, p. 99).[17]

The next property is inseparability, which means a simultaneous production and consumption. This property is typical for traditional services. But not for digital products. Software or other digital product can be produced and used over different periods of time.

Finally, perishability is a characteristic of traditional services, when it is impossible to store or save the result obtained. New storage methods such as public or private cloud storage offer the possibility of storing digital products infinitely.

Another classification of digital products are given by Hui and Chau (2002, pp. 74-75). They subdivide digital products into three categories: *tools and utilities, content-based digital products, and online services*. Tools and utilities are tend to accomplish some specific goals of the users (e.g. online players, antivirus programs, or office programs). Content-based products provide the user with some information (e.g. electronic newspapers, research reports, databases, music or videos). Online services include web-sites or apps/software, which serve as providers of some service. The difference between the first and the last category is that by using online services users do not purchase the product itself but use it to transfer or post some information.

Let us specify what *free* digital products are. They can be accessed and used without any monetary payments. The above-mentioned classification of digital products

[17] However, in our opinion, this is true when looking at a digital product in terms of production/cost. If we consider digital products from the customer's point of view, then they, on the contrary, are very heterogeneous. There are different product categories that cover different consumer needs.

may also be applied to the case of free digital products. This classification is a classification *in terms of product category*. The first category, *free tools and utilities*, comprises different freeware that can be downloaded and used free of charge (e.g. Avira - completely free antivirus program, Handbrake - the open source video transcoder etc.). The second category is *free content-based digital products*. Social-media platforms (Facebook, Instagram etc.), education/research (World Bank Databank, OECD Online Library, Wikipedia etc.), entertainment (Spotify etc.) are examples of the second category. WhatsApp or Skype can serve as examples of *free online services*.

The second useful dimension of the classification is *in terms of economic characteristics*. Free digital products have the following economic characteristics:

- *near-public goods or quasi-public goods*. Pure public goods are normally characterized by the two main features: non-rivalry and non-excludability. The former means that the product can be consumed by all customers without diminishing its value. The latter means that all people may use it without exclusion of another person or group of people. Near-public goods have semi-non-rivarly and semi-non-excludability features. On the one hand, free digital products satisfy the characteristic of non-rivarly due to near zero marginal costs of its usage. All free digital products may be used without any limit. However, if too many people want to use a free digital product at the same time, there may be an overload such that some users will not be able to access the product, and therefore competition in consumption will arise. On the other hand, the feature of non-excludability also applies. Users may use free digital products without any restriction. However, in order to access this free digital product one need to have a general access to the Internet and this requires some payment. If this fee is too high for a particular group of users, these users may not be able to use it. Moreover, it is technically possible for the suppliers of the specific digital products to exclude certain groups of people. But they have no incentive to do this.

- *non-market nature*. Demand for digital products is not directly observable as in the case of traditional market products. There is no obvious market price for free digital products. This is a characteristic they share with environmental goods.

- *quasi-heterogeneous nature*.[18] From a production/cost perspective (i.e. from a supplier point of view), free digital products are quite homogeneous. All are based on some standardised computer code and may be easily reproduced (Micken et al., 2020, p. 99). From a consumer perspective, they show a high degree of heterogeneity and may be divided into a lot of subcategories. Consequently, their economic value from the consumer's point of view, may be measured in different ways. For example, to measure the economic value of a free product such as Handbrake, one may record the number of downloads and the time spent on the software. While evaluating the use of social media platforms also requires not only the time spent on these platforms, number of visits but also information such as the value of personal data that users share with social media.

- *networking effect and shareability* of free digital products. Network effects are inherent in all digital products. The more they are accessed and used, the more valuable they become for their users.[19] Shareability (i.e. the capability of being shared) is applicable more to free digital products (they create the ideal environment for app/content to be distributed to users as much as possible). These features of free digital products may influence the slope of the demand curve of the access/usage of such products. It has been shown in the literature that the presence of network effects[20] in that products may lead to unconventional demand curves, can be represented by an inverted U-shape (e.g. the case of digital telecommunication goods, see Baraldi (2008), Rohlfs (1974)).

[18]In this paper, quasi-heterogeneous products mean that they have characteristics of both homogeneous and heterogeneous products.

[19]Buxmann (2002), Linde et al. (2012) conclude, that the more users use the product, the stronger the network effects.

[20]The literature usually differentiates between *direct and indirect network effects* (Linde et al. (2012), (Baraldi, 2008, p. 5). *Direct network effects* mean that the value of digital products increases the more users use the product. Social-media platforms are typical ones, having this characteristic (Facebook, Instagram). *Indirect network effects* mean that the value of a digital product increases as the number of additional, complementary goods and services of that digital product increases. Typical examples here are the telecommunications mobile network and the online messenger market (e.g. WhatsApp, Viber, etc.). The more users have WhatsApp, the more users will have to buy a sim card with a phone number to use WhatsApp and vice verse. Network effects are also called in the literature as "demand-side economies of scale". A detailed information about the nature and types of network effects in case of digital products one may find in Linde et al. (2012).

Based on these characteristics and features, many methods for assessing the economic value of free digital products have been applied and compared. The main theoretical and practical results are discussed in the next section.

3.3 Economic Valuation of Digital Products

3.3.1 Methods for Economic Valuation of Non-market Products

Various methods can be used for the monetary valuation of products for which there is no clear market price. Some of these methods will be considered in this subsection. The table 4[21] below gives a short overview and relationship between the two major groups of the methods (direct/indirect and stated/revealed):

Table 4: Methods of valuing non-market goods/services.

	Direct	**Indirect**
Revealed	Parallel private markets Simulated markets	Hedonic pricing Travel cost method
Stated	Contingent valuation Spend more-same-less survey questions	Conjoint analysis Choice modelling (discrete choice experiments) Experimental auctions of accesing WTA/WTP category

Discrete choice experiments are one of the possible methods to determine the consumers willingness-to-accept (WTA) and willingness-to-pay (further - WTP). They are also more often considered as methods to estimate the monetary value of non-market products. In this paper, this group of methods is identified as the group of *indirect methods* of economic valuation. Another group is called *direct methods* and includes methods that allow a direct observation of the value of non-market products (e.g. competitive market prices or contingent valuation method with open-ended format of questions) (National Research Council, 2005, p. 101). The division into direct and indirect methods is based on the nature of the data collected. *Direct methods* assume that the economic assessment is based on a direct survey of potential customers/users and their response, how they evaluate a particular product. *Indirect methods* consist of

[21] Own creation based on Sudman et al. (1989, p. 75).

a mixture of survey data and other relevant variables/attributes for the assessment, as well as only such variables (e.g. time spent accessing the product, time spent using the product, other possible costs regarding the accessing/using the product).

The other classification which is used in the literature is the division of all valuation methods of non-market products into *stated and revealed preference methods* (Sudman et al. (1989), Freeman et al. (2003), Baker and Ruting (2014), OECD (2018), NZIER (2018)). The *stated preference technique* investigates and determines the value of non-market products by asking people directly how much they value a particular product (Baker and Ruting, 2014, p. 24). Three main approaches here are *contingent valuation method*, *conjoint analysis*, *choice modelling* (NZIER, 2018, p. 14). *Revealed preference technique* investigates and determines the value of non-market products by analysing the consumer behaviour associated with the use of the product (NZIER, 2018, p. 12). This group of methods includes methods such as travel cost and hedonic pricing methods. An important difference between stated and revealed methods is the market type that is used for research. In the case of the stated preference technique, the whole process takes place in a simulated market: consumers answer some hypothetical questions. The revealed preference technique analyses actual market behaviour. All of the above-mentioned methods are applied to the non-market products and originally come from environmental economics. The popularity of these methods, however, makes it possible to introduce them into other areas of the economy (e.g. healthcare, transport and, more recently, the information economy and its digital products).

Contingent valuation method (further - CV) is a stated direct preference method that is based on a survey or questionnaire of people's responses to a hypothetical situation. The idea was introduced by Ciriacy-Wantrup (1947). The first empirical application of this method was made by Davis, R.K. (1963) in his dissertation (Smith (1997, p. 3); Richard T. Carson and W. Michael Hanemann (2005, p. 830)). CV usually asks the respondents about the maximum amount of money they are able to pay for a non-market product. The construction of the questions may differ. The following formats are possible to use: open-ended type survey questions, bidding games, payment card, single-bounded dichotomous choice formats and double-bounded dichotomous choice formats (OECD, 2018, pp. 95-99). One of the main advantages of CV method is its flexibility. Both existing and non-existing products can be assessed. Moreover, different kinds of values are possible to evaluate (use/non-use values). At the same time, the CV method has many critics in the academic literature. The following biases are usually mentioned (Meyerhoff, 1999, pp. 48-56):

- Hypothetical bias. This kind of bias reflects the hypothetical nature of surveys. As a result, individuals more often as usual tend to overestimate their value of a non-market product.
- Strategic bias arises if the respondent does not respond honestly and behaves in accordance with a certain strategy in order to subsequently gain a product advantage. A typical example of such behaviour is "free rider" problem.
- Payment vehicle bias means that the result may depend on the form/instrument of the payment that is assigned from the beginning (e.g. taxes or donations).
- Starting point bias. The initial amount of money may strongly influence the final result of the product evaluation.
- Part-whole-bias or Insensitivity to scope, which means that the respondents are insensitive to the changes in the scope of the product to be valued. They are not ready to pay more for more products. Their valuations do not vary significantly with changes in the scope ((OECD, 2018, pp. 106-107)).
- Information influence or framing bias. The results of the survey depend on the quality of information provided about the product. Information about product characteristics, product substitutes or other relevant information that gives more insight into the product has the potential to change the value of the product or how respondents perceive its value.

To eliminate some of the biases, one may also apply some laboratory experiments in the form of experimental auctions (iterated, second-price, sealed-bid auctions and group consensus mechanisms (Gregory and Furby, 1987, p. 275)).

Conjoint analysis (further - CA) is an example of the indirect stated preference methods. CA is based on the direct survey as CV method. The distinguishing feature of the CA is that the respondents have the possibility to choose among the alternatives with different characteristics and/or costs. Compared to the CV method, where the product is assessed as a whole, the CA method evaluates the product in terms of its attributes or characteristics that compose the product and influence it. The value of the product consists of the value of its components. Therefore, this method belongs to indirect methods. Smith (1997, pp. 53-54) highlights two features that distinguish the CA method from the CV methods. The first one is that a "specific reference situation is given and often the adjustments are made to non-monetary components of each outcome/scenario". The second one is "how the indifference judgment is made to recover measures of economic values".

The next popular approach of the stated preference technique is the *choice modelling* (further - CM). However, some researchers refer this method to the conjoint analysis group (OECD, 2018, p. 15). The distinguishing feature of the CM is its multi-attributiveness. The attributes of the product and costs related to it are described precisely. The most popular method inside the group of *choice modelling* group methods is the *discrete choice experiments* (further - DCE). This is the method most commonly used now to evaluate the value of free digital products.

CA and CM share with CV method the pros and cons. All methods are flexible and able to estimate future changes as well as non-use values (OECD, 2018, p. 139). The fragility of CA and CM lies precisely in the product attributes specified in the survey: more detailed product information provides more reliable results and an estimated value of WTP (Su et al., 2017). At the same time, the more complex the alternatives among which respondents can choose, the more difficult it is for respondents to make objective choices. In addition, not all potential product attributes may be included, and this leads to the bias in the final results of the estimated WTP.[22]

Revealed preference methods describe not a hypothetical market but real people's behaviour and based on real market data. *Hedonic pricing* is one of the widely used methods in this group. This method decomposes the product's price into a number of its attributes/characteristics that could potentially affect the final price of the product. For example, Byrne et al. (2018) applies this method to the case of cloud computing. The prices of cloud computing services of the largest providers as well as the characteristics (the amount of memory, the amount of disk storage in GB etc) which may

[22]More strengths and weaknesses of each of the methods one may find in Sudman et al. (1989) and OECD (2018).

potentially influence these prices are collected and analysed. However, this method is more difficult to apply for non-market products which do not have the market price. In addition, it is difficult to find any traded product that could serve as a potential replacement for a non-traded product and have a good explanatory power Gregory and Furby (1987, p. 274). Other weaknesses of the hedonic pricing technique include the difficulty of collecting reliable market data, difficulties in the determination of the attributes of the product etc. (Sudman et al., 1989).

Travel cost method (further -TCM) is another example of the revealed preference methods. This method measures the value of a non-market product by evaluating the people's visitation rate, including the time spent on visits and the costs associated with using the product. TCM determines not only the value of the product from the consumer's point of view, but also the implicit price of a non-market product. This method is primarily used for some recreational travel activities as well as cultural events. Information on visits is usually collected through surveys. One of the main disadvantages of TCM is that it estimates only the use value of the product and not the other types of values, in particular a non-use value. Moreover, the inclusion of the time costs is also a highly controversial variable and is the subject of ongoing discussion in the scientific literature. Although the inclusion of time in the model is controversial, it is also crucial for an adequate assessment of the value of the product, especially the digital one. That is why this article also considers time costs variable as one of the variables determining the value of free digital products.

3.3.2 A Review of the Results of Estimating the Economic Value of Free Digital Products from the Literature

The digitalisation and rapid growth of various digital products have a major impact on consumer behaviour and the economy as a whole. For the compilation of official statistics, two major challenges arise: How to measure the contribution of digitalisation to gross domestic product (GDP) and how to compute the consumer price index, which in turn measures inflation and is used to calculate real GDP. For a solution one can use the micro-data of consumers. They allow for an evaluation of the welfare gains consumers receive from the use of digital products. The first theoretical and empirical investigations start from the estimating of the demand and value of the priced digital goods and services for consumers, such as information technology hardware (Brynjolfsson, 1996), broadband access to the Internet vs. cable (see e.g. Rappoport et al. (2003), Goolsbee and Petrin (2004)). In particular, Brynjolfsson (1996, pp. 283-287) identifies three possible methods of evaluating the value of the IT hardware. These methods include the determination of consumer surplus derived from demand. Pricing of data traffic, including other services, such as mobile, fixed-line and/or broadband data plans or cloud computing are also of interest for research (e.g. Zhang (2016), Byrne et al. (2018)). The determination of the price, demand, and value of these priced digital products (including also partially priced or so-called "freemium"[23] products) is usually based on certain standard economic (neoclassical model of pricing) and econometric methods (tobit-logit model; hedonic regressions etc.).

Recently, however, research is predominantly concerned with free digital products. Post (2009) is one of the pioneers to extend the neoclassical model of pricing to the case of digital products, including also free digital products. An economic model for pricing digital products is based on several assumptions: the goal of the firm is to maximize its profit and the consumers tend to maximize their utility; very low or zero marginal costs of producing digital products; uniqueness and heterogeneity of digital products (Post, 2009, p. 1). Post (2009) defines that the final price of the digital product depends on the price itself (it can be also negative or zero), advertisement fees and marginal costs of production. Advertising is one of the dominant factors in determining the final price, the consumed amount and the demand curve for the product. The revenue that producers receive from advertising is one of the easiest ways to estimate the value of free digital products (Ahmad et al., 2017, p. 26). However, advertising revenue alone does not show the full benefit of free digital products to consumers. Other indicators must be included as well, such as the time spent using this product (Brynjolfsson et al., 2020, p. 27) and/or the data that consumers disclose in order to access various digital products (Ahmad et al., 2017, pp. 26-32).

Another way to assess the value of free digital products is to analyse the time usage of these products. For this purpose, Brynjolfsson and Oh (2012) developed a model for estimating the value of free digital services on the Internet with the inclusion of

[23] "Freemium" products are digital goods and services that at the beginning of their use must be provided without subscription fees or payment, but have limited possibilities in comparison with the premium version (Belleflamme, 2016, p. 8).

a time variable.[24] In the case of free digital products, the most important variable determining its value will be time ("user attention").

The other example of the economic approach to the determination of the price of free digital products is to apply the standard index number theory, in conjunction with Hicksian reservation price methodology (Brynjolfsson et al. (2020) and Diewert et al. (2019)). The authors use this methodology[25] to measure the consumer benefits of free digital products. The same methodology they use also for completely new and/or new free products. The final reservation price for free products is derived as "twice the estimated compensation price from the choice experiment" (Brynjolfsson et al., 2020, p. 29). Finally, these determined prices for free (new) goods, classified as so-called shadow prices, can be included in the cost-of-living index (using superlative price indices) or the consumer price index. However, the relevance as well as the need for such an inclusion, is controversial aspect and causes a lot of discussion in the literature but is not in focus of this paper (see e.g. Reinsdorf and Schreyer (2019, pp. 13-16)). The main drawback of the Hicksian reservation price methodology are certain assumptions which must be met.[26]

Returning to the main focus of this paper, in order to assess the value and price of free digital products, one may name the following, already mentioned, possible method: Choice experiments. Some of the pioneers in applying this method to free digital products are Brynjolfsson et al. (2019a,b).[27] Moreover, for the first time, they use massive online discrete choice experiments that allow them to reach many more potential users of free digital products in real time. In their work, they Brynjolfsson et al. (2019a, pp. 24-45) estimate not only consumer welfare of digital products, but also expand their evaluation by determining their contribution to GDP.[28] The exper-

[24] Other works, which estimate the value of the Internet using a time variable are presented by Goolsbee and Klenow (2006) and Pantea and Martens (2014).

[25] The reservation price is the price of the (new) free product which is available in the periods $1,...,T$ but not available in the period 0. The demand for such a product in the period 0 is equal 0 (Brynjolfsson et al., 2020, p. 26). The absence of a product or a product that is too expensive, resulting in zero consumption and therefore zero demand for it, are two equivalent concepts from a welfare viewpoint (IMF, 2018, p. 21).

[26] E.g. latent preferences that can also imply the same utility function. This may not always be the case (Brynjolfsson et al., 2020, p. 27).

[27] Another example of applying method of choice experiments to a free digital product is the work of Sobolewski (2021). Free digital maps and navigations systems are to be evaluated here from the consumer's point of view.

[28] By using two approaches: Total income approach (for more information, please see Brynjolfsson et al. (2019a, pp. 22-23)) and reservation price methodology

iment is conducted for two countries: USA (only Facebook, based on real representative sample of the US internet population) and the Netherlands (Facebook, Instagram, Snapchat, Skype, WhatsApp, digital Maps, LinkedIn, Twitter; estimations are based on the laboratory results). The results for the USA are as follows (Brynjolfsson et al., 2019a, pp. 24-31):

- the median willingness-to-accept (further - WTA) price for not using Facebook for one month is \$42.17 $\approx$ €34.76,
- the impact of Facebook on the real GDP growth is 0.11 percentage points on average per year from 2004.

The results for the Netherlands are the following (Brynjolfsson et al., 2019a, pp. 32-39):

- the median willingness-to-accept (WTA) price for not using Facebook for one month is €96.80, for not using WhatsApp is €535.73 and Instagram €6.79,
- assuming a user-population of 10 million, the impact of Facebook on the real GDP growth is 0.5 percentage points on average per year, the impact of WhatsApp is 4.10, Instagram is 0.07 for the periods 2004-2010.

There are also other studies evaluating the monetary value of Facebook. In particular, Herzog (2018) finds that the value of Facebook is €28.26 per week.[29] The discrete choice methodology is also applied here. Another example of determining the value of Facebook is presented in Mosquera et al. (2019). The results are based on a field experiment using an incentive-compatible procedure developed by Becker et al. (1964). They find that one week of Facebook is worth \$67, with a medium value of \$40 (Mosquera et al., 2019, pp. 577, 584-587). Corrigan et al. (2018) on the basis of experimental auctions (Vickrey second-price approach) access the WTA category. It shows how much money in exchange for giving up Facebook over a certain period of time bidders are willing to accept. The mean WTA per week is worth \$38.83, with a median of \$15.00.[30]

[29]The sample consists of 80 respondents and 14 nationalities, 50% of whom are Germans.

[30]The sample consists of one hundred twenty-two Facebook users on the campus of a Midwestern liberal arts college (Corrigan et al., 2018, p. 4).

We can see how other characteristics of free digital products influence their value and the determination of such definitions as value, price and costs in the following section 3.4.

3.4 Value, Price and Cost: Differences and Relationship

"The need of a clearer, more consistent, and more generally accepted terminology in economics is felt by all economists today"

Frank A. Fetter "The Definition of Price", Dec. 1912

When selecting the method to be used, terms such as value, price and cost should first be defined. Let us start with a short history of value and price, explaining briefly how they were used and what role the term "cost" plays here (Table 5 gives a short overview of the most important points). Then we show the difference and relationship of them and apply them to the case of free digital products.

Over time, the understanding and definition of value and price has constantly changed, with no obvious separation for a long time. In ancient times, theories of value and price did not yet exist, but there were early attempts to recognise their role. So, with the development of Roman law, the concept of the "just price"[31] gets a legacy. The roots of the "just price" go back to the Greek and Italian philosophers: Aristotle and Aquinas (Elder-Vass, 2019, p. 1486; Jaffe and Lusht, 2003, p. 7). Aristotle, however, did not introduce the term of the "just price", but explained the importance of the just exchange: The exchange is just if it is proportionate equal (Aristotle, 2009). Consequently, "the value is expressed by the proportion in which things exchange for each other" Sewall (1901, p. 2). And money, serves as an intermediate that measure all things, "makes goods commensurate and equates them" (Aristotle, 2009). As Sewall (1901, p. 2) mentions "Money price is the expression of the value". That is why we see here how value and price correlated with each other at that time and were almost synonymous. The uncertainty and ambiguity of the distinction between price and value persisted also in Roman times Sewall (1901, p. 3).

[31] The "just price" is based on the ethical norms, such as "justice", "fairness". It is the "price, corresponding to the true value of the object" (Kaulla, 1940, p. 35). Depending on the philosopher and/or economist, the just price definition may be interpreted a bit differently.

Table 5: Development of the definitions of “value”, “price” and “cost” over time.

Time	Authors	**Interpretation of definitions**
Ancient times (3000 BCE (before current era) - 500 CE (current era))	Roman Law, Greek and Italian philosophers (Aristotle, Aquinas etc.)	value = price; the goal - justification of the price (value); the term “cost” was not determined.
Middle ages (500 - 1500 CE)	J. Locke, J. Law, J. Steuart	value ≠ price; the goal - explanation of the value and price; exchange value (price) and natural value (value); value is based on the costs required to produce a commodity.
Early modern time period (1500 - 1815 CE)	W. Petty, A. Smith, D. Ricardo	“objective” theory of value, based on the cost approach; “value in exchange” and “value in use”; value is the “natural price”, which may deviate from its market price. Costs determine the “natural prices”.
Late modern time period(1815 - 1945 CE)	K. Marx, W. Jevons, F. von Wieser, C. Menger, L. Walras, A. Marshall	value ≠ price; “objective” and “subjective” theories of value; combination of the two theories.
Contemporary time period (1945 - now)	A. J. Jaffe, K.M. Lusht, RICS, USPAP	a clear distinction of terms and interpretation of definitions according to economic area and type of the product

In the Middle Ages, the need to develop a theory of value and its definition increased. Some early attempts were made to link the value of a product to the amount of labour required to produce it (e.g. A. Magnus, Aquinas) (Sewall, 1901, pp. 13-37; Jaffe and Lusht, 2003, p. 11). In the post-middle ages, the focus shifted from justification to explanation of value. One of the first economists to contribute to the development of the theory of value was W. Petty. He distinguishes between two kinds of value: exchange value, or price, and natural value. The exchange value, or price is called then as extrinsic or accidental value. The natural value or intrinsic value is determined by the amount of labour and land required to produce the product (Sewall, 1901, pp. 70-74). The important conclusion is that the natural value may not be equal to its extrinsic value, i.e. its price. So, we see that price and value are beginning to separate. Other economic writers, who distinguish value and price, are J. Locke, J.

Law and J. Steuart. J. Steuart viewed value as the intrinsic value of a commodity, based on the costs required to produce it. In contrast, price is determined by market forces (Jaffe and Lusht, 2003, p. 21).

The cost approach to the definition of value was continued by classical economists such as A. Smith and D. Ricardo. Such a theory of value is called in the literature "*objective*" theory of value (King and McLure, 2014). First of all, A. Smith distinguishes between "value in use" and "value in exchange".[32] Therefore, A. Smith makes suggestions in the direction of the "subjective" concept of value. Secondly, he makes labour the base of value of goods and argues that this is "the only universal, as well as the only accurate measure of value, or the only standard by which we can compare the values of different commodities at all times, and at all places" Smith (1776, Chapt. 5, Book 1). This is the kernel on which the natural real value or price is based. In the times of capitalism, however, A. Smith proposes to define the natural price of a good using the adding-up principle. The natural price or value consists of the sum of the costs not only of labour, but also of the cost of land and capital needed to produce the commodity.[33] He admits that the market price may deviate from its natural level. Hence, A. Smith also distinguishes between natural and market prices. Moreover, the term "price" is based upon the term "value", whereas both terms refer to the term "cost".

D. Ricardo elaborated a more systematic and expanded theory of value. At various stages of his work he confirms and criticises the views of A. Smith. But in the end Ricardo follows the assumption that the cost of production determines the natural prices of goods rather than the interplay of supply and demand (Jaffe and Lusht, 2003, pp. 26-28), (Bloch, 2020, p. 60). Moreover, with regard to the value of goods, he states that "the ratio between the direct and indirect labour requirements[34] of two commodities determines their relative value" Signorino (2011, p. 3).

[32]"Value in use expresses the utility of some particular object", while "value in exchange the power of purchasing other goods which the possession of that object conveys" Smith (1776, Chapt. 4, Book 1).

[33]"When the price of any commodity is neither more nor less than what is sufficient to pay the rent of the land, the wages of the labour, and the profits of the stock employed in raising, preparing, and bringing it to market, according to their natural rates, the commodity is then sold for what may be called its natural price" (Smith, 1776, Chapt. 7, Book 1).

[34]"Not only the labour applied immediately to commodities affect their value, but the labour also which is bestowed on the complements, tools, and buildings, with which much labour is assisted" Ricardo (1817).

K. Marx is another representative of "objective" theory of value. According to K. Marx "Value of any article is the amount of labour socially necessary, or the labour time socially necessary for its production" Marx (1887). A detailed review of K. Marx's work is not the purpose of this paper. However, one should mention, that he distinguishes between value and prices. Focusing on the capitalist era, he assumes that "prices are symbols of the relative worth of commodities and assume a money form" (Nicholas, 2011, p. 29). Price "is the value expression in money; ... is like relative value in general, expresses the value of a commodity" (Marx, 1887). As Elder-Vass (2019, p. 1487) mentions: "Marx did attempt to find a way to reconcile his theory of value with a theory of price".

Another interpretation of the terms "value" and "price" is presented by the school of marginalists. They tend to see the term "value" as part of a "*subjective*" theory based on the utility of the product to the individual. The predecessors of the marginal utility school are F. Galiani, Lloyd, Senior, Dupuit, Say (Jaffe and Lusht, 2003, p. 37). W. Jevons, F. von Wieser, C. Menger and L. Walras are among the main representatives of marginalism, who shifted the focus of the cost-based value concept to the utility value concept.

W. Jevons (1888)[35] in his work "*The Theory of Political Economy*" makes a statement that "value depends entirely upon utility". W. Jevons formulates the relationship between cost, value and utility as follows: "Cost of production determines supply. Supply determines final degree of utility. Final degree of utility determines value". C. Menger determines the term "value" from the subjective point of view as well:"Value is thus the importance that individual goods or quantities of goods attain for us because we are conscious of being dependent on command of them for the satisfaction of our needs" Menger (1976, p. 115). As Elder-Vass (2019, p. 1488) concludes, marginalists equated the terms value and price, treating them as synonymous.

[35]"*Prevailing opinions make labour rather than utility the origin of value; and there are even those who distinctly assert that labour is the cause of value. I show, on the contrary, that we have only to trace out carefully the natural laws of the variation of utility, as depending upon the quantity of commodity in our possession, in order to arrive at a satisfactory theory of exchange, of which the ordinary laws of supply and demand are a necessary consequence. Labour is found often to determine value, but only in an indirect manner, by varying the degree of utility of the commodity through an increase or limitation of the supply.*"

Finally, A. Marshall, as a neoclassical economist, seeks to combine objective and subjective theories of value. He integrates the results of marginalists into the general theory of demand and supply (Jaffe and Lusht, 2003, p. 40).

After a brief historical overview, we see that the definition and interpretation of both terms "value" and "price", played a huge role and were often very related to each other. It is very difficult to extract and present any unified definition of value, price and cost. F. Fetter (1912) analyses a variety of definitions of the term "price" and finds about 117 definitions as of 1912. He identifies four groups of definitions according to their relationship to value. Table 10 in the Appendix F gives a short overview of these various types of definitions. Jaffe and Lusht (2003, p. 5) also confirm this conclusion with regard to the concept of value and value theory: "The subject of value is so broad that it is virtually impossible to harness the entire scope of all literature on the topic, despite our best efforts".

Also different economic areas interpret the terms "value" and "price" differently (e.g. a special interest to the terminology exists in the appraisal practice[36]). The term "value" in modern practice apparently identifies mainly a subjective point of view and reflects the worth of a product according to the opinion of different people.

Considering the nature of free digital products, it is important to provide clear definitions of the terms "value", "price" and "cost" correctly, taking into account their characteristics and modern perspective. The present paper defines these three terms as follows:

[36]Three main value variations are used in the real estate world (Jaffe and Lusht, 2003, pp. 46-47):

- Value as price at which property is sold
- Value which reflects the market value of the property
- Value in terms of a subjective value to owner.

The Royal Institute of Chartered Surveyors(RICS) in the United Kingdom (RICS, 1996; Jaffe and Lusht, 2003) defines the first type of value as "Price" - is the actual observable exchange price in the open market; the second type as "Market value" - is an estimation of the price that would be achieved if the property were to be sold in the market; the third type is specified as "Worth" and reflects the perception of the capital sum that investor would be prepared to pay (or accept) for the stream of benefits that he expects to be produced by the property. According to the US Uniform Standards of Professional Appraisal Practice (USPAP) Appraisal Foundation (2020, p. 5): "value" is "the monetary relationship between properties and those who buy, sell, or use those properties, expressed as an opinion of the worth of a property at a given time.", "price" is "the amount asked, offered, or paid for a property". Finally, "cost" is defined "as the amount required to create, produce, or obtain a property".

- **Price** is the total amount of money a company demands from its users in the marketplace when providing a particular digital product. The implicit name of price is *the market price*. In the case of free digital products the market price is zero.

- **Value** is a monetary expression of an opinion on the worth of free digital products. From one side, value may be calculated on the basis of all costs regarding the access and usage of free digital products. Another interpretation of value is that it is a monetary expression of willingness to pay or accept some amount of money in order to access/use the free digital product by the user.

  This total amount of money can vary from person to person and depends on various factors. These can be:

  - objective or countable characteristics, such as e.g. a subscription fee (in case of free digital products it is equal zero), wage levels, total costs of Internet access etc.
  - subjective factors, such as the time spent using the digital product and evaluating it, some personal interest in the product, or the level of development of the networking effect/shareability etc.

  In the present paper, value is defined from the consumer's point of view, i.e. in terms of the consumer surplus. Consumer surplus is defined as the difference between the willingness to pay and the actual market price (payment).

- **Cost** is the amount of resources (tangible or intangible) needed to access and use a free digital product. Costs in the present paper are defined as "usage costs", and consist of four components: adjusted subscription fee, opportunity costs of time, attention factor and the value of personal data. These components will be defined more detailed in the next sections.

In the case of free digital products, usage costs determine a **shadow price**[37] for the product. It serves as a price for our users, what they are willing to pay to use this product.

[37] A shadow price is defined in our paper as the approximate economic price of free digital products based on their costs and/or value, that have no real market price.

Facebook may serve as a good example of a free digital product, and help with the interpretation of the definitions "price", "value", and "cost". We do not have to pay anything for the use of Facebook on the market. That is why, the price (market price) of Facebook for consumers is zero. The value of Facebook is, however, greater than zero. Value may be based on the cost approach. Possible costs of the access and usage of Facebook include the following objective and subjective characteristics: Total Internet access costs adjusted by the number of MB/GB spent on Facebook website/app; time spent using Facebook; personal data that users provide to Facebook during registration and use; attention to advertisements, which trigger additional clicks and lead to the disutility from unfulfilled newly created desires.[38] In this way, the costs serve as a shadow price for the user and represent the consumer's willingness to pay for Facebook, based on which the user determines the value of the product. Hence, overall, the value of Facebook can be roughly converted into the monetary form.

That is why, the price may be equal to, greater than or less than the value. Conversely, the value may be equal to, greater than or less than the price. A shadow price of the product is determined by the usage costs and represent the value of the product. However, costs may or may not fully justify prices and values. Other factors may also play a role in defining these terms. It also depends on the method used to determine price and value (subjective/objective theory). In case of digital products, the typical pricing strategies are: for free, freemium, subscription model or dynamic pricing (for additional information see (Krämer and Kalka, 2017)).[39] The pricing strategy for free digital products is called "for free" (Krämer and Kalka, 2017) and aims to create value for consumers. However, this does not mean that the company is making a loss with this pricing strategy, i.e. by setting the price at "zero level". The profit comes from charging third parties. Another two possible aims of this strategy, as mentioned by Krämer and Kalka (2017) are creation of a new market and the usage of the product as an integral part of the product range. Hence, the price in the case of free digital

[38]When scrolling the page in Facebook, Facebook always shows some targeted ads based on your interests. At some point this can overwhelm the users, especially if they have already bought that product and have no interest in it. See also the discussion about this in Fraumeni (2019, p. 347).

[39]There are a lot of pricing strategies that currently exist on the market, depending on the market itself and its features. E.g. cost-plus pricing (probably the oldest and most classic strategy, based on cost plus mark-up); customer-driven or value-based pricing, which generally sets the price according to the value of the product to the customer; share-driven pricing, based on the share of the company on the market and many others (Nagle and Mueller, 2005).

products is zero, i.e. has a value of zero. But this does not mean that the value of a free digital product is zero. Moreover, from the user's point of view, even with "zero price", the final price and value of using this product may not be completely zero. Some costs associated with the use of digital products may not be directly observable, or may be tangible or intangible. Although the market price of free digital products is zero, the implicit price and the value of the product may be higher than zero.

One of the purposes of this paper is to determine the so-called shadow price and the value of free digital products, using a usage cost model. Further explanation of this method can be found in the following section 3.5.

3.5 Usage Cost Model

3.5.1 A Short Overview of the Travel Cost Method

A common feature of natural assets and free digital products is that they are non-market products and have characteristics of public goods. As mentioned in the previous sections, there are a lot of methods and techniques used to assess the economic value of such products. One of such economic valuation approaches is the Travel Cost Model (TCM). The main feature of TCM is that travel and time costs represent the cost part and form the "implicit" price of visiting a site, which affects the value, or monetary willingness of the consumer to pay for visiting that site. This willingness to pay can therefore be estimated based on the number of visits, travellers make based on their level of expenditures. TCM has a count-data nature in terms of the dependent variable (integer variables), that is why count data models are used for the estimation.

The idea of TCM was initially proposed by Hotelling in 1947 in his Letter to the National Park Service (Hotelling, H., 1947; Smith, 1989). The aim of his proposal is to determine the value of national parks depending on their visitations rate. Hotelling emphasizes so-called "zones" (areas, which are grouped, depending on the distance to the park/travel place). The further away the travel object is, the more expensive it is for the consumer (group of consumers of one particular zone) and, consequently, the more travel costs has the consumer. Those, who are close enough to the travel object, receive a consumer surplus, when visiting the object. The demand curve has a downward slope. A travel good is assumed to be a normal good with a fundamental relationship between price and quantity consumed. The more expensive the good, the less it will be consumed. Hence, this model considers the quantity of visits to one or another place as the quantity demanded and, the travel costs - as the "price" of visits. Later, Clawson and Knetsch (2011)[40] in 1967 independently developed and applied this method, which nowadays has the name of "zonal travel cost method" (ZTCM). Today, there are two main varieties of TCM: Zonal TCM (ZTCM) and Individual TCM (ITCM). Instead of using average data of some particular group of population (zone), ITCM (Burt and Brewer, 1971; Špaček and Antoušková, 2013) is dedicated to the characteristics of the individual demand. There are many extensions of ITCM, such as those that include not only travel time, transportation costs, but also socio-economic characteristics (Špaček and Antoušková, 2013).

3.5.2 Introduction to the Usage Cost Model

The proposal of this paper is to modify, adapt and apply the principle and mechanism of the TCM to the case of free digital products. The modified model is denoted as usage cost model (UCM). We change the first word from "travel" to "usage", because of the specifics of digitization. When visiting a place of rest (park, lake, etc.) consumers bear the costs associated with the trip, i.e. transportation costs (distance and cost of fuel, time required for travel). When accessing a free web-page, search engine or app, consumers pay the costs associated with usage itself. These costs consist of a subscription fee for Internet access, the time taken to access/visit a free digital good/service, the value of their personal information given to one or another free digital product and the attention to undesired information (e.g. the adverts that create unfulfilled desires).

The objective of the UCM is to determine a consumer demand for free digital products, to define and to calculate the costs of using a free digital product and their "implicit" prices, to identify the consumer surplus and consequently to assess the value of such products.

The basic principle of the TCM is applied to the UCM. Based on the cost/benefit analysis, the cost side is represented by the usage costs, which users incur to visit a free digital product. The usage costs are the "implicit" price of visits. The benefit side is determined by the willingness to pay to visit a free digital product, which can be

[40]The second edition, originally was published in 1967.

measured by the number of sessions (visits) to the free digital product that users make at different levels of usage costs. The number of sessions (visits) is considered as the quantity demanded. So, why was the number of sessions (visits) chosen as the quantity demanded, and as the dependent variable? First of all, the number of sessions (visits) is considered as an important analytical measure of all digital products.[41] As Spintech (2015) notes: "It is a fundamental measurement of digital product's reach and growth". Google (2021) defines session (alternative name visit) as follows: "A session is a group of user interactions with your website that take place within a given time frame. For example a single session can contain multiple page views, events, social interactions, and e-commerce transactions". The number of sessions helps to understand a true usage of digital products (Klipfolio Inc, 2021). It also shows how "sticky" the digital product is (Spintech, 2015). By "stickiness" it is meant "whether your digital product is worthy of multiple visits from users" (Spintech, 2015). It also shows the loyalty of the users (regular and occasional users). The more interesting the digital product is for the user, the more often the user will visit it. Finally, including the fact that TCM originally uses a count variable as the dependent variable, the basic principles of this model construction apply to UCM as well.[42] This paper aims to include another independent variable besides money and time budget in the analysis of the value of free digital products. In contrast to environmental, health or other similar non-market products that produce both use and non-use value[43], free digital products produce only use value. It is therefore logical to apply here methods that directly investigate

[41]The most important metrics for measuring the success of digital products are: users, sessions, pages per session, devices, source/medium and channels, pageviews, unique pageviews, average time on page, landing pages, exit pages, bounce rate (Day, 2021).

[42]The duration of session (number of minutes, a user spends on the usage of digital product) is not used here as a dependent variable because, from our point of view, it illustrates the cost side of the analysis (can be converted into money the user would have earned if he or she had not used the digital product) and is part of the total usage costs variable that affects the number of sessions (visits) to the free digital product. Furthermore, it is not consistent with the original principle of the TCM model, which is based on the countable nature of the dependent variable. However, it can be seen as a possible measure of the value of the free digital product, but using a different valuation model (method). In particular, as already mentioned, Goolsbee and Klenow (2006); Brynjolfsson and Oh (2012) construct such models and calculate the welfare gain as well. The models show the relationship between time spent on the internet and the individual's wage, which predicts that the higher paid users spend less time on the internet compared to the lower paid users. Brynjolfsson and Oh (2012, p. 7) also include the quality of the Internet in their analysis.

[43]These goods usually in the literature consist of use and non-use values. Use values arises from the direct consumption of the product. Non-use value, on the other hand, is assigned to a product that is not used, but is saved for future use(Baker and Ruting, 2014, p.12).

the use value of a product. These methods are revealed preference methods. Therefore, this paper explores the possibility of applying UCM.

Usually the demand curve shows how many units of a product will be purchased based on the particular price. In general, this approach to determining demand is appropriate when applied to the "standard" market goods and services, where the price is settled and known. The lower the price, the larger the demand. However, such a result is not quite clear in the case of free digital products, which first of all, are not "standard" products, but non-market products and, secondly, are close to public goods. We assume, however, that there is an inverse relationship between price and the number of free digital products consumed. The higher the shadow price per visit (usage costs per visit), the fewer visits to free products are made, and vice versa, the lower the usage costs per visit, the more visits to the free digital product are made.

The convex falling demand curve for free digital products is represented in Figure 7. In contrast to the classical microeconomic theory, where a demand curve represents an inverted demand function, the depicted demand curve, represents the (non-inverted) demand function, i.e. the x-axis shows the price (usage costs per one visit) and the y-axis the quantity (number of sessions (visits)).

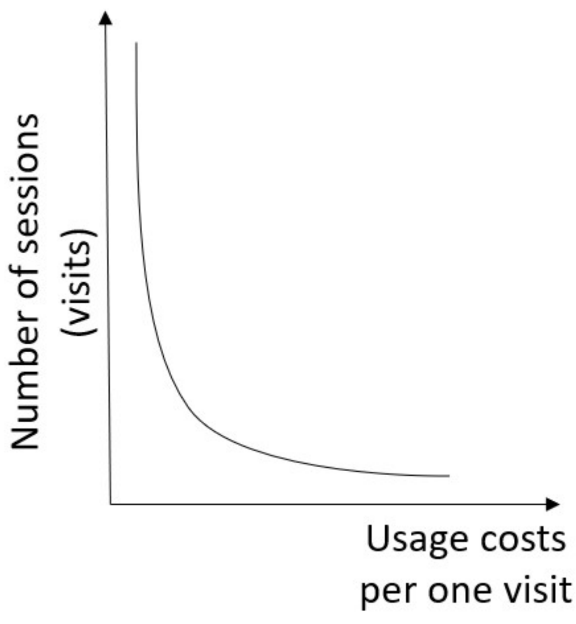

Figure 7: An illustrative expected demand function for a free digital product.

The more the user pays for the usage of free digital products, the more often he/she tends to visit the product, because there is an intention to do this/a necessity: the increased subscription fee results in more sessions, otherwise the user will choose the cheapest tariff with a limited amount of data included; or if the user is ready to share more his/her personal data, the number of his/her sessions will also increase (because every time the user visits a digital product, he/she shares his/her personal information, preferences, ideas etc.); or the more sticky the user to the free digital product, the more the user is willing to spend their extra attention on various minor things unrelated to the main product, and hence, the number of sessions increases as well; the more valuable/more sticky is the free digital product to the user, the more often the user will tend to use the product, the more sessions will be generated. More visits result in a cheaper cost per visit for the user. A higher price per visit leads to less demand (e.g. less visits).

A more detailed argumentation regarding the inverse relationship between the number of visits and the usage costs per visit is discussed in the Sections 3.5.3.2, 3.5.3.3, 3.5.3.4, for each component of the usage costs separately.

One of the aims of this paper is to present a logical economic explanation of the relationship between the selected exogenous and endogenous variables. The existence of an inverse relationship between the usage costs per visit (shadow price) and the number of visits (sessions, quantity demanded) will also be investigated by using an empirical application (see Section 3.6). The applicability of the UCM will then be analysed.

3.5.3 Specification of the Usage Cost Model

As pointed out above, the two classical variations of the TCMs are the ZTCM and ITCM. They can be adopted to UCMs. We start from the ZTCM, in our case: zonal UCM (ZUCM). ZTCM assumes that the costs of consumers depend on the distance to their location. Consequently, all consumers are grouped in certain zones depending on their remoteness. The ICT development runs throughout the whole world. In case of access to free digital products we do not have such distances: each consumer regardless of his/her location can access this free digital product from anywhere in the world (see Figure 8):

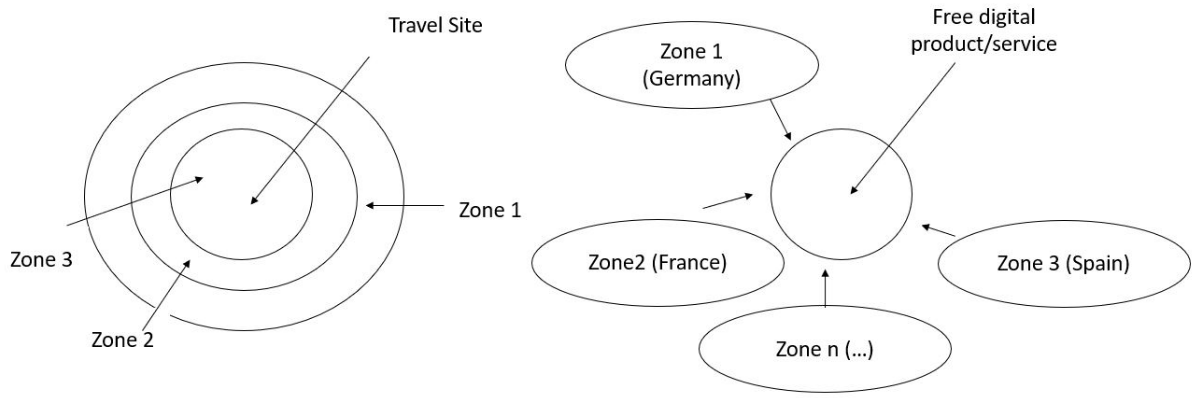

Figure 8: Zonal usage cost model.

However, the consumption of free digital products is not for free because a usage cost arises. It comprises at least four components. Besides the cost of Internet access, there is the cost of time, which will be spent by using the free product, the cost of our personal data and the cost associated with attention to undesired information, when using free digital products. The sum of these cost components one may denote as usage costs. A general representation of the ZUCM looks like this:

$$V_i/P_i = f(UC_i, S_i, e_i), \tag{52}$$

where

V_i is the number of sessions/visits from zone i to some given free digital product during a given period of time (day, week, month, year),

P_i is the population of zone i,

UC_i is the usage cost from zone i of some given free digital product,

S_i is socio-economic characteristics of the population living in zone i using a given free digital product,

e_i is an error term.

After the determination of the zones' demand, a total demand for free digital product may be derived. After this step one may identify the consumer surplus and the value of usage of the free digital product. Using ZUCM, it is possible to estimate the value of a free digital product based on official publicly available data.

An alternative is to work with the *individual UCM* (IUCM). This type differs from the ZUCM in that it uses survey data of the characteristics of each individual. On this basis it constructs an individual demand and after that, a cumulative demand of all individuals for some given free digital product and determines its value. The general IUCM looks as follows:

$$V_i = f(UC_i, S_i, e_i), \tag{53}$$

where

V_i is the number of sessions/visits of individual i to some given free digital product during a given period of time (day, week, month, year),

UC_i is the usage cost of some given free digital product by individual i,

S_i is socio-economic characteristics of the individual i using a some given free digital product ,

e_i is an error term.

The advantage of the IUCM is that it more accurately measures the value of a free digital product, as each person's data is used and analysed. However, it is not possible to gather the data from all individuals of the world. What is feasible is to survey a certain number of individuals in a country and make an approximate assessment of the value of the product from the perspective of the country's population. Another option is to determine the value of a free digital product for a particular group of people (e.g. scientists, young/old or married/unmarried people etc.). This will help to see how this particular group of people values these free digital products.

Another modification of the TCM is the single-site model. This model is merely an extended version of IUCM, when (53) is applied to various sites. Single-site travel cost models are usually applied to the individual data, however, in the case of limited data, the zonal version may also apply. The single site model is used to determine the value for those places that have several substitutes (Špaček and Antoušková, 2013, p. 2852). In case of the TCM, the higher the cost of reaching the substitute, the more trips taken to the initial site Parsons (2003, p. 294). A single-site usage cost model can be specified as follows:

$$V_i = f(UC_i, UCS_i, S_i, e_i), \tag{54}$$

where

V_i is the number of sessions/visits of individual i to some given digital product during a given period of time (day, week, month, year),

UC_i is the usage cost of a given free digital product by individual i,

UCS_i is the usage cost of the substitute to the initial given free digital product by individual i,

S_i are socio-economic characteristics of the individual i using a given free digital product,

e_i is an error term.

By incorporating usage cost to other alternatives, the model now accounts for substitutes.

In case of the zonal single-site usage model the dependant variable V_{ij} is replaced by the variable V_i/P_i.

Because of the focus on a specific small group of people and the evaluation of the value of specific free digital products, this paper, including its empirical application, focuses on the IUCM. Other options may be considered in the future for a more extensive research.

3.5.3.1 Description of Necessary Data

The *dependent variable* V_i is the number of sessions/visits to some given free digital product by individual i during a given period of time. Let us take Facebook, Instagram and WhatsApp as examples of free digital products. In the ZUCM, where zones are countries, the official available data of monthly/weekly/daily active Facebook users for that country can be used. The zone's population is also taken into consideration. In the IUCM, a survey or experiment should be conducted within a country or potential group of people to collect information on how often/how many times they visit Facebook or another free digital product.

The next variables to be considered are: *usage costs* and *secondary variables*. Usage costs have the following components:

1. adjusted subscription fee for Internet access (SF)
2. opportunity costs of the time spent (OC)
3. value of personal data of users (PD)
4. attention factor (A)

3.5.3.2 Adjusted Subscription Fee for Internet Access

A subscription fee differentiates between the type of usage device as follows: access to the free digital product from any kind of mobile phone or from the computer/laptop. It is also assumed that Internet access can be provided via mobile (wireless) network and/or fixed broadband Internet. In the case of ZUCM, the average costs for each internet type for each country may be taken from the official publicly available reports of the costs.[44] In the case of IUCM, it is necessary to conduct a survey or experiment within the country or a potential group of people to collect information on how much they spend on subscription fees, the type of network and tariff used. The subscription fee must then be adjusted to reflect the megabytes (MB)/time spent using the free digital product and the intensity of it.

Figure 9 provides the SF transformation and conversion mechanism to be applied to the IUCM. All users pay a certain subscription fee to have a general Internet access. This subscription fee is based on a specific tariff that includes a certain number (or unlimited) of MB/GB, minutes, SMS/MMS and other mobile/computer usage units per a certain period of time (week, month, year). Based on this information, one may extract the cost of 1 MB/GB or other necessary usage units per week/month/year. Furthermore, it is possible to determine the average data usage (MB/GB) from a particular free digital product.

[44] E.g. Reports of the UK company Cable.co.uk (Howdle, 2020a,b).

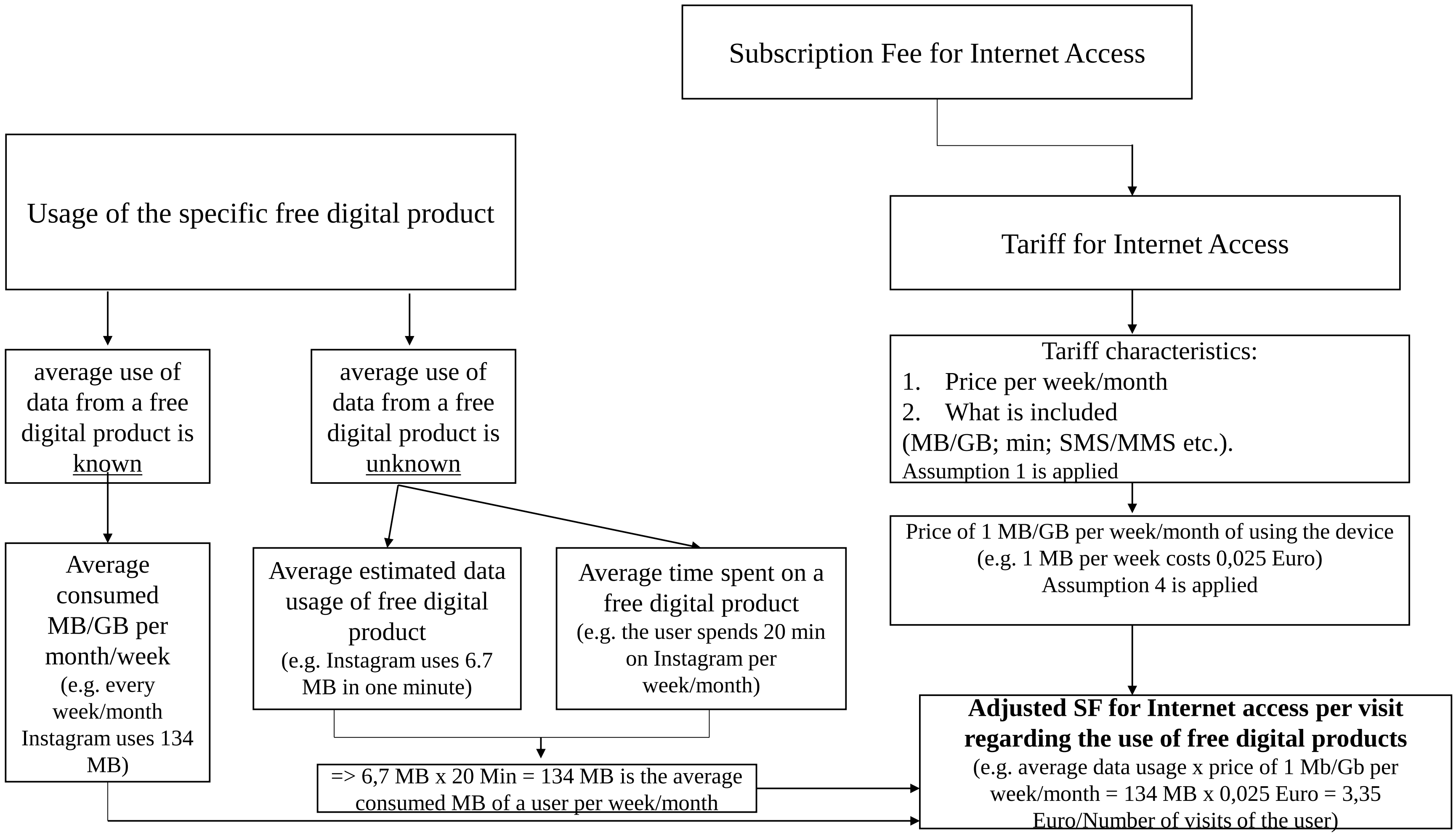

Figure 9: Subscription fee transformation and conversion mechanism to be applied to the UCM.

This can either be done through information collected in the app/website of that free digital product or based on the minutes spent in the free digital product, which can be converted into MB/GB. Finally, multiplying the average data usage and the price of 1 MB/GB and dividing it by the number of sessions (visits) gives the adjusted SF for internet access per visit relative to usage of that particular free digital product. The following assumption are to be applied to this mechanism and the UCM in general:

1. Individuals buy a tariff and "use" it completely, which includes deliberate reserves.

2. Anything included in some tariff (data traffic, minutes, sms/mms etc.) is considered to be of the same value in the model.

3. The users distribute all data traffic to all possible products in use (e.g. WhatsApp, Facebook, Instagram, other products, perhaps there are also reserves left).

4. A person's visits are considered to be uniform. Users are assumed to have some adjusted subscription fee costs[45] for using a product.

5. It is assumed as well, that the demand curves for each user between the adjusted SF of using the free digital product per visit and the number of visits has an inverse relationship.

6. Users are rational: normally, if they expect to consume a lot/to make a lot of visits, they buy a more expensive tariff (with more options included). If they are not interested in intensive use, they buy a cheaper tariff. This means, however: for those users who consume a more expensive tariff and visit the free digital product more often, the total costs of using the free digital product are higher, but the adjusted SF of product use per visit is lower than for those people who buy a cheaper tariff and visit the free digital product only a few times.

Let us consider a person A, who pays for the tariff 6,99 Euro per month. This tariff consists 4 GB Internet, flat rate calls and SMS flat rate in all German networks. The costs of 1 GB, hence, are: (6,99/3)/4 = 0,58 Euro per GB. The person has 60 visits per month of WhatsApp and consumes around 0,36 GB of traffic per month for WhatsApp.

[45] These costs are measured on the basis of observations, actual user behaviour. See also Figure 9.

The costs of WhatsApp usage for this person is: 0,21 Euro (0,36x0,58) and the adjusted SF per visit for the person A is 0,0035 Euro per month (0,21/60 Visits per Month). At the same time, if this person decides to make more visits (less visits), his/her adjusted SF per one visit for WhatsApp will consequently decrease (in case of 90 visits per Month: 0,21/90 = 0,0023 Euro per visit) (increase, in case of 30 visits per Month: 0,21/30 = 0,0069 Euro per visit). This means: the more visits a user makes, the cheaper it costs per visit for the user.

3.5.3.3 Opportunity costs of time

The next step is to define the opportunity costs of the time spent on visiting free digital products. The appropriate treatment of time is critical. There is a lot of discussion regarding this question in the literature.

The first attempts to assess time are based on the traditional neoclassical model of labour-leisure choice. This model is based on the assumption, that consumers always tend to maximize their utility. Time is limited and usually divided between working and non-working (leisure) time. Consequently, the aim of consumers is to allocate their working and leisure time in such a way as to obtain maximum utility. This is achieved when "the opportunity cost of an hour of leisure is equal to one's wage rate - the extra earnings a worker can take home from an extra hour of work" (Ehrenberg and Smith, 2017, p. 174). Put differently, the marginal rate of substitution between labour and leisure is equal to the wage rate. This may be interpreted as the value of time DeSerpa (1971, p. 831). Cesario and Knetsch (1970) and Cesario (1976) among the first researchers to investigate time bias as part of the TCM. When applied to TCM, they make a suggestion that "the effect of time on visit rates is likely to be of equal or even greater importance than the actual monetary cost incurred". The correction has been made by including the time variable as a separate variable in the estimated model. Also McConnell (1975, p. 332) concludes that "ignoring the opportunity cost of time, underestimates the marginal value of the recreation". Cesario and Knetsch (1970, p. 702) argue that people do make trade-offs between time and money costs and hence, they link time costs to money costs.

DeSerpa (1971, p. 833)) distinguishes between two values of time: the value of time in terms of the scarcity value, as a resource[46], and "the value of saving time", as a commodity.[47] Wilman (1980) investigates these two kinds of time values and comes to the conclusion, that "travel time" one should estimate in terms of "the value of saving time" and "on-site time" - in terms of the scarcity value. According to Cesario (1976, p. 34), the scarcity value of time represents the lower bound of the value of saving time.

There are also some differences of how fixed-time workers and workers with flexible work schedule value their time. Again, this is one of the critical moments when the wage rate is used as the proxy of the opportunity costs of time. In particular, Bockstael et al. (1987, p. 295-296) confirms that wage rates can be applied for those employees who have flexible working hours, but for those who have a fixed working day - their working hours have no relationship to the individual's wage rate. The present paper supports the view of Grazia, S. de (1962) and Amoako-Tuffour and Martinez-Espineira (2008), who suggests that non-working time is not free and already paid by the length of time devoted to work (Amoako-Tuffour and Martinez-Espineira, 2008, p. 4). And this is especially true for those who have a fixed work schedule. Parsons (2003, p. 285) mentions that it is common to relate the value of time to the wage earned by a an individual. Despite many criticisms of the value of time as a certain percentage/fraction of the wage rate (Smith et al. (1983); Shaw (1992)), it must be recognised that the wage-based approach is still one of the most widespread approaches of valuing the time (Cesario (1976, p. 34); Feather and Shaw (1999, p. 50); Brouwer (1999, p. 100); Borzykowski et al. (2017, p. 156); Amoako-Tuffour and Martinez-Espineira (2008, p. 11)). This relation is usually deviates from the one-third to the full wage.

Another possible approach is to construct the fitted regression and impute a wage for individuals Parsons (2003, p. 285), by using a hedonic method (Feather and Shaw, 1999). This approach has also its drawbacks and is complicated as an empirical ap-

[46]"The general value of time is determined by its use in alternative activities." (Shaw, 1992, p. 69). This equals the marginal rate of substitution between labour and leisure. The main assumption here is that time is limited and must be allocated.

[47]Is determined as difference between the value of alternative use and the value of time in any particular use (DeSerpa, 1971, p. 833). The time is defined as "a means for the consumption of market goods and services, just as money is a means for the purchasing. In its role as a commodity, time in a specific activity is not the same commodity as time in another activity"(Hensher, 1997, p. 252). The goal is to show difference and heterogeneity between different activities. This means that if work is disliked more than travel, then the value of travel time is higher than the wage rate (Nordström et al., 2019, p. 214).

proach (Borzykowski et al., 2017, p. 156). Other alternative approaches include asking the consumers/users directly via stated preference techniques (e.g. CV method) about their WTP value of time. These methods are criticised for the hypothetical bias. This mix of revealed and stated preference techniques is also applicable in the literature, e.g. a combination of TCM and CV method (Czajkowski et al., 2019). In addition, it is important to mention that the majority of time research, in particular in transportation and recreation economics, is related to the "travel" time to the site and not "on-site" travel time.

When considering a standard TCM, there is a debate on the inclusion of "on-site" time costs into the model. For a long time this type of costs was either completely ignored or considered to be endogenous (McConnell, 1992, p. 918). Parsons (2003, p. 286) advises to include time travelling to the site as well as "on-site" travel time. This difference is also important in the context of the UCM. When using some free digital products, the user does not have the so-called "travel" time costs. Due to the high speed of current internet connections, counting the time required to access digital products is no longer relevant. What is important is the inclusion and calculation of the costs of using this free digital product "on-site", i.e. the actual time spent using it. Therefore, the UCM will include only the "on-site" time usage costs. We calculate the opportunity cost (OC) of "on-site time" as one-third of the individual's income (wage),following the recommendations of Parsons (2003, p. 285), Cesario (1976), Amoako-Tuffour and Martinez-Espineira (2008) and Czajkowski et al. (2019, p. 964). Using the logic of Borzykowski et al. (2017, p. 156), we define the formula for the calculation of the OC as follows:

$$OC_i = C_i \cdot T_i$$

$$C_i = \frac{1}{3} \cdot Income_i \cdot \frac{1}{H_i \cdot 60}$$

where

OC_i is the opportunity cost of the time spent on the given free digital product by the individual i per one visit,

T_i is "on-site" time spent by using a given free digital product by the individual i,

C_i corresponds to the individual opportunity cost of time per minute on a given free digital product,

$Income_i$ is the monthly income of the individual i declared in the questionnaire,

H_i is the average number of working hours per week/per month of the individual i declared in the questionnaire.[48]

We also assume here that the demand curve for the free digital product, showing the relationship between the opportunity cost per visit and the number of visits, has a downward slope. The more valuable a user's time, the more that time is worth, and consequently the fewer visits that user will make.

3.5.3.4 Value of personal data and user's attention

The next component of the usage costs is the value of personal data of users (PD).[49] Each day consumers exchange their personal information on the access and usage of free digital products. Hence, the costs of selling this personal data should be also included in the model. It is also one of the most complex and controversial topics in recent literature and can be seen as an independent research issue. Moreover, inclusion of this sub-variable into the model depends strongly on the contribution of the personal data in the free digital product itself. As mentioned earlier, free digital products have some particular features and one of them is the quasi-heterogeneous nature of products. E.g. Facebook is a company that is closely related to the collection and processing of personal data. In contrast, companies that offer free translation services (DeepL, Google Translator) have less access to personal data if you are not registered there. In any case, one can continue to use these free translation services without registering. This cannot be done with Facebook, if one wants to see the full information of some pages.

Personal data cover not only personal data, which users give voluntary to the company, but also the user's attention. The latter is actively used by companies offering free digital products when displaying targeted ads to the user. The attention factor represents the number of undesired information, which we receive using the free dig-

[48] This is multiplied by 60, because H_i is represented in hours and since our final variables (C_i and OC_i) are in minutes, we convert hours to minutes (1 hour = 60 minutes).

[49] This paper uses the following definition of the personal data:"Personal data - is any information relating to an identified or identifiable individual (data subject)" (OECD, 2013b, p. 13)

ital products. For most free digital products providers, advertising can serve as one of the main sources of income. However, exactly how the attention factor can be assessed is not clear. One way is to calculate the number of clicks on different ads while using the free digital product and then calculate the monetisation of those clicks.[50] In general, the calculation of the value of personal data and the attention factor are mixed with each other.

There are two main approaches to measuring the monetary value of personal data: based on market valuation (firm's perspective) and based on individual valuation (individual perspective) (OECD, 2013a, pp. 5-12). To estimate the value of personal data from the firm's perspective one may look at the main financial indicators of the market valuation: the stock value of a firm (market capitalisation), the revenues of the firm and average revenues per data record/user (profits minus costs) (OECD (2013a, pp. 19-22); Lieshout (2014, pp. 5-6)). Among these three methods, the third one - average revenue per record/user, may give the best estimate. This is especially true for those companies that generate income mainly through the use and processing of personal data (e.g. Facebook). To use the prices of data brokers or cyber-crime (illegal) markets is another possible option to estimate the value of personal data. Finally, the economic cost of a data breach may also reflect the value of personal data.

All proposed methods have their benefits and drawbacks. One of the main advantages is the ease of identification. The disadvantage is the complexity and breadth of the financial indicators, which can eventually cover not only the value of personal data, but also many other components. It is therefore important first to determine the company's sources of income, whether they are based primarily on personal data or other sources.

The second point of view in the literature, is to define the value of personal data from the individual perspective. One reason for this is that people assess personal data themselves in different ways. As a OECD Report mentions (OECD, 2013a, p. 30), the assessment may be done with respect to the "individual valuation of personal data" and with respect to the "individual valuation of privacy". This means an assessment of the WTA some money in return to provide companies with personal data and of the WTP in order to preserve their privacy. The estimation of WTA and WTP is mostly

[50]Normally this is achieved through a metric such as CPC (cost-per-click).

done by using on-site or off-site surveys. Based on survey data such methods as conjoint analysis, discrete choice experiments, generalised second-price auction mechanism (Li et al., 2021) combined with regression analysis etc. are used to determine the value of personal data for an individual. One of the advantages of assessing the value of personal data from an individual point of view is to obtain a more accurate result from individuals. One of the huge drawbacks of this approach is the dependence of the results on certain behavioural factors, i.e. the condition, mood of the individual and current situation. Moreover, the final outcome of a person's decision depends largely on the formulation of the question. The hypothetical bias is present as well. Hence, the results may be biased and subjective. In the case of ZUCM, it is strongly recommended to use more standardized methods for valuing the personal data, i.e. from the market perspective. In the case of the IUCM, an individual evaluation is likely to prevail. Nevertheless, one should take into consideration, which individuals are to be evaluated. When it comes to a particular homogeneous "focus" group of individuals/users, then using the first approach, from the market perspective, is more logical. In the end, one should admit, that most of the valuations of personal data in the current literature are carried out within the US market of personal data. Europe and other markets have a more limited number of assessments.

We assume that when the demand curve for a free digital product is shown, where the relationship between the costs (value) of personal data and the amount of demand for the free digital product is presented, the following logic applies: the higher the costs associated with personal data/the more valuable the personal data to the user, the less the user will visit the product. Consequently, here we also have a falling demand curve.

After a full description of all usage cost components, one may define total usage costs (UC_i):

$$UC_i = SF_i + OC_i + PD_i + A_i \tag{55}$$

where

SF_i is the adjusted subscription fee for Internet access in the zone i/of the individual(user) i regarding the use of a given free digital product,

OC_i is the opportunity costs of the time spent on some given free digital product in the zone i/ by individual (user) i,

PD_i is the value of personal data of the user i (in the zone i) when using a given free digital product,

A_i is the attention factor of the user i (in the zone i) when using a given free digital product.

In the empirical part of our UCM, we consider it is more logical to represent the components not as their sum, but as individual variables. Because of the homogeneity of the personal data variable within the group, we do not include this variable in our empirical application. The attention factor is also not included due to lack of data.

Moreover, usage costs may not fully explain the demand for free digital product. Demand usually also depends on such variables as age, marital status, family size, and the usage costs of possible substitutes of the free digital product. The age of the potential user may affect the frequency of use of free digital products. In the case of social networking products (Facebook, WhatsApp, Instagram, etc.), young people are expected to use them more often and more intensively than older people. Herzog (2018, pp. 2, 13) examines the value of Facebook by measuring WTP and WTA and finds that young people tend to give Facebook more value than older people. Marital status and/or family size also has the potential to affect the value of free digital products. Married people with children are among those who are expected to use the free digital products less intensively. Finally, it is also possible to take into account the value of substitutes, which may affect the value of free digital products. Other possible secondary variables are occupation and level of education. The inclusion of secondary variables may vary and depend on the type of free digital products, country/regional differences, focus-groups and other circumstances and attributes.

3.6 Usage Cost Model in Practice

3.6.1 A General Estimation Plan of the Model

In this following section a small simulation of the IUCM is conducted. According to Parsons (2003), all TCM models considered in section 3.5.3 follow a common estimation plan with eight steps. This plan also applies to the UCM:

1. Define the digital product to be valued
2. Define the goal of usage and the frequency of usage
3. Develop a sampling strategy
4. Specify the model
5. Design and implement the survey
6. Measure the usage costs
7. Estimate the model
8. Calculate the usage value of digital products

Step 1 begins with the definition of the concrete digital products to be valued. This step is very important due to the heterogeneity of free digital products. In our case we want to estimate the value of the following free digital products: *Facebook*, *Instagram* and *WhatsApp*. *Facebook* is a free social-media networking website, which provides the possibility for people to exchange their thoughts, photos, links etc. It also serves as a comprehensive platform for marketing, i.e. helps businesses attract target customers. *Instagram* is a free social network, which allows people to share and exchange their photos and videos as well as to promote their products. *WhatsApp* is one of the free leading messengers, which helps people to communicate with each other, send messages, audio and video. All products considered are free of charge.

Step 2. In general, all free digital products have the same single dominant usage goal as platform visitation. A platform visitation goal may also be divided into more detailed usage goals. In case of Facebook these could be: communication with friends, sharing photos and videos, publishing some advertisements etc. And for all these detailed goals one my construct an individual demand curve. This paper assumes that the consumers have one general usage goal and estimates an aggregated demand curve. It is better to use revealed preference techniques such as hedonic pricing or stated preference methods such as CV or CM to examine attributes of free digital products and their impact on the final price of it. The usage of the free digital products is investigated via the number of sessions (visits) of these products within the week and month (why we choose the number of sessions and why this is a measure of determining the usage of the free digital products was described in section 3.5.2).

Step 3. According to Parsons (2003), two basic sampling strategies in the TCM exist: on-site and off-site sampling. In case of a UCM on-site sampling may be done only by the company-owner of the free digital product. When the user visits the web-site, an additional window on this web-page will invite a user to complete a survey/questionnaire. Or it may be done as well by monitoring the usage of the products by using built-in metrics in the products. We do not have access to these metrics. Hence, this paper chooses an off-site survey. The survey can be conducted personally/by phone/via on-line form. We conduct the survey online. The survey has been conducted in November and December 2020 in Trier, in particular in the University of Trier. Overall, 41 persons participated in the survey. This focus group represents the group of scientists and their usage of such digital products as Facebook, Instagram and WhatsApp.

Step 4 describes all variables which are used in the model to estimate the usage value of free digital products.

- The dependent variable is the variable *Sessions/Visits:* V_i, which shows the number of accesses/visits to Facebook, Instagram or WhatsApp by one individual i per week(month) to the given free digital product. This variable consists in the survey of both, certain numbers (e.g. 1,2,3 etc.) and binned (e.g. 10-20, 21-30, 31 and more) data. In the case of the binned data with closed intervals the bin midpoint method is applied[51]. In the case of open intervals, the lower limit of the open category applies (Faia (1981, p. 1098), Parker and Fenwick (1983, p. 873), Celeste and Bastos (2013, p. 169)).

- Independent variables include usage costs UC_i and some additional secondary socio-economic variables of a person i. *Usage costs* consist of these three variables. They are: *a subscription fee for Internet access for individual i* (SF_i), *opportunity costs of the time spent by the individual i in Facebook, Instagram or WhatsApp* (OC_i), *value of personal information of user i in case of Facebook, Instagram and WhatsApp*

[51]This method suggests to assign within each bin $b = 1,..,B$ the n_b visits to the bin midpoint $m_b = (l_b + u_b)/2$ (von Hippel et al., 2017, p. 5).

(PD_i). Due to the limited possibilities of measurement of the attention factor [52], this variable is not included in the current empirical simulation.

An adjusted subscription fee for Internet access for individual i (SF_i) per one visit is the adjusted cost of the Internet access for individual i with regard to the consumption of some given digital product per one visit. In order to calculate this type of cost correctly, two details must be considered. Internet access can be either mobile or fixed-line. The tariff used and its characteristics is also of interest. The other point is that the total fee for Internet access must be adjusted to the number of MB or GB/time spent directly on Facebook, Instagram or WhatsApp, and their intensity. Additional questions in the survey are to be set to clarify all these aspects.

Opportunity costs of the time spent by the individual i per one visit on Facebook, Instagram or WhatsApp OC_i reflect time costs of usage these free digital products. This is probably one of the most controversially discussed questions in the literature (see Section 3.5.3.3). The time spent on Facebook, Instagram or WhatsApp could be investigated in another activity, it is the time of so-called lost opportunities. The spent "on-site" time must be reflected in the total usage costs. In our practical application, we use the relationship between the wage rate and "on-site" time and define "on-site" time usage costs as the product of one third of the wage rate per user and time spent on Facebook, Instagram or WhatsApp. The wage rate is determined in the survey through an income question. The data consists of binned data. The same methods to the binned data apply here as in the case of the variable V_i.

Our third variable is *the value of personal data of users, PD_i*. Our practical application may use, for example, the average revenue per user (ARPU) of Facebook, Instagram and WhatsApp in Europe.[53] An alternative is to use the calculated value of personal data for Germany, based on the paper by Prince and Wallsten

[52] E.g., there is no way to accurately measure the number of clicks on the targeted ads. This can be done explicitly by the owner of the free digital products.

[53] ARPU per month for Facebook and Instagram is calculated by the author on the basis of the available public information, published by Facebook and Statista (see Facebook Earnings Presentation Q4 2020 (Facebook, 2021), Statista (2021)). ARPU is calculated by dividing the total revenue received per user by average number of user during a certain period of time. For Facebook this value is equal €4.68 and for Instagram €2.59. An average revenue per user for WhatsApp is taken as €0.26 per month (see Trefis Team (2020)).

(2020). In this paper they calculate and distinguish for the first time the value of personal data across different countries to reflect the regional peculiarities. The value is calculated there using an individual approach and based on discrete choice surveys, determining the WTA. These two approaches may be used and chosen without conducting an individual evaluation because our simulation assumes that there is a certain "focus" group of people whose preferences and conditions can be assessed as quite similar. The assumption is therefore made that the evaluation of personal data is homogeneous. However, this variable will not be included in the final empirical application in this paper, as it is the same for all users and does not make any difference.

The following secondary variables are used as well in our practical application. They are: age, gender, marital status and family size (presence of children). The value of substitutes is not taken into account here. Facebook in Germany is one of the leaders in this kind of social platform. It cannot be compared to Twitter or Telegram. Recently, however, Instagram is gaining popularity in this area as well. A direct relationship between Facebook and Instagram was not found. That is why the value of Facebook without substitutes is to be estimated. A possible substitute for WhatsApp is Viber. Viber, however, is not as popular in Germany as it is in other countries. Thus, in the case of WhatsApp, its estimated value is assessed as well without any substitutes.

Step 5 is to design and implement the survey. The survey we conduct measures the usage costs of a user. The survey is based on the questionnaire. The questionnaire can be found in the Appendix G. This is an off-site survey that was conducted on-line. The survey is divided into three parts:

- *introductory material*. Here one can read a few introductory remarks, explanations of the purpose of the study and instructions for completing the questionnaire. The survey provides full confidentiality.

- *collection of the relevant general and some socio-demographic characteristics of the user*: age, gender, marital status, family size, approximate average income per month, working hours per week.

- *collection of the information regarding the user's usage of Facebook, Instagram and WhatsApp*. The aim is to summarise the necessary information about the frequency of use and MB(GB)/time spent by a person on Facebook, Instagram and WhatsApp per week. It is also important to know the type of network used: mobile or fixed-line and its provider (Vodafone, Deutsche Telekom etc.). This helps to assess the Internet subscription fee more accurately.

Step 6 measures the usage costs based on the results of the survey.

Step 7 aims to estimate the model. Linear and non-linear forms of estimations are used in the literature. However, a linear form tends to show a poor goodness of fit in comparison to non-linear models (Vicente and Frutos, 2011, p. 46). Moreover, linear models allow to predict negative and/or non-integer frequency (Borzykowski et al., 2017, p. 152). Hence, most researchers prefer for the estimation of TCM counts models such as Poisson or Negative Binomial models (Creel and Loomis (1990, p. 435); Hellerstein and Mendelsohn (1993); Borzykowski et al. (2017, p. 152); Parsons (2003, p. 286)). The number of sessions/visits of an individual i of Facebook, Instagram and WhatsApp is a nonnegative integer. For this reason, the focus is on using count models to evaluate the proposed UCM. The models are estimated by maximum likelihood.

Several possible modifications of count models are carried out. First, a simple *Poisson regression* is constructed. A dependent variable, V_i is assumed to have a Poisson distribution with parameter λ, which is related to the regressors x_i (these are our explanatory variables such as usage costs of Facebook, Instagram and WhatsApp UC_i and secondary socio-economic variables S_i). The variable V_i takes integer values from 0 to infinity. Hence, the probability that the user i visits Facebook, Instagram or WhatsApp in a certain period of time can be specified as (Greene, 2012, p. 844):

$$Prob(V = v_i \mid x_i) = \frac{e^{-\lambda_i}\lambda_i^{v_i}}{v_i!} \tag{56}$$

where λ_i reflects the expected number of sessions/visits of Facebook, Instagram or WhatsApp per a certain period of time.

The relation between λ_i and the explanatory variables x_{ij} can be parameterized by the following semi-log models:

$$ln(\lambda_i) = x_i'\beta = \beta_{uc_i} UC_i \qquad or$$
$$ln(\lambda_i) = x_i'\beta = \beta_{sf_i} SF_i + \beta_{oc_i} OC_i \qquad and/or$$
$$ln(\lambda_i) = x_i'\beta = \beta_{uc_i} UC_i + \beta_{s_i} S_i \qquad or$$
$$ln(\lambda_i) = x_i'\beta = \beta_{sf_i} SF_i + \beta_{oc_i} OC_i + \beta_{s_i} S_i$$

The special feature of this model is that it is assumed, that the (un)conditional mean and the (un)conditional variance of the dependent variable are equal. The violation of this property is called "over-dispersion" (Greene, 2012, p. 844). Poisson models often suffer from failure to meet this property. Hence, this feature that is sharply criticized. "Over-dispersion" is the form of heterogeneity (Creel and Loomis, 1990, p. 436) and describes the empirical finding that observed variation is higher than would be expected (Dormann, 2016, p.1). Hence, the *Negative Binomial model* is also constructed. This model relaxes the assumption of the "over-dispersion". To eliminate this problem, an individual, unobserved effect is introduced into the conditional mean in the model. So, the models can be now specified as follows Greene (2012, p. 846):

$$ln(\mu_i) = x_i'\beta + \epsilon_i = \beta_{uc_i} UC_i + \epsilon_i = ln(\lambda_i) + ln(u_i) \qquad or$$
$$ln(\mu_i) = x_i'\beta + \epsilon_i = \beta_{sf_i} SF_i + \beta_{oc_i} OC_i + \epsilon_i = ln(\lambda_i) + ln(u_i) \qquad and/or$$
$$ln(\mu_i) = x_i'\beta + \epsilon_i = \beta_{uc_i} UC_i + \beta_{s_i} S_i + \epsilon_i = ln(\lambda_i) + ln(u_i) \qquad or$$
$$ln(\mu_i) = x_i'\beta + \epsilon_i = \beta_{sf_i} SF_i + \beta_{oc_i} OC_i + \beta_{s_i} S_i + \epsilon_i = ln(\lambda_i) + ln(u_i)$$

where ϵ_i reflects either the specification error term or the unobserved heterogeneity of the user, an unobserved choice of the individual i to sessions/visits Facebook, Instagram or WhatsApp. The Negative Binomial probability of visiting/assessing a free digital product with mean λ_i and variance $\lambda_i + \alpha_i \lambda_i^2$ may be written as (Creel and Loomis, 1990, p.436):

$$Prob(V = v_i \mid x_i) = \frac{\Gamma(v_i + (1/\alpha_i))}{\Gamma(v_i + 1)\Gamma(1/\alpha_i)} (\alpha_i \lambda_i^{v_i})(1 + \alpha_i \lambda_i)^{-(v_i + 1/\alpha_i)} \tag{57}$$

where Γ indicates the gamma function. However, there are also other specifications for probability functions (see Greene (2012, p. 848)).

Both above-mentioned models allow zero visitation rates. A lot of on-site surveys have the problem that non-visitors are not observed. That means that these samples have no zero observations. Hence, the estimation may give some biased results. Therefore, in such cases the behaviour of the selected individuals is modelled by the Zero-Truncated Poisson (ZTP) or Zero-Truncated Negative Binomial (ZTNB) models (Englin and Shonkwiler (1995), Borzykowski et al. (2017, p. 152)). The ZTNB model corrects not only the over-dispersion, but also truncation and endogenous stratification (Englin and Shonkwiler (1995, p. 107), Amoako-Tuffour and Martinez-Espineira (2008, p. 18)).

Our analysis is based on the Poisson and Negative Binomial models since the applied data includes the zero visitation rates as well. The selection between two models can be made with the Pearson χ^2 dispersion statistics (the dispersion ideally should be 1), log likelihood ratio tests and the Akaike Information Criteria (further - AIC) (the lower is better).

Step 8 is the final step and consists of the following positions:

- determine the estimated parameters of regressions run in *Step 7*, including the so-called shadow price for the user;
- model a predicted demand function of the relationship between the sessions/visits of Facebook, Instagram and WhatsApp on the basis of the estimated parameters;
- calculate the total consumer surplus of the product and then the individual consumer surplus per user per week and month, which reflects their value of the free digital product.

3.6.2 Empirical Application: Results

Four models have been constructed for Facebook, Instagram and WhatsApp (Poisson and Negative Binomial models respectively). In each case, two models focus on the relationship between usage costs (in particular, subscription fees (SF) and the opportunity costs (OC)) and the number of sessions/visits, the other two models include also additional variables (such as age, marital status, children and gender) that may influence the visit frequency of users. The final sample for all models consists of 41 observations.

Estimation results of the models (Poisson and Negative Binomial) for Facebook, Instagram and WhatsApp are presented in Tables 6, 7 and 8 respectively. All models are presented using the robust (sandwich) standard errors.The results of the estimation show a contradiction with our original hypothesis and economic justification.

Table 6: Facebook results.

	Dependent variable:			
	Visits			
	Poisson		*negative binomial*	
	(1)	(2)	(3)	(4)
SF	0.281	0.305	0.574	0.922
	(1.084)	(1.192)	(1.216)	(1.511)
OC	0.116	0.111	0.147	0.186
	(0.089)	(0.106)	(0.114)	(0.143)
Age		−0.045		−0.066**
		(0.035)		(0.032)
MS		0.334		0.193
		(0.433)		(0.425)
No Children		−0.802*		−0.898*
		(0.425)		(0.471)
Gender		−0.077		0.173
		(0.397)		(0.413)
Constant	3.539***	5.319***	3.500***	5.938***
	(0.231)	(1.378)	(0.242)	(1.277)
Observations	41	41	41	41
Log Likelihood	−1,058.246	−972.549	−187.068	−184.966
θ			0.540***	0.594***
			(0.116)	(0.130)
Akaike Inf. Crit.	2,122.491	1,959.098	380.136	383.932
Note: Heteroskedasticity-robust standard errors in parentheses			*p<0.1; **p<0.05; ***p<0.01	

Table 7: Instagram results.

	Dependent variable:			
	Visits			
	Poisson		*negative binomial*	
	(1)	(2)	(3)	(4)
SF	−0.452	0.658	3.979***	5.408***
	(1.244)	(1.335)	(1.317)	(1.336)
OC	0.349***	0.280**	0.824***	0.846***
	(0.110)	(0.112)	(0.132)	(0.129)
Age		−0.141**		−0.161***
		(0.068)		(0.037)
MS		0.023		−0.589
		(0.569)		(0.560)
No Children		−0.268		0.028
		(0.577)		(0.465)
Gender		−0.117		−0.282
		(0.469)		(0.509)
Constant	3.178***	7.699***	2.580***	7.940***
	(0.279)	(2.573)	(0.315)	(1.611)
Observations	41	41	41	41
Log Likelihood	−1,239.364	−1,052.048	−151.843	−149.147
θ			0.227***	0.263***
			(0.055)	(0.064)
Akaike Inf. Crit.	2,484.729	2,118.097	309.685	312.294
Note: Heteroskedasticity-robust standard errors in parentheses			*p<0.1; **p<0.05; ***p<0.01	

According to the hypothesis formulated in the Section 3.5.2, the relationship between the usage costs (including subscription fees and opportunity costs) and the number of sessions/visits is negative. According to the empirical results, on the contrary, the higher the usage costs (in particular, SF and OC), the higher the number of visits to Facebook, Instagram and WhatsApp (a positive sign of the coefficients SF and OC). For both, Poisson and Negative Binomial models, this trend applies.

Table 8: WhatsApp results.

	Dependent variable:			
	Visits			
	Poisson		*negative binomial*	
	(1)	(2)	(3)	(4)
SF	3.858^{***}	3.503^{**}	3.983^{**}	3.655^{*}
	(1.381)	(1.704)	(1.586)	(1.906)
OC	0.059^{**}	0.031^{*}	0.068^{**}	0.037^{*}
	(0.023)	(0.017)	(0.028)	(0.021)
Age		−0.003		−0.002
		(0.009)		(0.009)
MS		0.340^{*}		0.362^{*}
		(0.201)		(0.217)
No Children		−0.110		−0.142
		(0.163)		(0.190)
Gender		0.115		0.115
		(0.120)		(0.131)
Constant	4.492^{***}	4.374^{***}	4.482^{***}	4.369^{***}
Observations	41	41	41	41
Log Likelihood	−546.073	−504.037	−221.724	−220.395
θ			2.699^{***}	2.901^{***}
			(0.635)	(0.691)
Akaike Inf. Crit.	1,098.147	1,022.073	449.448	454.790
Note: Heteroskedasticity-robust standard errors in parentheses			*p<0.1; **p<0.05; ***p<0.01	

In almost all models (except Poisson and Negative binomial models for Facebook, and Poisson models for Instagram for the coefficient SF), the coefficients of SF and OC are statistically significant. This results in an upward slope of the demand curve for visits of Facebook, Instagram and WhatsApp.

Other variables are also included in the regression to broaden the analysis. The variable *Age* presents the negative sign in all models for all considered digital products. However, only for Facebook and Instagram is the coefficient statistically significant, suggesting that the older a person, the less visits has the product. The next variable is marital status MS. This is a dummy variable (married - 0, not married -

1). In almost all models (except WhatsApp), there is no significant impact of marital status on visits. The variable *No children* is also a dummy variable (0 - with children, 1 - without children) is not significant in the case of Instagram and WhatsApp.

The results of the regressions for Facebook show a negative significant at the five percent level association between the existence of children and the number of visits of Facebook. People without children are more likely to visit Facebook than people with children. The last variable *Gender* shows no significant impact on visits for all digital products considered in all models. This means that there are no gender differences when visiting Facebook, Instagram or WhatsApp.

We argue that the results of this empirical application are inconsistent and cannot be taken to calculate the consumer surplus of a free digital product, and hence to determine the value of this product. First, the number of observations is only 41, which is clearly insufficient for a fully coherent empirical study. Secondly, our considered group is quite homogenous; the third component of the usage costs (the value of personal data) is the same for all our users and hence, can not make a contribution to the total usage costs. Hence, this variable, as well as the variable "attention factor" are not included into empirical analysis. Thirdly, the list of observations includes people who demonstrate abnormal behaviour, i.e. have low usage costs per one visit but do not visit as much, and conversely, have high usage costs per one visit but visit a lot.

If the results were consistent, it would be possible to determine the predicted demand curves of Facebook, Instagram and WhatsApp visits. And then calculate the consumer surplus of the products and their value.

One important issue is also the choice between Poisson and Negative Binomial models. As already mentioned in Section 3.6, one of the main drawbacks of Poisson regression is the assumption that the mean and variance of the dependent variable are equal. It is assumed that the residual deviance to degrees of freedom should be 1, otherwise the regression has overdispersion. To check the existence of overdispersion, different tests are available (e.g. Pearson χ^2 and Pearson dispersion statistics [54]). The results of the test are provided in Table 9. The results show the presence of overdispersion (greater than 1) for all Poisson models.

[54]In R function P_{disp} package MASS.

Table 9: Test results for overdispersion.

Model, digital product	Pearson χ^2	Dispersion
Poisson, Facebook	2180.40	57.38
NB, Facebook	32.15	0.85
Poisson, Instagram	2766.06	72.79
NB, Instagram	28.46	0.75
Poisson, WhatsApp	575.26	16.92
NB, WhatsApp	17.89	0.47

Overall, Negative binomial models have significantly better dispersion values than Poisson models. The same holds true for the likelihood ratio tests (their values can be founded in Tables 6, 7 and 8 respectively). Therefore, in all cases Negative binomial models are superior to Poisson models. In order to account for the problem of over/underdispersion, heteroskedasticity-robust standard errors are applied for all the models.

The other criteria for the choice between the models is the Akaike Information Criteria, stated in Tables 6, 7 and 8. The Negative binomial models in all three tables have a lower AIC than the Poisson models. Consequently, all above-mentioned criteria provide clear evidence in favour of the Negative binomial models.

Finally, the consumer surplus may be determined. Consumer surplus is theoretically defined as the area under the demand curve and the market price of the product. In our case, the real market price of free digital products is zero. However, we determined the shadow price, the actual price, which users pay for the product. Hence, the area between the demand curve and the shadow price may be calculated and defined as a consumer surplus.

The UCM has its own advantages and disadvantages. The advantages include that the model is based on people's observed behaviour and includes all possible components that influence the user and represent a shadow price of usage. In addition, all of today's free digital products have the necessary statistics about the number of visits/sessions, traffic consumed, time spent, etc., making it easier to collect and analyse user behaviour and thus calculate the willingness to pay for a free digital product. The disadvantages include a number of assumptions under which the model is supposed to work. These include the uniformity of visits within a user based on traffic consumed, the rationality of users and the calculation of the opportunity cost of time. The calculation of the value of personal data and the variable of the attention factor is an additional hurdle.

Nevertheless, the empirical results in this paper show the inconsistency, we would not completely reject this method as a possible method of economic evaluation of free digital products. We propose to conduct a broader empirical application with many more observations and more heterogeneous respondents to draw a final conclusion on the applicability of UCM.

3.7 Concluding Remarks and Further Considerations

This paper explores different methods of economic valuation and pricing of free digital products and proposes an alternative way to calculate the economic value and a shadow price of free digital products.

First of all, the features of free digital products have been listed. Based on the listed features, a usage cost model has been proposed. A theoretical framework has been described. Overall, the model consists of three steps: assuming and determining the shape of the demand curve, determining the "shadow" price of free digital products and the consumer surplus, and estimating their value.

It has been assumed that such products have a classical relationship between price and quantity demanded. Consequently, the demand curve should have a standard convex falling form.

The proposed usage cost model (UCM) considers the number of sessions (visits) to a free digital product and the usage costs as the demanded quantity and "price" of visits, respectively. The usage costs consist of the sum of variables such as, the adjusted subscription fee for Internet access per one visit, the opportunity costs of time per one visit, the value of personal data and the attention factor.However, due to the homogeneity of the personal data variable and the lack of data regarding the variable "attention factor", these two variables are not included in our empirical application later. Additional socio-economic variables (age, gender, marital status, presence of children) have also been included in the analysis. The purpose of this study is to determine whether the proposed model and method are applicable to the economic valuation of free digital products.

To demonstrate how the model works in practice, a survey within a small focus group has been conducted. Based on the results of this survey, the model has been implemented. As examples of free digital products have been chosen Facebook, Instagram and WhatsApp. Two types of regressions have been carried out: Poisson and Negative Binomial regressions. However, the empirical results show an inconsistency and hence are not used to calculate the consumer surplus and the value of free digital products.The possible reasons for this, as well as the advantages and disadvantages of the model, are discussed. One of the main advantages of this type of model is that it is based on revealed rather than hypothetical preferences, which can help to estimate economic value more objectively.

This study has introduced the model, which to the best of our knowledge, has not been implemented and tested yet in the case of free digital products. This provides an opportunity to test this model for the economic valuation and pricing of other free digital products.

This work has its limitations, which are relevant for future research. It is important to state that the goal of the paper was, first of all, to formulate a theoretical framework and to introduce and incorporate an alternative measure of the value of free digital products. The empirical part is represented by a small sample size to show how the model works. For future research, a broader survey is recommended in order to obtain more reliable and consistent estimates, and to make a final conclusion about the applicability of the model. In addition, more precise calculation of variables such as the value of personal data and the attention factor is needed. Moreover, due to the specific characteristics of the free digital products, listed in Section 3.2, it makes sense to study other examples of free digital products and the relationship between the price and quantity demanded. Another interesting area for future research would be to use a random utility model[55] instead of a modified version of TCM to construct a consumer demand and calculate the value of free digital products.

[55] Additional information about this kind of model one may find in Parsons (2003, pp. 296-324). The consumer uses the product, evaluating not only the usage costs but also the quality of the product.

Appendix F

Table 10: Groups of price definitions.

Source: Own creation based on Fetter (1912).

Variety	Objective-exchange-value definitions	Subjective-value definitions	Ratio-of-exchange definitions
Non-monetary definitions	Price is defined "in terms of value in the sense of purchasing power", not connected with the money expression. A. Smith (1776); T. Malthus (1820); J. Mill (1821) etc. According to A. Smith (1776): "The real price of everything really costs to the man who wants to acquire it, is the toil and trouble of acquiring it".	Price is defined "in the sense of desirability, estimation, subjective value", not connected with the money expression. J. P. Say (1803); C. Menger (1871); F. C. Hicks (1901) etc. According to Hicks (1901): "Value when measured is expressed in terms of the measure or unit of comparison, and this expression is price. Price, then, may be defined as value expressed in terms of a measure" (Fetter, 1912, p. 791).	Ratio-of-exchange definitions. Price is defined "in terms of value in the sense of a mere ratio of exchange, or bare mathematically", not connected with the money expression. G. Gunton (1900), I. Fischer (1908). According to Fisher (1906): "Price - a ratio of exchange. The value of goods (wealth, property, service) is the product of their quantity multiplied by the price" (Fetter, 1912, p. 797).
Monetary definitions	Price is defined "in terms of value in the sense of purchasing power", related to monetary value. Ricardo (1817); W. Perry (1878); Leroy-Beaulieu (1887), Marshall (1890) etc. According to Leroy-Beaulieu (1888): "When value is estimated in money it takes the name price" (Fetter, 1912, p. 789). It makes sense to add K. Marx here, because he interpreted price in monetary terms as well.	Price is defined "in the sense of desirability, estimation, subjective value", related to monetary value. J. Kudler (1845); E. B. Andrews (1888); A.S. Johnson (1909) etc. According to Gide (1891): "Value is desirability. The price of an object .. is its value expressed in terms of money" (Fetter, 1912, p. 792). Marginalists are another example in this category. Depending on the time period of the commodity (short or long), the price can be influenced either by supply or demand. The Marshall demand curve shows prices, which are the amount of money that the consumer is willing to pay to purchase a certain amount of a good.	Ratio-of-exchange definitions. Price is defined "in terms of value in the sense of a mere ratio of exchange, or bare mathematically", related with the money expression. W. Jevons (1871), E. Rambaud (1895), H. Davenport (1908). According to Jevons (1888): "Value implies, in fact, a relation. Value ...not an object, but a quality of an object". E. Rambaud (1895) continues his idea and defines the price as the monetary form of value, i.e. a ration of exchange (Fetter, 1912, p. 794).

Appendix G

Survey

Introduction

Thank you for participating in this survey. We are researching the economic value of Facebook, Instagram and WhatsApp. This survey is used for our research paper. Please read each question carefully. It will take about 10 minutes to answer the questions. All data is processed and considered anonymous and confidential.

General questions:

1. What is your gender?
   (a) Male
   (b) Female
   (c) Other/Prefer not to disclose
2. How old are you?
3. Are you married?
   (a) Yes
   (b) No
4. How many children do you have?
   (a) 0
   (b) 1
   (c) 2
   (d) 3 or more
5. How many hours per week do you work?
   (a) 40
   (b) 30
   (c) 20

(d) 10

(e) other, specify:

6. Please indicate your total gross income (before taxes and other deductions) for the last month:

(a) 1000 - 1500

(b) 1501 - 2000

(c) 2001 - 2500

(d) 2501 - 3000

(e) 3001 - 3500

(f) 3501 - 4000

(g) 4001 - 5000

(h) 5001 - 6000

(i) 6001 - 8000

(j) 8001 - 10000

(k) 10001 and more

7. What is your average cost of **mobile**[56] Internet access per month (in the case of fixed tariffs - the average cost of the total subscription fee)?

(a) I do not use mobile internet

(b) less than 10 Euro

(c) 10

(d) 11-20

(e) 21-30

(f) 31-40

(g) 41-50

(h) 51 and more

8. Which provider do you use in case of mobile Internet?

(a) I do not use mobile internet

(b) Vodafone

(c) Deutsche Telekom

(d) O2 – Telefonica

[56]Mobile Internet access means the access to the Internet from mobile devices (smartphones, tablets, portable modem, USB wireless modem etc.) through a mobile network.

(e) 1&1

(f) Unitymedia

(g) Other, specify:, ____

9. If possible, could you please name the tariff that you use for mobile internet? What is included in this tariff (how many minutes/megabyte or gigabyte, or you have unlimited minutes/megabyte)?

   ____

10. What is your average cost of **fixed-line**[57] Internet access per month (in the case of fixed tariffs - the average cost of the total subscription fee)?

    (a) I do not use fixed-line internet

    (b) less than 10 Euro

    (c) 10

    (d) 11-20

    (e) 21-30

    (f) 31-40

    (g) 41-50

    (h) 51 and more

11. Which provider do you use in case of fixed-line Internet?

    (a) I do not use fixed-line internet

    (b) Vodafone

    (c) Deutsche Telekom

    (d) O2 – Telefonica

    (e) 1&1

    (f) Unitymedia

    (g) Other, specify:, ____

[57]Fixed-line Internet access means the access to the Internet via cable, which uses metal wire or optical fiber telephone line. This type is typically used at home and/or offices.

12. If possible, could you please name the tariff that you use for fixed-line internet? What is included in this tariff (how many megabyte/gigabyte or you have unlimited megabyte/gigabyte)? ______________________________

Usage of Facebook:

1. Do you have an account on Facebook?
   (a) Yes
   (b) No
2. Over the last week how many times have you visited your account on Facebook?
   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5
   (f) 6
   (g) 7
   (h) 8
   (i) 9
   (j) 10 -20
   (k) 21- 30
   (l) 31 or more
3. How many minutes per visit do you spend on Facebook?
   (a) 1-5
   (b) 6-10
   (c) 11-15
   (d) 16-20
   (e) 21-30
   (f) 31-40
   (g) 41-50
   (h) 51-60
   (i) more than 60
4. What type of device and network connection do you primarily use to access Facebook?
   (a) Smartphone/mobile network
   (b) Computer, Laptop/fixed-line network
   (c) Computer, Laptop/mobile network

Usage of Instagram:

1. Do you have an account on Instagram?

(a) Yes

(b) No

2. Over the last week how many times have you visited Instagram?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6
(g) 7
(h) 8
(i) 9
(j) 10 -20
(k) 21- 30
(l) 31 or more

3. How many minutes per visit do you spend on Instagram

(a) 1-5
(b) 6-10
(c) 11-15
(d) 16-20
(e) 21-30
(f) 31-40
(g) 41-50
(h) 51-60
(i) more than 60

4. What type of device and network connection do you primarily use to access Instagram?

(a) Smartphone/mobile network

(b) Computer, Laptop/fixed-line network

(c) Computer, Laptop/mobile network

Usage of WhatsApp:

1. Do you have an account on WhatsApp?

(a) Yes

(b) No

2. Over the last week how many times have you visited WhatsApp?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6
(g) 7
(h) 8
(i) 9
(j) 10 -20
(k) 21- 30
(l) 31 or more

3. How many minutes per visit do you spend on WhatsApp?

(a) 1-5
(b) 6-10
(c) 11-15
(d) 16-20
(e) 21-30
(f) 31-40
(g) 41-50
(h) 51-60
(i) more than 60

4. What type of device and network connection do you primarily use to access WhatsApp?

(a) Smartphone/mobile network
(b) Computer, Laptop/fixed-line network
(c) Computer, Laptop/mobile network

5. What is your average data usage (in MB or GB) of Facebook, Instagram and WhatsApp over the last month?[58]

(a) Facebook: ________________________

[58]*In case if you use it primarily on your mobile device:

(a) **Android** system: Open the *Settings* of your mobile device, followed by *Connections* and then *Data Usage*. After that select the menu *Mobile Data Usage* and find how many megabyte/gigabyte you monthly spend on Facebook, Instagram and WhatsApp.

(b) **iPhone** system: Open the *Settings* of your mobile device, followed by *Cellular* and then *Cellular Data*. There you are able to find how many megabyte/gigabyte you monthly spend on Facebook, Instagram and WhatsApp.

In case if you use it primarily on your computer via fixed-line device:

(a) **Windows** system: Open the *Settings* of your computer/laptop, followed by *Network and internet*. Click *Data usage*. After that select the menu *Usage details* and find how many megabyte/gigabyte you monthly spend on Facebook, Instagram and WhatsApp.

(b) Instagram: ____________________

(c) WhatsApp: ____________________

Thank you for taking the time to complete this survey.

(b) **iOS** system: Open the *Settings* of your mobile device, followed by*Cellular* and then *Cellular Data*. There you are able to find how many megabyte/gigabyte you monthly spend on Facebook, Instagram and WhatsApp.

Appendix H

Table 11: List of variables defined

Variable	Definition
UC	Usage costs by individuals using Facebook, Instagram or WhatsApp. Usage costs consist of the sum of the following three components: a subscription fee for Internet access, the opportunity costs of the time spent using free digital products, and the value of personal data of users.
Visits	Dependent variable. Number of sessions/visits to the free digital product by users during the month.
Age	Age of users using Facebook, Instagram and WhatsApp.
MS	Marital Status. Married = 0, unmarried = 1.
Children	The presence of children. If user has children = 0, no children = 1.
Gender	Male = 0, Female = 1.

References

Ahmad, N., Ribarsky, J., and Reinsdorf, M. (2017). Can potential mismeasurement of the digital economy explain the post-crisis slowdown in gdp and productivity growth? *OECD Statistics Working Papers*.

Amoako-Tuffour, J. and Martinez-Espineira, R. (2008). Leisure and the opportunity cost of travel time in recreation demand analysis: A re-examination.

Appraisal Foundation (2020). *Uniform standards of professional appraisal practice, 2020-2021*. Washington, DC. Available at URL: `http://web1.amchouston.com/flexshare/003/AIHouston/2020.21USPAP.pdf` [date of access: 11 Januar 2021].

Aristotle (2009). *Aristotle: Nicomachean Ethics*. Oxford University Press.

Baker, R. and Ruting, B. (2014). Environmental Policy Analysis: A Guide to Non-Market Valuation. 2014 Conference (58th), February 4-7, 2014, Port Macquarie, Australia 165810, Australian Agricultural and Resource Economics Society.

Bank of Japan (2019). Corporate Goods Price Index (2015 base). Available at: `http://www.boj.or.jp/en/statistics/pi/cgpi_2015/index.htm` (accessed July 2019).

Baraldi, A. L. (2008). Network Externalities and Critical Mass in the Mobile Telephone Network: a Panel Data Estimation. MPRA Paper 13373, University Library of Munich, Germany.

Becker, G. M., Degroot, M. H., and Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science*, 9(3):226–232.

Belleflamme, P. (2016). The economics of digital goods: A progress report. *Review of Economic Research on Copyright Issues*, 13:1–24.

Bloch, H. (2020). Price theory, historically considered: Smith, ricardo, marshall and beyond. *History of Economics Review*, 75(1):50–73.

Bockstael, N. E., Strand, I. E., and Hanemann, W. M. (1987). Time and the recreational demand model. *American Journal of Agricultural Economics*, 69(2):293–302.

Bortkiewicz, L. v. (1923). Zweck und Struktur einer Preisindexzahl, erster Artikel. *Nordisk Statistik Tidskrift*, 2:369–408.

Borzykowski, N., Baranzini, A., and Maradan, D. (2017). A travel cost assessment of the demand for recreation in Swiss forests. *Review of Agricultural, Food and Environmental Studies*, 98(3).

Brouwer, W. (1999). Time and time costs in economic evaluation: taking a societal perspective. Dissertation. .

Brynjolfsson, E. (1996). The contribution of information technology to consumer welfare. *Information Systems Research*, 7(3):281–300.

Brynjolfsson, E., Collis, A., Diewert, W. E., Eggers, F., and Fox, K. J. (2019a). GDP-B: Accounting for the Value of New and Free Goods in the Digital Economy. NBER Working Papers 25695, National Bureau of Economic Research, Inc.

Brynjolfsson, E., Collis, A., Diewert, W. E., Eggers, F., and Fox, K. J. (2020). Measuring the impact of free goods on real household consumption. *AEA Papers and Proceedings*, 110:25–30.

Brynjolfsson, E., Collis, A., and Eggers, F. (2019b). Using massive online choice experiments to measure changes in well-being. *Proceedings of the National Academy of Sciences*, 116(15):7250–7255.

Brynjolfsson, E. and Oh, J. (2012). The attention economy: Measuring the value of free digital services on the internet. *International Conference on Information Systems, ICIS 2012*, 4:3243–3261.

Bureau of Labor Statistics (2018a). Import, Export Price Indexes. Available at: `https://www.bls.gov/mxp/ippfaq.htm` (accessed February 2019).

Bureau of Labor Statistics (2018b). Terms of Trade Indexes. Available at: `https://www.bls.gov/mxp/terms-of-trade.htm` (accessed February 2019).

Bureau of Labor Statistics (2020). Handbook of Methods. International Price Program: Overview. Available at: `https://www.bls.gov/opub/hom/ipp/home.htm` (accessed November 2020).

Burt, O. R. and Brewer, D. (1971). Estimation of net social benefits from outdoor recreation. *Econometrica*, 39(5):813–827.

Buxmann, P. (2002). Strategien von standardsoftware-anbietern: Eine analyse auf der basis von netzeffekten. *Schmalenbachs Zeitschrift für betriebswirtschaftliche Forschung*, 54.

Byrne, D., Corrado, C., and Sichel, D. E. (2018). The Rise of Cloud Computing: Minding Your P's, Q's and K's. NBER Working Papers 25188, National Bureau of Economic Research, Inc.

Celeste, R. K. and Bastos, J. L. (2013). Mid-point for open-ended income category and the effect of equivalence scales on the income-health relationship. *Revista de Saude Publica*, 47:168 – 174.

Cesario, F. J. (1976). Value of time in recreation benefit studies. *Land Economics*, 52(1):32–41.

Cesario, F. J. and Knetsch, J. L. (1970). Time bias in recreation benefit estimates. *Water Resources Research*, 6(3):700–704.

Ciriacy-Wantrup, S. V. (1947). Capital Returns from Soil-Conservation Practices. *American Journal of Agricultural Economics*, 29(4 (II)):1181–1196.

Clawson, M. and Knetsch, J. (2011). *Economics of Outdoor Recreation*. New York: RFF Press,, USA.

Corrigan, J. R., Alhabash, S., Rousu, M., and Cash, S. B. (2018). How much is social media worth? estimating the value of facebook by paying users to stop using it. *PLOS ONE*, 13(12):1–11.

Creel, M. and Loomis, J. (1990). Theoretical and empirical advantages of truncated count data estimators for analysis of deer hunting in california. *American Journal of Agricultural Economics - AMER J AGR ECON*, 72.

Czajkowski, M., Giergiczny, M., Kronenberg, J., and Englin, J. (2019). The individual travel cost method with consumer-specific values of travel time savings. *Environmental and Resource Economics*, 74:961–984.

Davis, R.K. (1963). The value of outdoor recreation: an economic study of the maine woods. Dissertation.

Day, M. (2021). 10 google analytics metrics you absolutely must track. Available at URL: https://www.impactplus.com/blog/marketing-metrics-on-google-analytics [date of access: 22 December 2021].

de Haan, J., Balk, B., and Hansen, C. (2010). Retrospective Approximations of Superlative Price Indexes for Years Where Expenditure Data Is Unavailable. In: Biggeri L., Ferrari G. (eds.), Price Indexes in Time and Space. Contributions to Statistics, 25-42, Physica-Verlag, Heidelberg. Available at URL: https://doi.org/10.1007/978-3-7908-2140-6_2 [date of acces: 29 September 2021].

DeSerpa, A. C. (1971). A theory of the economics of time. *The Economic Journal*, 81(324):828–846.

Deutsche Bundesbank (2021). Deutsche Bundesbank Statistics. Available at: https://www.bundesbank.de/en/statistics/exchange-rates. (accessed June 2021).

Diewert, E., Fox, K. J., and Schreyer, P. (2019). Experimental Economics and the New Commodities Problem. Microeconomics.ca working papers, Vancouver School of Economics.

Dormann, C. F. (2016). Overdispersion, and how to deal with it in R and JAGS. Available at URL: https://biometry.github.io/APES/LectureNotes/2016-JAGS/Overdispersion/OverdispersionJAGS.pdf/ [date of access: 19 April 2021].

Dridi, J. and Zieschang, K. (2004). Export and Import Price Indices. *IMF Staff Papers*, 51(1):157–194. DOI: https://doi.org/10.2307/30035869.

Egner, U. (2003). Umstellung des Verbraucherpreisindex auf Basis 2000. *Wirtschaft und Statistik*, (5):423–432.

Egner, U. (2013). Verbraucherpreisstatistik auf neuer Basis 2010. *Wirtschaft und Statistik*, (5):329–344.

Egner, U. (2019). Verbraucherpreisstatistik auf neuer Basis 2015. *Wirtschaft und Statistik*, (5):86–106.

Ehrenberg, R. and Smith, R. (2017). *Modern Labor Economics: Theory and Public Policy*.

Elbel, G. and Egner, U. (2008). Verbraucherpreisstatistik auf neuer Basis 2005. *Wirtschaft und Statistik*, (4):339–350.

Elder-Vass, D. (2019). No price without value: towards a theory of value and price. *Cambridge Journal of Economics*, 43(6):1485–1498.

Englin, J. and Shonkwiler, J. S. (1995). Estimating social welfare using count data models: An application to long-run recreation demand under conditions of endogenous stratification and truncation. *The Review of Economics and Statistics*, 77(1):104–112.

Eurostat (2017). Euros or dollars? EU trade in goods by currency. Available at: `https://ec.europa.eu/eurostat/en/web/products-eurostat-news/-/DDN-20170627-1` (accessed November 2020).

Facebook (2021). FB Earnings Presentations. Available at URL: `https://investor.fb.com/investor-events/default.aspx` [date of access: 15 April 2022].

Faia, M. A. (1981). Selection by certification: A neglected variable in stratification research. *American Journal of Sociology*, 86(5):1093–1111.

Feather, P. and Shaw, W. (1999). Estimating the cost of leisure time for recreation demand models. *Journal of Environmental Economics and Management*, 38(1):49 – 65.

Fetter, F. A. (1912). The definition of price. *The American Economic Review*, 2(4):783–813.

Fisher, I. (1906). *The nature of capital and income*. New York, The Macmillan Company; London, Macmillan and Co., ltd.

Fraumeni, B. (2019). *Measuring Economic Growth and Productivity: Foundations, KLEMS Production Models, and Extensions*. Academic Press. 554 p.

Freeman, A., Herriges, J., and Kling, C. (2003). *The Measurement of Environmental and Resource Values: Theory and Methods*.

Gehle, S. (2013). Methodenvergleich zwischen Preisindizes und Durchschnittswertindizes im Außenhandel. *Wirtschaft und Statistik*, 1(10):930–940. Available at: `https://m.bpb.de/system/files/dokument_pdf/destatis_Methodenvergleich_Indizes_Aussenhandel.pdf` (accessed June 2021).

Gide, C. (1891). *Principles of Political Economy*. D.C. Heath and Company.

Google (2021). How a web session is defined in universal analytics. Available at URL: `https://support.google.com/analytics` [date of access: 22 December 2021].

Goolsbee, A. and Klenow, P. J. (2006). Valuing consumer products by the time spent using them: An application to the internet. Working Paper 11995, National Bureau of Economic Research. Available at URL: `https://www.nber.org/papers/w11995` [date of access: 20 July 2021].

Goolsbee, A. and Petrin, A. (2004). The consumer gains from direct broadcast satellites and the competition with cable tv. *Econometrica*, 72(2):351–381.

Grazia, S. de (1962). *Of Time Work and Leisure*. The Twenty Century Fund, New York 1962.

Greene, W. H. (2012). *Econometric Analysis*. Prentice Hall, 7 edition.

Greenlees, J. S. and Williams, E. (2010). Reconsideration of Weighting and Updating Procedures in the US CPI. *Jahrbücher für Nationalökonomie und Statistik (Journal of Economics and Statistics)*, 230(6):741–758.

Gregory, R. and Furby, L. (1987). Auctions, experiments and contingent valuation. *Public Choice*, 55(3):273–289.

Hansen, C. B. (2007). Recalculations of the Danish CPI 1996 - 2006. Paper presented at the 10th Meeting of the International Working Group on Price Indices (Ottawa Group), Ottawa, October 9–12, 2007.

Hellerstein, D. and Mendelsohn, R. (1993). A Theoretical Foundation for Count Data Models. Technical report.

Hensher, D. A. (1997). Behavioral Value of Travel Time Savings in Personal and Commercial Automobile Trave. In Greene, D. L., Jones, D. W., and Delucchi, M. A., editors, *The Full Costs and Benefits of Transportation*, pages 245–278, Berlin, Heidelberg. Springer Berlin Heidelberg.

Herzberg, J., Knetsch, T. A., Schwind, P., and Weinand, S. (2021). Quantifying Bias and Inaccuracy of Upper-level Aggregation in HICPs for Germany and the Euro Area. Discussion Paper 06/2021, Deutsche Bundesbank. Available at URL: https://www.bundesbank.de/resource/blob/861158/4cc8ed30bf2fce8ff50e08c7119b9cf3/mL/2021-03-16-dkp-06-data.pdf [date of acces: 29 March 2021].

Herzog, B. (2018). Valuation of Digital Platforms: Experimental Evidence for Google and Facebook. *Review of Financial Studies*, 6:87.

Hicks, F. C. (1901). *Lectures on the Theory of Economics*. University of Cincinnati Press.

Hotelling, H. (1947). Letter to the National Park Service. U.S. Department of the Interior, National Park Service and Recreational Planning Division, Washington, D.C; Reprinted in, An Economic Study of the Monetary Evaluation of Recreation in the National Park, 1949.

Howdle, D. (2020a). The cost of fixed-line broadband in 206 countries. Available at URL: https://www.cable.co.uk/broadband/pricing/worldwide-comparison/ [date of access: 11 November 2020].

Howdle, D. (2020b). Worldwide mobile data pricing: The cost of 1GB of mobile data in 228 countries. Available at URL: https://www.cable.co.uk/mobiles/worldwide-data-pricing/ [date of access: 11 November 2020].

Huang, N., Wimalaratne, W., and Pollard, B. (2017). The Effects of the Frequency and Implementation Lag of Basket Updates on the Canadian CPI. *Journal of Official Statistics*, 33(4):979–1004.

Hui, K. L. and Chau, P. Y. K. (2002). Classifying digital products. *Commun. ACM*, 45(6):73–79.

ILO et al. (2004). Consumer Price Index Manual: Theory and Practice. Technical report, ILO/IMF/OECD/UNECE/Eurostat/The World Bank, Geneva, International Labour Office. Available at URL: https://www.ilo.org/public/english/bureau/stat/download/cpi/cpi_manual_en.pdf [date of acces: 29 March 2021].

IMF (2009). Export and Import Price Index Manual: Theory and Practice. DOI: https://doi.org/10.5089/9781589067806.069.

IMF (2018). Measuring the Digital Economy. Available at URL: https://www.imf.org/en/Publications/Policy-Papers/Issues/2018/04/03/022818-measuring-the-digital-economy [date of access: 13 October 2020].

INSEE (2019). Producer price indices of french industry - all markets. Available at: https://www.insee.fr/en/metadonnees/source/fichier/MET_PPI_BASE2015_EN.pdf (accessed January 2020).

Istat (2019). Foreign trade and industrial import prices. Available at: https://www.istat.it/en/archivio/229630 (accessed January 2020).

Jaffe, A. J. and Lusht, K. M. (2003). *The History of Value Theory: The Early Years*, pages 3–44. Springer US, Boston, MA.

Jevons, W. S. (1888). *The Theory of Political Economy*. London: Macmillan, 3rd ed. Available at URL: https://oll.libertyfund.org/title/jevons-the-theory-of-political-economy#lf0237_label_033 [date of access: 06 Januar 2021].

Kaulla, R. (1940). *Theory of the just price; a historical and critical study of the problem of economic value, by Rudolf Kaulla. Translated from the German by Robert D. Hogg.* G. Allen and Unwin, ltd., London.

King, J. E. and McLure, M. (2014). History of the concept of value. Economics Discussion / Working Papers 14-06, The University of Western Australia, Department of Economics.

Klick, J. (2018). Improving Initial Estimates of the Chained Consumer Price Index, Monthly Labor Review. Available at URL: https://doi.org/10.21916/mlr.2018.6 [date of access: 29 March 2021].

Kling, R. and Lamb, R. (1999). It and organizational change in digital economies: A socio-technical approach. *SIGCAS Comput. Soc.*, 29(3):17–25.

Klipfolio Inc (2021). Sessions. Available at URL: `https://www.klipfolio.com/metrics/marketing/sessions` [date of access: 22 December 2021].

Koiso-Kanttila, N. (2004). Digital content marketing: A literature synthesis. *Journal of Marketing Management*, 20(1-2):45–65.

Krämer, A. and Kalka, R. (2017). *How Digital Disruption Changes Pricing Strategies and Price Models*, pages 87–103.

Leroy-Beaulieu, P. (1888). *Précis d'économie politique*. Paris: Ch. Delagrave. Available at URL: `https://gallica.bnf.fr/ark:/12148/bpt6k112933x/f386.item.texteImage` [date of access: 08 Januar 2021].

Li, X.-B., Liu, X., and Motiwalla, L. (2021). Valuing personal data with privacy consideration. *Decision Sciences*, 52(2):393–426.

Lieshout, M. (2014). *Marc van Lieshout (2015). The Value of Personal Data. In: Jan Camenisch, Simone Fischer-Hubner, Marit Hansen (eds). Privacy and Identity Management for the Future Internet in the Age of Globalisation; pp. 26-38; London: Springer Verlag; 2015.*, pages 26–38.

Linde, F., Kock, M., and Gorges, A. (2012). Network effects of digital information goods a proposal for the operationalization of direct and indirect network effects. *International Journal of Business Research*, 3:12 p.

Lippe, P. v. d. and Mehrhoff, J. (2010). Unit Value Bias Reconsidered. *Paper presented at the 11th Meeting of the Ottawa Group*. Available at: `https://www.econstor.eu/bitstream/10419/37526/1/VfS_2010_pid_211.pdf` (accessed June 2021).

Marx, K. (1887). *Capital, Volume I*. Progress Publishers, Moscow, USSR. Available at URL: `https://www.marxists.org/archive/marx/works/download/pdf/Capital-Volume-I.pdf` [date of access: 05 Januar 2021].

McConnell, K. E. (1975). Some problems in estimating the demand for outdoor recreation. *American Journal of Agricultural Economics*, 57(2):330–334.

McConnell, K. E. (1992). On-site time in the demand for recreation. *American Journal of Agricultural Economics*, 74(4):918–925.

Menger, C. (1976). *Principles of Economics*. Ludwig von Mises Institute. Available at URL: http://www.hacer.org/pdf/Menger04.pdf [date of access: 06 Januar 2021].

Meyerhoff, J. (1999). Ökonomische Bewertung ökologischer Leistungen. Available at URL: http://www.jav.tu-berlin.de/fileadmin/a0731/uploads/publikationen/edocuments/1999_Meyerhoff_bfg.pdf [date of access: 26 October 2020].

Micken, K. S., Roberts, S. D., and Oliver, J. D. (2020). The digital continuum: the influence of ownership, access, control, and Cocreation on digital offerings. *AMS Review*, 10(1):98–115.

Mosquera, R., Odunowo, M., McNamara, T., Guo, X., and Petrie, R. (2019). The Economic Effects of Facebook. Available at URL: https://ssrn.com/abstract=3312462 [date of access: 15 October 2020].

Nagle, T. T. and Mueller, G. (2005). The strategy and tactics of pricing: A guide to growing more profitably.

National Research Council (2005). *Valuing Ecosystem Services: Toward Better Environmental Decision-Making*. The National Academies Press, Washington, DC.

Nicholas, H. (2011). *Marx's theory of price and its modern rivals / Howard Nicholas*. Palgrave Macmillan Basingstoke.

Nordström, M., Hansson, S. O., and Hugosson, M. B. (2019). The philosophical dimensions of urban transportation. *Essays in Philosophy*, 20:206–229.

NZIER (2018). What's the use of non-use values? Non-Use Values and the Investment Statement. Available at URL: https://www.treasury.govt.nz/sites/default/files/2018-08/LSF-whats-the-use-of-non-use-values.pdf [date of access: 26 October 2020].

OECD (2013a). Exploring the Economics of Personal Data: A Survey of Methodologies for Measuring Monetary Value. OECD Digital Economy Papers 220, OECD Publishing.

OECD (2013b). The OECD Privacy Framework. Available at URL: https://www.oecd.org/sti/ieconomy/oecd_privacy_framework.pdf [date of access: 26 September 2020].

OECD (2018). Cost-Benefit Analysis and the Environment. Available at URL: https://www.oecd-ilibrary.org/content/publication/9789264085169-en [date of access: 26 October 2020].

OECD (2021). Terms of trade (indicator). Available at: https://data.oecd.org/trade/terms-of-trade.htm. (accessed June 2021).

ONS (2017). Producer price indices QMI. Available at: https://www.ons.gov.uk/economy/inflationandpriceindices/methodologies/producerpriceindicesqmi (accessed January 2020).

Pantea, S. and Martens, B. (2014). The value of the internet for consumers. Available at URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2446962 [date of access: 20 July 2021].

Parker, R. N. and Fenwick, R. (1983). The pareto curve and its utility for open-ended income distributions in survey research. *Social Forces*, 61(3):872–885.

Parsons, G. (2003). The travel cost model, chapter 9. *Primer for Nonmarket Valuation*, pages 269–329.

Peter, F. (2009). Die Indizes der Außenhandelspreise auf Basis 2005. *Wirtschaft und Statistik*, (7):711–23. Available at: https://www.destatis.de/DE/Methoden/WISTA-Wirtschaft-und-Statistik/2009/07/aussenhandel-basis-2005-072009.html (accessed June 2021).

Peter, F. (2014). Die Indizes der Außenhandelspreise auf Basis 2010. *Wirtschaft und Statistik*, (4):262–82. Available at: https://www.destatis.de/DE/Methoden/WISTA-Wirtschaft-und-Statistik/2014/04/aussenhandel-basis-2010-042014.pdf?__blob=publicationFile&v=4 (accessed June 2021).

Peter, F. (2019). Die Indizes der Außenhandelspreise auf Basis 2015. *Wirtschaft und Statistik*, (3):36–53. Available at: https://www.

destatis.de/DE/Methoden/WISTA-Wirtschaft-und-Statistik/2019/03/aussenhandels-grosshandelsverkaufspreise-basis-2015-032019.pdf?__blob=publicationFile (accessed June 2021).

Post, G. (2009). An economic model for pricing digital products. HICSS '09, page 1–10, USA. IEEE Computer Society.

Pötzsch, K. (2004). Die Indizes der Außenhandelspreise auf Basis 2000. *Wirtschaft und Statistik*, (5):511–524. Available at: https://www.statistischebibliothek.de/mir/servlets/MCRFileNodeServlet/DEAusgabe_derivate_00000018/1010200041054.pdf;jsessionid=CE66647CC6224FAB686F48868950B3B7 (accessed June 2021).

Prince, J. and Wallsten, S. (2020). How much is privacy worth around the world and across platforms? *SSRN Electronic Journal*.

Rappoport, P., Kridel, D., Taylor, L., Alleman, J., and Duffy-Deno, K. (2003). Residential demand for access to the internet 1. *Emerging Telecommunications Networks*, 2:55–72.

Reinsdorf, M. and Schreyer, P. (2019). Measuring consumer inflation in a digital economy. *Statistics and Data Directorate Working Papers, Working Paper No. 101*.

Ricardo, D. (1817). *The principles of political economy and taxation*. Rod Hay's Archive for the History of Economic Thought, McMaster University, Canada. Available at URL: https://www.marxists.org/reference/subject/economics/ricardo/tax/index.htm [date of access: 04 Januar 2021].

Richard T. Carson and W. Michael Hanemann (2005). Chapter 17 Contingent Valuation. In Karl-Gran Mler and Jeffrey R. Vincent, editor, *Valuing Environmental Changes*, volume 2 of *Handbook of Environmental Economics*, pages 821 – 936. Elsevier.

RICS (1996). *RICS Appraisal and Valuation Manual*. Royal Institution of Chartered Surveyors.

Rohlfs, J. (1974). A theory of interdependent demand for a communications service. *The Bell Journal of Economics and Management Science*, 5(1):16–37.

Sewall, H. R. (1901). The theory of value before adam smith. *Publications of the American Economic Association*, 2(3):1–128.

Shapiro, C. and Varian, H. R. (1998). *Information Rules: A Strategic Guide to the Network Economy*. Harvard Business School Press, USA.

Shaw, W. D. (1992). Searching for the Opportunity Cost of an Individual's Time. *Land Economics*, 68(1):107–115.

Signorino, R. (2011). *David Ricardo on natural and market prices*. Available at URL: https://mpra.ub.uni-muenchen.de/39212/ [date of access: 04 Januar 2021].

Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations*. McMaster University Archive for the History of Economic Thought. Available at URL: https://geolib.com/smith.adam/woncont.html [date of access: 04 Januar 2021].

Smith, V. (1989). Taking stock of progress with travel cost recreation demand methods: theory and implementation. *Marine Resource Economics*, 6:279–310.

Smith, V. (1997). Pricing what is priceless: A status report on non-market valuation of environmental resources. *Environmental Economics*.

Smith, V. K., Desvousges, W. H., and McGivney, M. P. (1983). The opportunity cost of travel time in recreation demand models. *Land Economics*, 59(3):259–278.

Sobolewski, M. (2021). Measuring consumer well-being from using free-of-charge digital services. the case of navigation apps. *Information Economics and Policy*, page 100925.

Spintech (2015). 7 Website Analytics That Matter Most. Available at URL: https://www.spinutech.com/digital-marketing/analytics/analysis/7-website-analytics-that-matter-most/ [date of access: 22 December 2021].

Statista (2021). Projected revenue of Instagram from 1st quarter 2017 to 4th quarter 2020. Available at URL: https://www.statista.com/statistics/448157/instagram-worldwide-mobile-internet-advertising-revenue/ [date of access: 22 April 2021].

Statistics Canada (2019). International Merchandise Trade Price Index. Available at: https://www23.statcan.gc.ca/imdb/p2SV.pl?Function=getSurvey&SDDS=2203 (accessed January 2020).

Statistisches Bundesamt (2004). Preisindizes für die Einfuhr/Ausfuhr, Thematische Veröffentlichungen, Fachserie 17, Reihe 8.1/8.2 Preise, 1995-2004.

Statistisches Bundesamt (2017). Preise und Preisindizes für gewerbliche Produkte (Erzeugerpreise), Fachserie 17, Reihe 2. Available at: https://www.statistischebibliothek.de/mir/receive/DESerie_mods_00000239 (accessed April 2020).

Statistisches Bundesamt (2018). Preise: Verbraucherpreisindex für Deutschland. Statistisches Bundesamt, Wiesbaden.

Statistisches Bundesamt (2019). Index der Ein- und Ausfuhrpreise - Zusammenfassungen, Terms of Trade, Fachserie 17, Reihe 8.1.

Statistisches Bundesamt (2021). Genesis Database: Database of the Federal Statistical Office of Germany). Available at: https://www-genesis.destatis.de/genesis/online. (accessed June 2021).

Su, L., Adam, B., Lusk, J., and Arthur, F. (2017). Anchoring, Information, and Fragility of Choice Experiments: An Application to Consumer Willingness to Pay for Rice with Improved Storage Management. *Journal of Agricultural and Resource Economics*, 42(2):255–274.

Sudman, S., Mitchell, R., and Carson, R. (1989). Using surveys to value public goods: The contingent valuation method. *Contemporary Sociology*, 20:243.

Trefis Team (2020). How Much Revenue Can Facebook's WhatsApp Generate In The Next Five Years? Available at URL: https://www.forbes.com/sites/greatspeculations/2016/03/03/how-much-revenue-can-facebooks-whatsapp-generate-in-the-next-five-years/?sh=2c5104bd40ea [date of access: 21 April 2021].

Vicente, E. and Frutos, P. (2011). Application of the travel cost method to estimate the economic value of cultural goods: Blockbuster art exhibitions. *Hacienda Pública Española*, 196:37–63.

von Auer, L. and Shumskikh, A. (2022a). Retrospective Computations of Price Index Numbers: Theory and Application. Review of Income and Wealth. DOI: https://doi.org/10.1111/roiw.12627.

von Auer, L. and Shumskikh, A. (2022b). Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction. *Journal of Official Statistics*, 38(1):107–126.

von Auer, L. and Wengenroth, J. (2021). Consistent Aggregation With Superlative and Other Price Indices. *Journal of the Royal Statistical Society: Series A*, 184(2):589–615.

von Hippel, P. T., Hunter, D. J., and Drown, M. (2017). Better Estimates from Binned Income Data: Interpolated CDFs and Mean-Matching. *Sociological Science*, 4(26):641–655.

Wilman, E. A. (1980). The value of time in recreation benefit studies. *Journal of Environmental Economics and Management*, 7(3):272–286.

World Bank (2021). World Bank Commodity Price Data (The Pink Sheet). Available at: `https://www.worldbank.org/en/research/commodity-markets`. (accessed June 2021).

Zeithaml, V., Parasuraman, A. P., and Berry, L. (1985). Problems and strategies in service marketing. *Journal of Marketing*, 49:33–46.

Zhang, L. (2016). Smart data pricing in wireless data networks: an economic solution to congestion. Available at URL: `https://theses.lib.polyu.edu.hk/handle/200/8653` [date of access: 09 October 2020].

Špaček, J. and Antoušková, M. (2013). Individual single-site travel cost model for Czech paradise geopark. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 61(7):2851–2858.