Dissertation

Trust-region Methods for Flow Control based on Reduced Order Modelling

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Summary

The discretization of optimal control problems governed by partial differential equations typically leads to large-scale optimization problems. We consider flow control involving the time-dependent Navier–Stokes equations as state equation which is stamped by exactly this property.

In order to avoid the difficulties of dealing with large-scale (discretized) state equations during the optimization process, a reduction of the number of state variables can be achieved by employing a reduced order modelling technique. Using the snapshot proper orthogonal decomposition method, one obtains a low-dimensional model for the computation of an approximate solution to the state equation. In fact, often a small number of POD basis functions suffices to obtain a satisfactory level of accuracy in the reduced order solution. However, the small number of degrees of freedom in a POD based reduced order model also constitutes its main weakness for optimal control purposes. Since a single reduced order model is based on the solution of the Navier–Stokes equations for a specified control, it might be an inadequate model when the control (and consequently also the actual corresponding flow behaviour) is altered, implying that the range of validity of a reduced order model, in general, is limited. Thus, it is likely to meet unreliable reduced order solutions during a control problem solution based on one single reduced order model.

In order to get out of this dilemma, we propose to use a trust-region proper orthogonal decomposition (TRPOD) approach. By embedding the POD based reduced order modelling technique into a trust-region framework with general model functions, we obtain a mechanism for updating the reduced order models during the optimization process, enabling the reduced order models to represent the flow

dynamics as altered by the control. In fact, a rigorous convergence theory for the TRPOD method is obtained which justifies this procedure also from a theoretical point of view.

Benefiting from the trust-region philosophy, the TRPOD method guarantees to save a lot of computational work during the control problem solution, since the original state equation only has to be solved if we intend to update our model function in the trust-region framework. The optimization process itself is completely based on reduced order information only.