

Analysis of Portfolio Risk of Private Investors

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Abstract

This dissertation includes three research articles on the portfolio risks of private investors. In the first article, we analyze a large data set of private banking portfolios in Switzerland of a major bank with the unique feature that parts of the portfolios were managed by the bank, and parts were advisory portfolios. To correct the heterogeneity of individual investors, we apply a mixture model and a cluster analysis. Our results suggest that there is indeed a substantial group of advised individual investors that outperform the bank managed portfolios, at least after fees. However, a simple passive strategy that invests in the MSCI World and a risk-free asset significantly outperforms both the better advisory and the bank managed portfolios.

The new regulation of the EU for financial products (UCITS IV) prescribes Value at Risk (VaR) as the benchmark for assessing the risk of structured products. The second article discusses the limitations of this approach and shows that, in theory, the expected return of structured products can be unbounded while the VaR requirement for the lowest risk class can still be satisfied. Real-life examples of large returns within the lowest risk class are then provided. The results demonstrate that the new regulation could lead to new seemingly safe products that hide large risks. Behavioral investors who choose products based only on their official risk classes and their expected returns will, therefore, invest into suboptimal products. To overcome these limitations, we suggest a new risk-return measure for financial products based on the martingale measure that could erase such loopholes.

Under the mean-VaR framework, the third article discusses the impacts of the underlying's first four moments on the structured product. By expanding the expected return and the VaR of a structured product with its underlying moments, it is possible to investigate each moment's impact on them, simultaneously. Results are tested by Monte Carlo simulation and historical simulation. The findings show that for the majority of structured products, underlyings with large positive skewness are preferred. The preferences for variance and for kurtosis are ambiguous.

Keywords: individual investor, portfolio management, private banking, mixture model, cluster analysis, Value at Risk, structured products, risk measure, skewness, kurtosis

Zusammenfassung

Diese Dissertation umfasst drei Forschungsarbeiten zum Thema Portfoliorisiken bei privaten Investoren. In der ersten Arbeit analysieren wir einen großen Datensatz von Private-Banking-Portfolios einer schweizerischen Großbank. Dieser Datensatz weist die Besonderheit auf, dass ein Teil der Portfolios von der Bank verwaltet wurde, während der andere Teil Beratungsportfolios waren. Um die Heterogenität der einzelnen Anleger zu korrigieren, setzen wir ein Mixture-Model und eine Cluster-Analyse ein. Unsere Ergebnisse legen nahe, dass es tatsächlich innerhalb der Beratungsportfolios eine gewichtige Gruppe von Einzelinvestoren gibt, die in der Lage waren, die Verwaltungsportfolios zu übertreffen, zumindest nach Abzug der Gebühren. Doch eine einfache passive Strategie, die in den MSCI World Index und eine risikofreie Anlage investiert, übertrifft signifikant sowohl die besseren Beratungsportfolios als auch die Verwaltungsportfolios.

Die neue Regelung der EU für Finanzprodukte (UCITS IV) schreibt den Value-at-Risk (VaR) als Maßstab für die Einschätzung von Risiken strukturierter Produkte vor. In der zweiten Forschungsarbeit diskutieren wir die Grenzen dieses Ansatzes und zeigen, dass in der Theorie die erwartete Rendite von strukturierten Produkten unbegrenzt sein kann, während die VaR-Anforderung für die niedrigste Risikoklasse immer noch erfüllt sein kann. Beispiele aus der Praxis mit großen Renditen innerhalb der niedrigsten Risikoklasse werden dargestellt. Die Ergebnisse zeigen, dass es durch die neue Regelung möglich ist, riskante Produkte als scheinbar sicher zu bewerben. Verhaltensorientierte Investoren, die Produkte nur auf Basis ihrer offiziellen Risikoklassen und ihrer erwarteten Renditen wählen, werden dementsprechend in suboptimale Produkte investieren. Um diese Einschränkungen zu überwinden, schlagen wir eine neues Risiko-Rendite-Maß für Finanzprodukte auf Basis des Martingalmaßes vor, welches solche Schlupflöcher schließen könnte.

Im Rahmen des Mean-VaRs diskutiert die dritte Forschungsarbeit die Auswirkungen der ersten vier Momente des Basiswerts auf das strukturierte Produkt. Durch die Expansion der erwarteten Rendite und des VaRs eines strukturierten Produkts mit den Momenten des Basiswerts, ist es möglich, die Auswirkungen aller Momente auf sie gleichzeitig zu untersuchen. Die Ergebnisse werden durch Monte-Carlo-Simulation und historische Simulation getestet. Die Ergebnisse zeigen, dass für die Mehrheit der

strukturierten Produkte Basiswerte mit großer positiver Schiefe bevorzugt werden. Die Präferenzen für Varianz und für Kurtosis sind mehrdeutig.

Stichwörter: Einzelinvestor, Portfoliomanagement, Private-Banking, Mixture-Model, Cluster-Analyse, Value-at-Risk, Strukturierte Produkt, Risikomaß, Schiefe, Kurtosis

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Completing a PhD study is a work that requires effort and persistence. As I submit this dissertation, I cannot help recalling both the challenging and the joyful moments during my PhD study over the past years. Most importantly, many people come to mind to whom I am indebted and would like to express my gratitude at the beginning of this dissertation.

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Trier

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Chapter 1

Introduction

Studying the financial issues faced by individuals, namely household finance, has become a growing and substantial research field of finance in recent years. Besides asset pricing and corporate finance, the two traditional fields of financial research, household finance offers researchers a series of new challenges as well as new opportunities. A particular, and probably the biggest challenge in household finance is that individual investors' behavior and their preferences implied by this behavior deviate to some extent from standard textbook theories. Those standard financial theories have already been established in academia for decades, possibly even for up to a century. As stated by John Campbell in his presidential address to the American Finance Association in 2006: "Many households seek advice from financial planners and other experts, yet some households make decisions that are hard to reconcile with this advice or with any standard model."

Financial researchers' responses to this challenge are generally divided into two camps. The first group tends to consider the behavior of private investors to be boundedly rational, "rational" in the sense of standard financial theories, e.g. maximizing the von Neumann-Morgenstern utility. Research in this group identifies "mistakes" or "behavioral biases" of private investors, focuses on what individuals should do and discusses how their "mistakes" can be reduced by, for example, advisors or by regulators. Their approach is called "normative", or "neoclassical".

The second group views the issue in a “positive” way, as described by Campbell (2006). Researchers in this group often take the behavior of private investors for granted. They study the psychological roots of the individual’s behaviors, induce theoretical forms of their preferences, explain financial markets and asset prices with these preferences and discuss their further implications. Often, their approach is called “behavioral”. Researchers such as Hersh Shefrin, believe that the future of financial research will be more and more behavioral or “behavioralized” (Shefrin, 2009). “Normative” or “positive” is only a rough categorization of two research ideologies. They do not totally oppose, or exclude each other and research frequently overlaps between the two.

According to this categorization, the present dissertation mostly falls into the first group, the “normative” one. We discuss the risk, and accordingly the risk-return tradeoff, of the private investor’s portfolio, particularly in the context of financial advisors and regulators. The central question is whether and how private investors can be helped by advisors and by regulators. This dissertation is based on three of my research articles written during my PhD study from 2010 to 2014 at the University of Trier. They are “Should Your Bank Invest for Your? Evidence from Private Banking Accounts” (Cao et al., 2011), “Risk Classes for Structured Products: Mathematical Aspects and Their Implication for Behavioral Investors” (Cao and Rieger, 2013) and “How does the Underlying affect the Risk-Return Profiles of Structured Products?” (Cao, 2013). In the first article, we study whether advice and delegation help private investors in their decisions. The second and the third articles discuss the implications of the EU regulation for structured products, which are a type of financial products involving derivatives and have been popular among retail investors.

Chapter 2 of this dissertation is devoted to the first article. We analyze a large data set of private banking portfolios in Switzerland of a major bank with the unique feature that parts of the portfolios were managed by the bank, and parts were advisory portfolios. To correct the heterogeneity of individual investors, we apply a mixture model and a cluster analysis. Our results suggest that there is indeed a substantial group of advised individual investors that outperform the

bank managed portfolios, at least after fees. However, a simple passive strategy that invests in the MSCI World and a risk-free asset significantly outperforms both the better advisory and the bank managed portfolios.

The second and the third articles are presented together in Chapter 3, due to their similar research contexts. The new regulation of the EU for financial products (UCITS IV) prescribes Value at Risk (VaR) as the benchmark for assessing the risk of structured products. The second article (Section 3.2) discusses the limitations of this approach and shows that, in theory, the expected return of structured products can be unbounded while the VaR requirement for the lowest risk class can still be satisfied. Real-life examples of large returns within the lowest risk class are then provided. The results demonstrate that the new regulation could lead to new seemingly safe products that hide large risks. Behavioral investors who choose products based only on their official risk classes and their expected returns will, therefore, invest into suboptimal products. To overcome these limitations, we suggest a new risk-return measure for financial products based on the martingale measure that could erase such loopholes.

Under the mean-VaR framework, the third article (Section 3.3) discusses the impacts of the underlying's first four moments on the structured product. By expanding the expected return and the VaR of a structured product with its underlying moments, it is possible to investigate each moment's impact on them, simultaneously. Results are tested by Monte Carlo simulation and historical simulation. The findings show that for the majority of structured products, underlyings with large positive skewness are preferred. The preferences for variance and for kurtosis are ambiguous.

Chapter 2

Performance analysis of private banking accounts

2.1 Introduction

In the first part of the dissertation, we want to study whether it is worthwhile for individual investors to entrust their money to a portfolio manager of a private bank (discretionary account), or whether it would be sufficient to obtain well-informed investment advice from the bank (advisory account). Individual investors are known to be prone to suboptimal investments. Much research has been done on this topic, particularly on the comparison with institutional investors, who usually do more research when making investment decision, often have a larger search set of assets for purchase and sale, and devote more time to searching. Individual investors typically lack either discipline or professional knowledge. They may be overconfident and are more likely to be influenced by attention and news. This research studies the performance of individual investors in an advisory context – that is, with the assistance of an advisory service provided by a bank – and compares their investment performances with the performances of bank managed portfolios. In such a situation, individual investors to some extent share the same information and knowledge as the bank providing the advice. We try to identify in this case, whether institutional investors outperform individual

investors.

Among the literature studying the relation between individual and institutional investors, Barber et al. (2009) find that individuals lose from the trade and institutions win, by studying a complete transaction data on the Taiwan Stock Exchange. Based on data of a discount brokerage firm from the U.S., Barber and Odean (2000) document that the net return of average household is poor. Households underperform a market index by 1.1 percent annually on average. Research with focus on the performance of mutual funds, which are one of the typical institutional investments in financial markets, mainly concludes that actively managed funds on average, underperform their passively managed counterparts. However, there is still some research on the value of actively managed funds. In a comprehensive analysis of the mutual fund industry, Wermers (2000) finds that from 1975 to 1994 mutual funds held stock portfolios that outperform a broad market index by 1.3 percent per year. He concludes that funds pick stocks well enough to cover their costs. Some recent research also suggests that some trades by individual investors are profitable. Coval et al. (2005) find strong persistence in the performance of individual investors' trade, indicating that some individuals are able to earn abnormal return. Ivković and Weisbenner (2005) find that both individuals and institutions appear to be able to exploit local information to gain excess return. Although under-diversification is usually thought to be one of the common mistakes of individual investors, e.g. Goetzmann and Kumar (2008); Ivković et al. (2008) find that some individuals concentrating on a few securities tend to outperform those diversifying across many stocks.

The features of individual investors' behaviors are summarized in De Bondt (1998), who describes four main anomalies: biased perceptions of price movements, biased perceptions of values, error in managing risk and return and inadequate trading practices. The author also gives typical examples for these anomalies:

- “people are optimistic in bull markets and pessimistic in bear markets.”
- “few individual investors have an adequate understanding or are capable of using the valuation techniques [...]”

- “many households are under-diversified.”
- investment “discipline is difficult to maintain” for individual investors, i.e. strategies are changed too often.

These facts have been studied in many follow-up works which can be divided into several strands: overconfidence, under-diversification, effect of attention, etc.

The overconfidence literature relates the performance of individual investors with psychology. Studies have found that people tend to overestimate the precision of their knowledge. Such overconfidence has been observed in many fields. A comprehensive review is given in Odean (1998). Financial researchers extend this theory to the study of investors. In Odean (1998), the author models overconfidence “as a belief that a trader’s information is more precise than it actually is” and then studies the relationship between overconfidence and trading volume, volatility, return, etc. He finds inter alia, that “overconfidence increases expected trading volume, increases market depth, and decreases the expected utility of overconfident traders”. Odean (1999) focuses on one particular group of investors, those with discount brokerage account. He finds that “not only do the securities that these investors buy not outperform the securities they sell by enough to cover trading costs, but on average the securities they buy underperform those they sell”. The author concludes that overconfidence may contribute to this fact, but that there could be other reasons as well. Barber and Odean (2000) and Barber et al. (2009) investigate individual investors directly. Both papers assert that trading results in large losses for individual investors and the authors consider overconfidence as one of the explanations.

The under-diversification literature studies the common phenomenon that individual investors hold under-diversified portfolios, which, according to financial theory, might be irrational. Goetzmann and Kumar (2008) study U.S. individual investors and find that “the level of under-diversification is greater among younger, low-income, less-educated, and less-sophisticated investors”. Moreover, the authors find that “under-diversification is costly to most investors, while a small subset of them under-diversify due to superior information” and that “the level of under-diversification is correlated with investment choices that are consis-

tent with overconfidence, trend-following behavior, and local bias”. Ivković et al. (2008) study the phenomenon that concentrated individual investors outperform more diversified ones and give information advantages as an explanation. Information advantage is also studied in Ivković and Weisbenner (2005) for explaining the local bias of individual investors.

A review of the literature concerning the effect of attention is given by Barber and Odean (2008) , who investigate the effect of attention and news on individual investors. One of their conclusions is that “the buying behavior of individual investors is more heavily influenced by attention than is the buying behavior of professional investors”.

In Campbell (2006), the author points out that there is heterogeneity in the above mentioned behavioral effects across individual investors. Heterogeneity of individuals’ investment performances is also confirmed in Coval et al. (2005). As related topics, financial sophistication and literacy of investors are further studied for example in Calvet et al. (2009), where the authors confirm that “richer, educated households of larger size are less prone to making financial mistakes than other households”. van Rooij et al. (2011) find that a majority of households possesses limited financial literacy, which differs on different education, age and gender.

The role of financial advisors has already attracted attentions from researchers. Allen (2001) points out that financial institutions create an agency problem, where the investment decision makers do not necessarily own the assets. Krausz and Paroush (2002) model financial advisors’ behavior when facing a conflict of interest between themselves and investors paying for both financial advice and execution as a joint product. Inderst and Ottaviani (2009) analyze in a theoretical perspective the inherent conflict between the task of prospecting for customers and the task of advising for the needs of the customers when searching for suitable products. Bergstresser et al. (2009) study broker-sold and direct-sold funds from 1996 to 2004 and find no substantial tangible benefits delivered by brokers. Moreover, broker-sold funds have lower risk-adjusted performance than direct-sold funds, even before fees, and funds with higher fees are sold more. Kramer

(2009) finds no evidence of significant out- or underperformance of advised investors in comparison with self-directed investors. Hackethal et al. (2011) find that advised portfolios deliver lower net return and lower risk-adjusted performance than self-managed portfolios on average and this phenomenon is stronger with bank advisors than with independent financial advisors. Bhattacharya et al. (2011) study the case of unbiased financial advices. They find that the portfolio efficiency of investors following the advice increases, but that financial advice is hardly followed by those who receive it and thus that advised portfolios on average show no improvement of efficiency. They conclude that unbiased financial advice is a necessary but not sufficient condition for individual investors' benefit.

Different from the above-mentioned studies, this research compares advised portfolios and bank managed portfolios within one setting. The financial advisor for one client is at the same time the portfolio manager, and thus the final decision maker for another client. Given that the same institution plays different roles simultaneously, the comparison in this case will be more direct.

The dataset we use for this research stems from the private banking department of a large bank in Switzerland with mainly international clients. This unique data encompasses 4,870 clients for the years 2005 and 2006. A client could choose between two different mandates: an advisory (non-discretionary) mandate or a discretionary mandate. With the advisory mandate, the client himself determines, which investment to make at what time. The bank consults the client with regard to an appropriate investment and carries out the relevant transactions. With the discretionary mandate, the investment of the client is mainly taken care of by the bank. The client and the bank make an agreement on the investment policy, which is implemented as precisely as possible afterwards. Therefore, the advisory mandate can be considered as an individual investment in an advisory context, while the discretionary mandate is an institutional investment. For both of the mandates, the clients have to pay fees periodically, where the fee for the discretionary mandate is higher than that for the advisory mandate. A distinguishing feature of our data is that it contains both types of clients. Each client in our dataset is marked as either having an advisory mandate or a discretionary mandate.

The goal of our study is to compare the performance of these two groups. To do so, the most natural question needs to be answered first: Does the bank do a better job than the individual investors themselves? To assess the performance of the bank (the discretionary mandate clients) and the advisory mandate clients, the annualized return, the annualized volatility, the Sharpe ratio, the Beta coefficient, Jensen's alpha and the Treynor/Black ratio are calculated from the data, taking into account the fees paid by clients. In the advisory mandate group, we additionally allow for heterogeneous investors. Some of them might have "strange" portfolios, e.g. because they use their bank account for hedging of (unknown) other positions or because they invest in a rather hazardous way: they might be either overconfident, under-diversified or easy to be influenced by attention and news, etc. As such accounts will inevitably worsen the average performance of the advisory mandate group, we have been looking for a method to exclude them from the analysis. To this end, we employ the mixture model and a cluster analysis to identify potential subgroups among the advisory mandate clients. The mixture model is a tool for examining and representing the presence of subgroups of individuals within an overall population, without requiring that an observable variable should identify the subgroup to which an individual observation belongs. Our algorithm is done in R with the package `mixtools`, see Benaglia et al. (2009).

We draw two main conclusions from the empirical results. First, there is a substantial group of advised individual investors that outperforms the bank managed portfolios, at least after fees. Second, neither the better advisory portfolio nor the discretionary portfolio can beat the market. An index portfolio performs the best in our sample.

The rest of the chapter is organized as follows. Section 2.2 discusses the performance difference between the two mandate groups and the difference among subgroups of the advisory mandate. Section 2.3 compares the performance of advisory and discretionary portfolios with a simple two-fund strategy. Section 2.4 concludes.

2.2 Does the bank do a better job than individual investors?

2.2.1 Advisory mandate vs. discretionary mandate

Advisory mandate and discretionary mandate are two different services of private banking for investors. The advisory mandate allows the clients to make all their own investment decisions, whilst they have the access to the bank's research advice and execution services. The discretionary mandate authorizes the bank to manage a client's investment based on his investment objectives. Clients can remain involved and will receive reporting regarding the positioning and performance of their investment portfolio. The decision-making responsibility will lie with the bank.

The advantage of a discretionary mandate is a saving of time by relying completely on the expertise of the bank. With the expertise of the bank, clients can use the time saved to pursue their other commitments. The advantages of an advisory mandate are flexibility and autonomy. Some clients may want contribute more to the investment process than others, and are willing to make decisions on their own. However, how closely the client follows the advice given by the bank is up to him and not measurable in our data. A large degree of heterogeneity is to be expected. In our sample, the fee is 0.6% p.a. for the advisory mandate and 1.2% p.a. for the discretionary mandate.

2.2.2 Performance measures

In order to measure the performance of the portfolios, we calculate the Sharpe ratio, the Beta, Jensen's alpha and the Treynor/Black ratio, in addition to annualized return and volatility from the dataset, taking into account the fees paid by clients.

The Sharpe ratio (Sharpe (1966)) measures the excess return per unit of risk in an investment,

$$SR_i = \frac{r_i - r_f}{\sigma_i}, \quad (2.1)$$

where r_i and σ_i are the expected return and the volatility of portfolio i . r_f is the risk-free interest rate.

The Beta is a parameter in the capital asset pricing model (CAPM),

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}, \quad (2.2)$$

where $\text{Cov}(r_i, r_M)$ is the covariance between portfolio i and the market portfolio. σ_M^2 is the variance of the market portfolio.

The Jensen's alpha for portfolio i is defined as

$$\alpha_i = r_i - [r_f + \beta_{i,m}(r_M - r_f)], \quad (2.3)$$

where r_M and $\beta_{i,m}$ are expected market return and the Beta of the portfolio, respectively (Jensen (1968)).

The Treynor/Black ratio for portfolio i is defined as

$$TB_i = \frac{\alpha_i}{\sigma_{\epsilon i}}, \quad (2.4)$$

where α_i is the Jensen's alpha, $\sigma_{\epsilon i}$ is the standard deviation of the residual (Treynor and Black (1973)).

2.2.3 Correcting for investors: mixture model and cluster analysis

In this research we allow the advisory clients to be heterogeneous. As mentioned before, some of them might have "strange" portfolios, e.g. because they use their bank account for hedging of (unknown) other positions or because they invest in a rather hazardous way: They might be either overconfident, under-diversified or easy to be influenced by attention and news or have other alternative investment motifs. To identify these subgroups among the advisory mandate clients, we employ two different methods: the mixture model and a cluster analysis.

Much of the theory of mixture models is based on the assumption that the

full sample consists of subsamples, but it is unknown which individual belongs to which subsample. Subgroups follow a particular form of distribution and quite often this form is assumed to be univariate or multivariate normal. Suppose the random variables X_1, \dots, X_n are a random sample from a finite mixture of $m > 1$ arbitrary distributions, which are called components. The density of each X_i may be written as

$$g_{\theta}(X_i) = \sum_{j=1}^m \lambda_j \phi_j(X_i), \quad X_i \in \mathbb{R}^r, \quad (2.5)$$

where $\theta = (\lambda, \phi) = (\lambda_1, \dots, \lambda_m, \phi_1, \dots, \phi_m)$ denotes the parameter and the λ_j are positive with $\sum_{j=1}^m \lambda_j = 1$. The densities ϕ_j are assumed to be drawn from some family F of density functions. By expressing the density of each observation with the sum of several normal densities, the full sample is decomposed into several subsamples, without the need of knowing which observation belongs to which subgroup. Thus, the mixture model is a helpful technique in our study, as we also don't know which of the investors have the above-mentioned alternative investment motif. One of the estimation procedures for mixture model is the Expectation Maximization (EM) algorithms, for more detail see Benaglia et al. (2009) and references therein.

Another approach we employ is a cluster analysis. Cluster analysis assigns a set of observations into subsets, according to their dissimilarities or more precisely, their distances from each other. A common choice to measure the dissimilarity is the (squared) Euclidean distance,

$$d(x_i, x_{i'}) = \sum_{j=1}^p (x_{ij} - x_{i'j})^2, \quad (2.6)$$

where x_i and $x_{i'}$ are two multivariate observations from the sample, with $x_i = (x_{i1}, x_{i2}, \dots, x_{ip})^T$ and $x_{i'} = (x_{i'1}, x_{i'2}, \dots, x_{i'p})^T$. The clustering is done in such a way that the observations within each subset are more closely related with each other than observations assigned to different subsets. For more detail see, e.g., Hastie et al. (2009). By doing so, it is possible to identify subgroups within the full sample. In contrast to the mixture model, a cluster analysis needs no assumption

Mandate	Variable	Observations	Min	Max	Mean	Median	Std. deviation
Full sample	Return	4,870	-0.525	0.592	0.057	0.043	0.062
	Volatility	4,870	0.001	0.732	0.047	0.031	0.053
Advisory	Return	2,962	-0.525	0.592	0.064	0.043	0.076
	Volatility	2,962	0.001	0.732	0.059	0.043	0.064
Discretionary	Return	1,908	-0.009	0.221	0.046	0.043	0.025
	Volatility	1,908	0.005	0.230	0.029	0.028	0.016

Table 2.1: *Descriptive statistics for annualized return and volatility.*

about the distribution of the subsamples and thus has more flexibility.

2.2.4 Empirical results

Data

The data covers the period from the beginning of 2005 to the end of 2006. (For data protection reasons it is not possible to study a sample of current accounts.) Only natural persons are included in the analysis, i.e. firm clients, foundations, employees and so on are not considered. Each observation represents a client. In order to avoid duplication, the analysis considers only clients who held the same mandate during the whole examination period. Those who switched mandate within the two years, for example, from the discretionary mandate to the advisory mandate, are not considered. The average age of the discretionary mandate client is 64. Their average investment is 446,000 CHF. For the advisory mandate client, their average age is 61 and their investment is 1,118,000 CHF on average. There are 4,870 observations, 1,908 with discretionary mandate and 2,962 with advisory mandate. The portfolios are held by international clients investing into Switzerland, thus we use USD instead of CHF as the benchmark currency. We use the 12 month LIBOR USD of the year 2005 and 2006 as the risk-free interest rate, which gives a average value of 4.72%. For the market portfolio, we use the MSCI World USD of the same period, with an annualized return of 12.638%.

Figure 2.1 gives a scatter plot of the sample. The descriptive statistics are summarized in Table 2.1. The full sample's annualized return ranges from -0.525

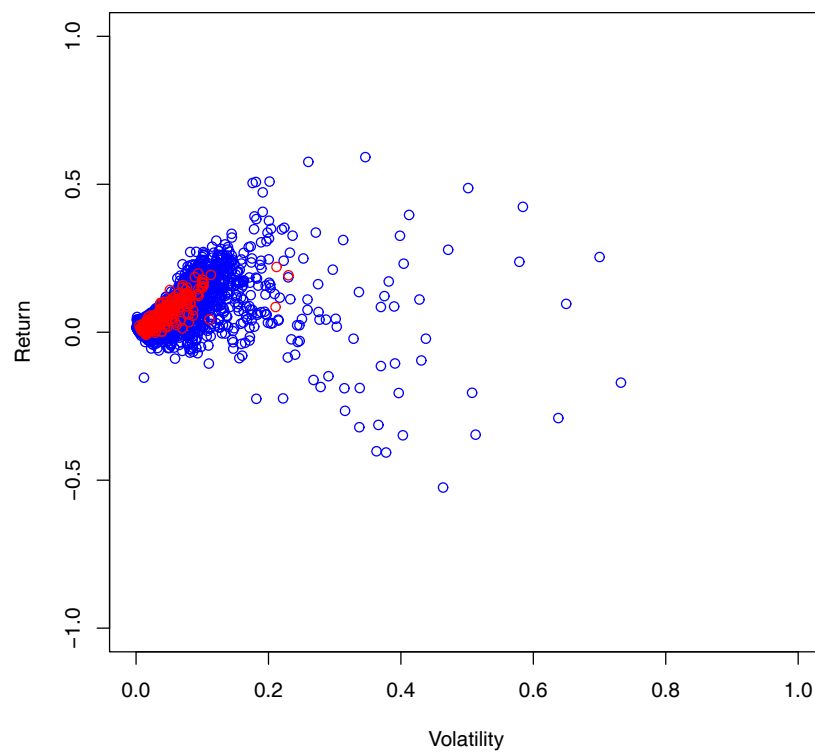


Figure 2.1: *Scatter plot of advisory mandate clients (blue) and discretionary mandate clients (red) for return and volatility.*

to 0.592, with a mean of 0.057 and a median of 0.043. The return of the advisory group varies between -0.525 and 0.592, with a mean of 0.064 and a median of 0.043. The discretionary group's return ranges from -0.009 to 0.221. The mean is 0.046 and the median is 0.043. The return of the advisory group is distributed more widely than that of the discretionary group. The standard deviation of the advisory group is 0.076, while the standard deviation of the discretionary group is 0.025 – thus, much lower than that of the advisory group. With a minimum of -0.525, the advisory clients have a much worse return than the discretionary mandate clients, whose minimal return is close to 0. However, the best performer of the advisory mandate clients exceeds the best of the discretionary mandate clients. With 0.592 the former has a surprisingly good annualized return, whereas the latter reaches a return of 0.221. While the mean return for the advisory mandate clients is higher than for the discretionary mandate clients, their respective medians, which are less likely to be influenced by outliers, are the same. The annual volatility of the full sample ranges from 0.001 to 0.732, both the minimum and the maximum are achieved by the advisory mandate clients. The mean and the median of the full sample's volatilities are 0.047 and 0.031, respectively. The discretionary mandate clients have a maximal volatility of 0.23, which is much lower than the advisory mandate client's maximum, while both groups have minimal volatilities close to 0. Both the mean and the median of volatilities for the advisory mandate clients are higher than for the discretionary mandate clients. The standard deviation of volatilities for advisory mandate clients is also much higher than for discretionary mandate clients. The former is 0.064 and the latter is 0.016. These facts support our assumption that the advisory clients are heterogeneous – more heterogeneous than the discretionary mandate clients. In sum, although the mean return of the advisory mandate clients is higher than that of the discretionary mandate clients, their medians of return are the same. No one outperforms the other. However, the discretionary group clearly has a lower volatility than the advisory group.

The Sharpe ratio, see Equation (2.1), the Beta, see (2.2), Jensen's alpha, see (2.3) and the Treynor/Black ratio, see (2.4), are given in Table 2.2. While

Mandate	Variable	Observations	Min	Max	Mean	Median	Std. deviation
Full sample	SR	4,870	-38.250	2.740	-0.721	-0.125	2.436
	β	4,870	-1.319	5.763	0.262	0.166	0.373
	α	4,870	-0.753	0.513	-0.011	-0.017	0.049
	TB	4,870	-39.370	2.632	-1.137	-0.641	2.364
Advisory	SR	2,962	-38.250	2.740	-0.931	-0.097	3.008
	β	2,962	-1.319	5.763	0.314	0.168	0.454
	α	2,962	-0.753	0.513	-0.009	-0.015	0.062
	TB	2,962	-39.370	2.632	-1.278	-0.524	2.941
Discretionary	SR	1,908	-4.985	1.868	-0.393	-0.167	0.960
	β	1,908	-0.177	2.232	0.181	0.165	0.160
	α	1,908	-0.070	0.086	-0.016	-0.019	0.015
	TB	1,908	-5.767	1.576	-0.917	-0.788	0.872

Table 2.2: *Descriptive statistics for Sharpe ratio (SR), Beta (β), Jensen's alpha (α) and Treynor/Black ratio (TB).*

the Sharpe ratio for advisory mandate clients ranges from -38.25 to 2.74, for discretionary mandate clients it varies between -4.985 and 1.868. The mean for the advisory group is -0.931, and -0.393 for the discretionary group. While the median for the advisory group is -0.097, that for the discretionary group is -0.167. The advisory mandate clients' Sharpe ratios are more widely distributed than the ones of discretionary mandate clients. For Beta, the values of the advisory group are more widely distributed than the ones of the discretionary group. Both the mean and the median of the former are higher than that of the latter. Jensen's alpha shows a similar result as Beta: Both the mean and the median of the discretionary group are lower than that of the advisory group, whose values are distributed more widely. Concerning the Treynor/Black ratio, the median of the advisory group is higher than the one of the discretionary group, while the mean of the former is lower than that of the latter. Judging from these performance measures, advisory mandate clients slightly outperform their discretionary mandate counterparts; however, the former are distributed more widely and have higher variation than the latter. Due to the ambiguous meanings of the negative Sharpe ratio and the negative Treynor/Black ratio, comparisons based on them are not so meaningful.

	Component 1	Component 2	Discretionary
Proportion	0.734	0.266	-
Mean	-0.017	0.015	-0.016
Median	-0.017	0.015	-0.019
Standard deviation	0.023	0.110	0.015

Table 2.3: *Result of mixture modeling for Jensen's alpha.*

Results from the mixture model approach

We then employ the mixture model to identify subgroups within the advisory mandate group based on Jensen's alpha. The Sharpe ratio and the Treynor/Black ratio are not considered in the analysis, because of their negative values. The procedure is done through the EM algorithm for normal mixtures. The modeling results are presented in Table 2.3. The density estimations for the components are given in Figure 2.2.

The advisory mandate clients are decomposed into two subgroups (components) by the algorithm. For comparison purposes, we again give the mean and median of the discretionary group next to the mixture modeling result in the table. Since the mixture model assume normal distributions, the mean is identical to the median for each component. The first component of the advisory group has a average Jensen's alpha of -0.017 and a standard deviation of 0.023. The second component's mean and standard deviation of Jensen's alpha are 0.015 and 0.11, respectively. The second component clearly outperforms the discretionary group by the mean and the median; however, the discretionary group has the lowest standard deviation. Welch's t -test on the two components gives a p -value much lower than 0.0001, indicating the mean Jensen's alphas of the two components are significantly unequal. The same test on the means between the second component and the discretionary group also delivers a significant result.

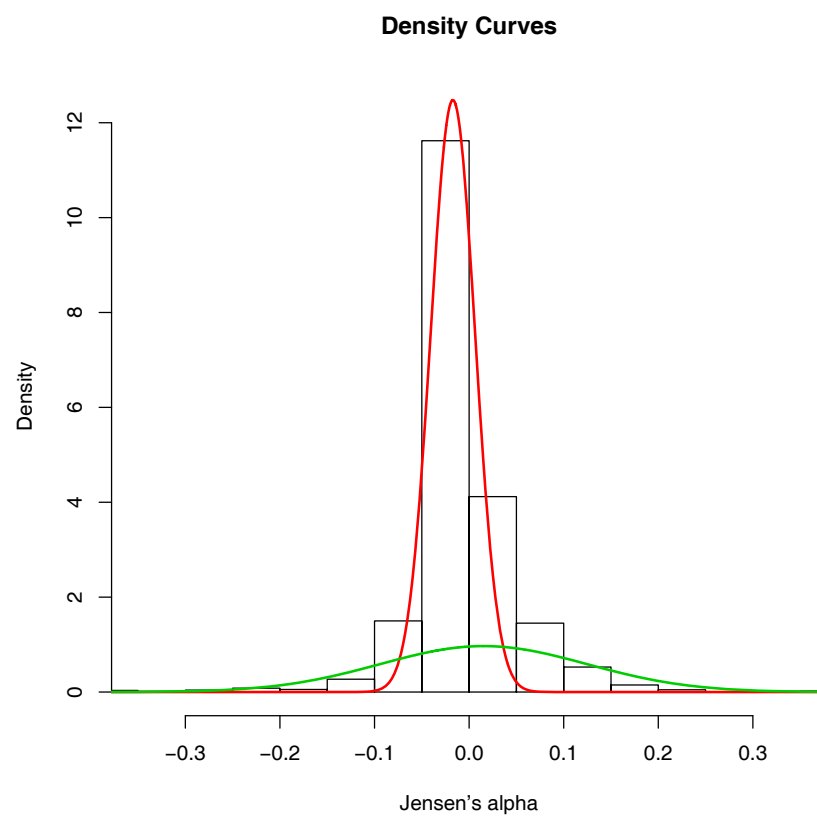


Figure 2.2: *Density estimations of two components for Jensen's alpha. Original data is presented as black histogram.*

Results from the cluster analysis

Moreover, cluster analysis is implemented in this study. We employ two-dimensional cluster analysis on return and volatility. The procedure is done with the Euclidean distance and hierarchical clustering. In the case of two clusters, one cluster has 2,943 observations and the other has 19 observations. The descriptive statistics are presented in Table 2.4, together with the descriptive statistics of the discretionary mandate group, for comparison purpose. A scatter plot is given in Figure 2.3, where the second cluster (Advisory II) represents the individuals with low return and high volatility. These investors corresponds to “weird” portfolios, i.e. portfolios with very poor performance suggesting other than usual investment motivations (hedging or “gambling”). In this case, the second cluster’s size might be too small for us to draw reasonable conclusions. In the case of three clusters, one cluster has 2,926 observations, another cluster has 19 observations and the third cluster has 17 observations, see Table 2.5 and Figure 2.4. Now the third cluster (Advisory III) represents the individual investors with high volatility and relatively high return. The second cluster, as before represents, “weird” portfolios. By excluding the second and the third clusters and focusing on the first cluster (Advisory I), we now have a relatively reasonable representation of the usual individual investors. The first cluster has the same median of return as the discretionary group, while the mean of the former is larger than that of the latter. Both the mean and the median of volatilities of the first cluster are higher than that of the discretionary group, respectively. Welch’s t -tests on returns, volatilities and Jensen’s alphas between the first cluster and the discretionary group all give p -values lower than 0.001, suggesting significantly unequal means of return, unequal means of volatilities and unequal means of Jensen’s alpha. In order to consider the influence of fee, we did the same tests on return (before fee) and Jensen’s alpha (before fee) between the first cluster and the discretionary group. The test on return (before fee) gives a p -value lower than 0.0001, while the test on Jensen’s alpha (before fee) delivers a p -value of 0.024.

To check robustness, we randomly sample 50% of the observations from each mandate and repeat several times the mixture model and cluster analysis. The

Mandate	Variable	Observations	Min	Max	Mean	Median	Std. deviation
Advisory I	Return	2,943	-0.154	0.592	0.066	0.043	0.071
	Volatility	2,943	0.001	0.700	0.057	0.043	0.057
	α	2,943	-0.397	0.513	-0.006	-0.015	0.054
Advisory II	Return	19	-0.525	-0.149	-0.270	-0.225	0.102
	Volatility	19	0.182	0.732	0.385	0.363	0.137
	α	19	-0.753	-0.177	-0.359	-0.320	0.143
Discretionary	Return	1,908	-0.009	0.221	0.046	0.043	0.025
	Volatility	1,908	0.005	0.230	0.029	0.028	0.016
	α	1,908	-0.070	0.086	-0.016	-0.019	0.015

Table 2.4: *Descriptive statistics of clusters and discretionary group for return, volatility and Jensen’s alpha (α).*

results are virtually the same as for the full sample. In sum, by mixture modeling Jensen’s alpha, we are able to identify subgroups among the individual investors. One subgroup is significantly better than the other. These “better” investors also significantly outperform the bank. By cluster analysis on return and volatility, we correct the whole group of individual investors by focusing on the main cluster. These investors have a significantly higher return and a higher volatility than the discretionary group. Judging from Jensen’s alpha, the main cluster of the individual investors significantly outperforms the bank. Even when differences in fee are not taken into account, the out-performance is significant, albeit only on a 5% level. This results is slightly puzzling, as it suggests that fees explain *most* of the performance difference, but probably not all.

2.3 Comparing performance with a passive investment

After comparing the performances of the advisory group with the discretionary group, it is natural to raise the question, how well the individual investors as well as the bank manage their investments, i.e. if they can beat the market. Our next step is to simply compare their performances with the two-fund strategy. A two-fund portfolio is constructed with a risk-free asset and risky assets to balance

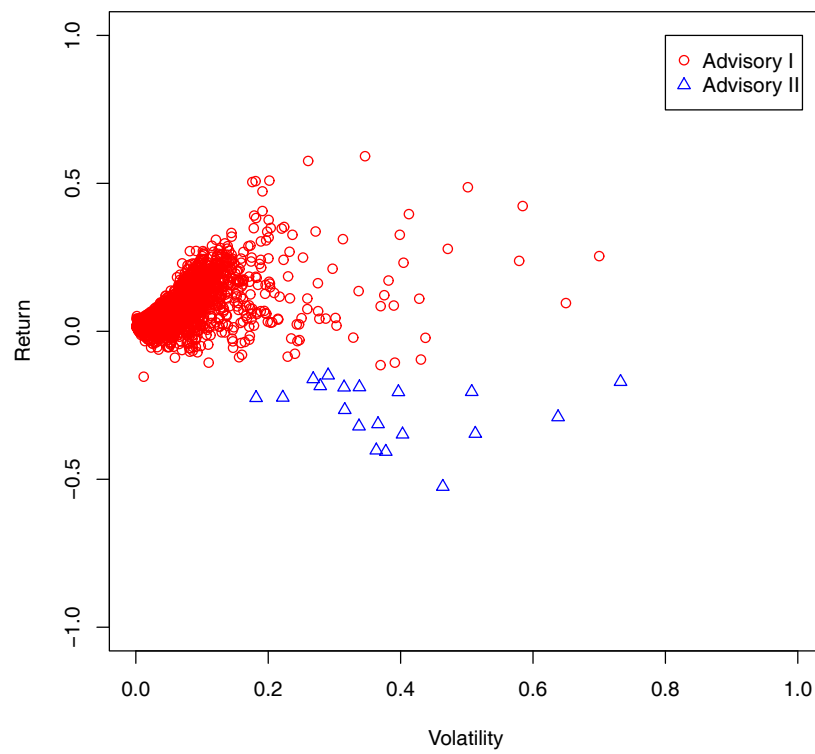


Figure 2.3: *Scatter plot of two clusters for return and volatility.*

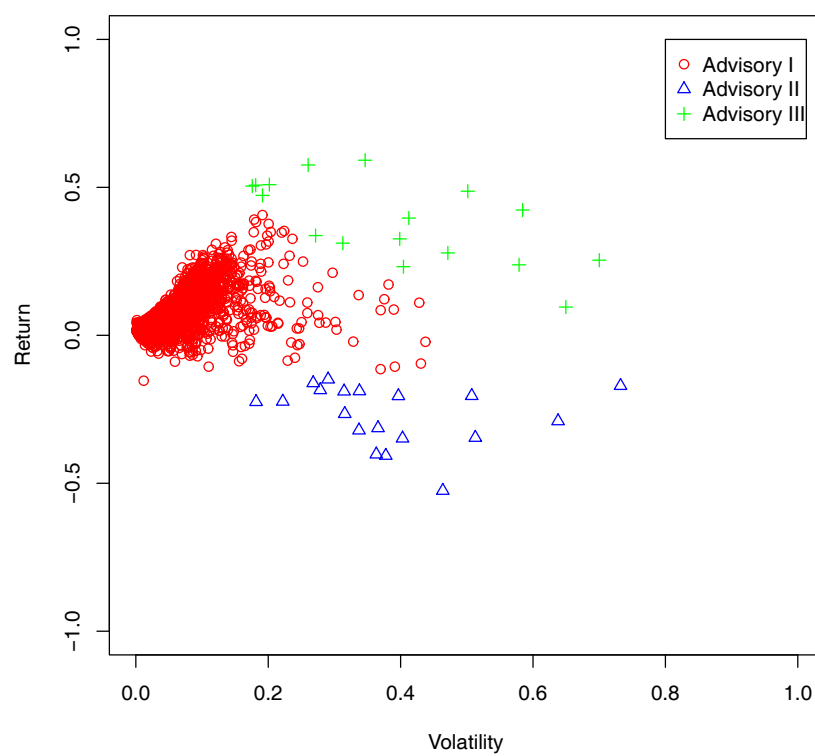


Figure 2.4: *Scatter plot of three clusters for return and volatility.*

Mandate	Variable	Observations	Min	Max	Mean	Median	Std. deviation
Advisory I	Return	2,926	-0.154	0.407	0.064	0.043	0.070
	Volatility	2,926	0.0001	0.438	0.055	0.042	0.050
	α	2,926	-0.397	0.388	-0.007	-0.015	0.050
Advisory II	Return	19	-0.525	-0.149	-0.270	-0.225	0.102
	Volatility	19	0.182	0.732	0.385	0.363	0.137
	α	19	-0.753	-0.177	-0.359	-0.320	0.143
Advisory III	Return	17	0.096	0.592	0.385	0.396	0.140
	Volatility	17	0.176	0.700	0.391	0.399	0.170
	α	17	-0.250	0.513	0.185	0.191	0.196
Discretionary	Return	1,908	-0.009	0.221	0.046	0.043	0.025
	Volatility	1,908	0.005	0.230	0.029	0.028	0.016
	α	1,908	-0.070	0.086	-0.016	-0.019	0.015

Table 2.5: *Descriptive statistics of clusters and discretionary group for return, volatility and Jensen's alpha (α).*

risk and return. If the market portfolio is chosen as the risky asset, then the allocation is along the capital market line, according to modern portfolio theory. In our analysis we choose the MSCI World ETF as the risky asset in our two-fund portfolio. Index ETFs (Exchange Traded Funds) are index funds that attempt to replicate a stock market index and can be traded in stock exchanges.

The MSCI World Index is listed in USD. We refer to the ETFs issued by financial institutions in the market for their fee requirements. An institution requires a fee of 0.4% p.a. for the MSCI World ETF. We then construct two-fund portfolios with different weights on risky asset, in our case the ETF, and the risk-free asset. Starting from 100% weight on the risky asset and 0% on the risk-free asset, to 75% weight on the risky asset and 25% on the risk-free asset, then to 50% weight on both, at last 25% weight on the risky asset and 75% on the risk-free asset. The risk-free asset in the MSCI World portfolio is the 12 months LIBOR USD. We choose the main cluster (the first cluster in the three clusters case of the cluster analysis) of the advisory group as a representation for individual investors. The annualized return (after fee), the volatility and Jensen's alpha of this two-fund portfolio with different portfolio weights are given in Table 2.6, together with the performances of the main cluster of the advisory group and the discretionary

Risky asset	Weight for risky asset	Return	Volatility	α
MSCI World ETF	100%	0.112	0.078	0
	75%	0.103	0.058	
	50%	0.084	0.039	
	25%	0.065	0.019	
Discretionary	-	0.043	0.028	-0.019
Advisory I	-	0.043	0.043	-0.015

Table 2.6: *Annualized return, volatility and Jensen’s alpha (α) for two-fund portfolio, discretionary group and the main cluster of advisory group. For discretionary group and main cluster of advisory group, the median is presented.*

group. Both the individual investors and the bank have a negative Jensen’s alpha. When the weight on the risky asset in the two-fund portfolio decreases to 25%, where its volatility is lower than the volatilities of the individual investors and the bank, the two-fund portfolio still delivers a higher return than the individuals and the bank do. Both the individual investors and the bank thus cannot beat the market.

Finally, it could of course be that non-standard investor preferences (e.g. larger degrees of loss aversion or specific investment goals) influence portfolio optimization in a way that makes simple risk-return optimization suboptimal and requires more sophisticated, asymmetric payoff profiles. Our analysis can *a priori* not exclude that this could explain why bank managed portfolios perform worse than passive investments when focusing solely on risk and return. Theoretical results (Hens and Rieger (2013)) however seem to suggest that this explanation would require highly non-standard risk preferences that are unlikely to hold for a larger number of investors. To sum up, it seems that our general result, namely that bank managed portfolios perform rather poorly as compared to passive investments and to a substantial group of advisory clients’ portfolios still holds.

2.4 Conclusion

We have analyzed a unique data set of private banking portfolios in Switzerland of a major bank. Parts of the portfolios were managed by the bank, parts were advisory portfolios. At first glance, individual investors and the bank did not clearly outperform each other by return and risk-adjusted performance, even when taking into account the higher fees for bank managed portfolios; however, bank managed portfolios on average did have a lower portfolio risk than individual investors' portfolios. This comparison, however, is not unbiased, since there might be groups of individual investors with non-standard investment goals, e.g. hedging or they might invest in a hazardous way.

We tried to correct for these effects in two different ways: First, we applied a mixture model approach on the risk-adjusted performances. Second, we conducted a cluster analysis on return and volatility. The results of our analysis seem to suggest that there is indeed a substantial group of advised individual investors that outperforms the bank managed portfolios, at least after fees.

This result itself does not necessarily mean that investors who entrust their money to a discretionary account make a bad decision: It seems likely that their financial skills are systematically lower than the skills of the investors who decide for an advisory mandate. Our final result, however, suggests that for both of these groups there is an easy way to improve their performances without the need of sophisticated financial abilities: We found that a simple passive strategy that invests in the MSCI World and a fixed interest asset (e.g. US government bonds) in our sample significantly outperformed both the better advisory and the discretionary portfolios.

Chapter 3

Risk classification for structured products

3.1 Introduction

The third chapter of the dissertation deals with the risk of structured products. Structured products (SPs) are a class of financial products which combine a set of elementary financial instruments, e.g. stocks and derivatives, in order to achieve specific investment purposes. Two typical examples are *capital protected products* and *discount certificates*: Capital protected products allow the investor to participate, to a certain degree, in the potential gains of a stock or an index (the *underlying*), whilst being protected against potential losses at the same time. This kind of products can be constructed by combining a call option with a fixed interest investment. Discount certificates offer shares of an underlying at a price below its current market price. In return, the investor must be prepared to accept a fixed maximum return (the cap). Investors will receive one share of the underlying per discount certificate if at maturity the underlying price is lower than the cap strike. If the underlying price at maturity is higher than or equal to the cap strike, investors will receive a cash settlement amount equivalent to the cap strike. A discount certificate can be constructed from the underlying and short calls. For an overview of structured products, see Blümke (2009).

According to C el erier and Vall ee (2013), assets under management (AUM) for retail structured products alone, is about 700 billion EUR in Europe in 2011, about 3% of all European financial savings or 12% of mutual funds' AUM. Although Europe is currently the world's largest market, the US and Asian markets are expanding rapidly. Due to structured products' relatively complicated structures and the occasional lack of transparency of their internal mechanisms, understanding their risk has always been an important issue for all investors, but especially for retail investors. One example is the case of the Hong Kong-listed company *Citic Pacific* and its 2 billion USD losses from the *accumulator*, a structured product that requires an investor to buy a specified amount of a security or currency at a fixed price, settled periodically, subject to certain conditions, as seen in a report by Santini (2008) from *The Wall Street Journal*.

In 2010, the European Commission has introduced a series of new directives, regulations and guidelines (European Commission (2010a,b), CESR (2010)) for the Undertakings for Collective Investments in Transferable Securities. This set of directives is usually referred to as "UCITS IV". Structured funds (another name for structured products) are one of the financial products considered by these regulations. The principle of the regulations is to use annual volatility as a risk measure to classify different products into different risk categories, in order to give investors a uniform view of their investment risk. The risk measurement for structured products is implemented through Value at Risk (VaR). Based on historical performance or simulation, the VaR of a structured fund is calculated at a given level. Then a corresponding volatility is computed from this VaR, based on the assumption that the return follows (log)-normal distributions. A similar approach based on VaR has also been adopted in Switzerland, see Swiss Structured Products Association (2013). VaR as a risk measure has been controversial for years in academia, see, e.g., Artzner et al. (1999) on the fact that VaR is not coherent. The financial crisis beginning in 2008 also revealed an abuse of VaR. One can naturally wonder, whether VaR is an appropriate risk measure for such complicated financial instruments as structured products and whether the newly introduced EU regulations can give a sufficient indication for the risk of structured

products.

Research literature on structured products has mainly focused on the seller's side, e.g., pricing and hedging of the products. Recently, attention has begun to be paid to the buyer's perspective. Branger and Breuer (2008) investigate the utility benefit that CRRA investors gain from investment certificates. Das and Statman (2009) propose a new framework of portfolio optimization, which differs from the mean-variance portfolio theory, and use this framework to analyze structured products. Breuer and Perst (2007) analyze the investors' utility for buying structured products in the cumulative prospect theory framework. Hens and Rieger (2013) conclude that, from a theoretical point of view, only behavioral factors like loss aversion, gambling to avoid sure losses, probability weighting and misestimation, overconfidence, etc. can explain the demand for the majority of structured products. Rieger (2012) conducts experiments and concludes that a systematic probability misestimation is the main driver for the attractiveness of some of the most popular structured products.

Portfolio choice under VaR or other downside risk measures is another strand of the literature. Basak and Shapiro (2001) discuss the optimal portfolio policy of a utility maximizing investor with the VaR constraint. Alexander et al. (2006) compare mean-VaR model to the mean-variance analysis. Benati (2003) solves the portfolio choice problem with coherent risk measure constraint by linear programming. Cui et al. (2013) compare different approximation methods of VaR estimation for portfolio with derivatives, where the analysis is mainly based on normal distributed risk factors.

In the second section (Section 3.2) of this chapter, we firstly analyze the problem from a theoretical perspective. We show that measuring the risk of structured products with VaR has limitations – in theory the expected return of the structured products can be infinite positive while the VaR requirement for the lowest risk class can still be satisfied. Then we directly use market data to give practical examples showing that this problem is not merely a theoretical one. Furthermore, we propose a new theoretical approach to measure the performance of financial products and apply this approach to market data.

The third section (Section 3.3) of the chapter will not focus on the design of the payoff function of the structured product, but on the underlying. The aim is to discuss whether and how the underlying's return distribution, especially its first four moments, can affect the risk-return profile of a structured product under the mean-VaR framework. The outcomes will be relevant for the product design from the seller's side and for the portfolio planning from the buyer's side, as well as for the risk classification from the regulator's side.

Studies on the preference for skewness and higher moments of the return distribution have begun from Kraus and Litzenberger (1976), where the authors state that positive skewness is preferred by investors. Their analysis is based on expanding the expected utility with Taylor series to cubic term. Scott and Horvath (1980) show that the preference direction for positive odd central moments is positive and for even central moments, it is negative. They expand the expected utility to higher order term. Since then, it has become common to carry out the discussion under the expected utility framework. Recent studies include: explaining underdiversification with a mean-variance-skew model (Mitton and Vorkink (2007)). Chang et al. (2013) derive skewness from option prices and investigate the impact of the implied skewness on underlying returns with Capital Asset Pricing Model (CAPM)-like models.

In the third section, we first show that there is no one-to-one relation between the expected return and the VaR of a structured product. Switching underlyings does affect the risk-return profile of a product. Then, we expand both the expected return and the VaR of a structured product with its underlying's first four moments. This allows us to discuss the impact of each moment on both the expected return and the VaR, simultaneously. The theoretical results are then tested by Monte Carlo simulations, where we consider the cases of normal distribution, t -distribution and NIG-distribution, as the distribution for the underlying log-return. Structured products considered in the simulations are tracker certificates, discount certificates and capped outperformance certificates. Simulation with real-world data is also carried out on three structured products. Underlyings are seven major European stock market indices.

3.2 Mathematical aspects and their implications

This section deals with the limitation of using VaR as the risk measure for structured products and its implications. It is organized as follows: Subsection 3.2.1 develops theoretical arguments and gives practical examples. Subsection 3.2.2 proposes a risk-return measure for financial products. Subsection 3.2.3 concludes.

3.2.1 VaR as risk measure for structured products

Theoretical limitations

Let the underlying price be a random variable X on a probability space (Ω, \mathcal{F}, P) , with the probability density function $f(x)$ and the cumulative distribution function $F(x)$. We normalize X_0 , the underlying price at time 0, to be 1 and further assume that X is nonnegative. The payoff (value) of the structured product is a function $y : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ of X . We assume that $y(x)$ is nondecreasing, see Rieger (2011). y_0 (the value of the structured product at time 0) is also normalized to 1. $\ell(x)$ is the Radon-Nikodym derivative of the martingale measure P^* with respect to the physical measure P . We assume that low payoffs are relatively more expensive than high payoffs, i.e., $\ell(x)$ is decreasing, and additionally assume that $\ell(x) \rightarrow 0$ for $x \rightarrow \infty$. R_f and r_f denote gross and net risk-free interest rate, respectively.

In order to attract investors, the issuer (bank or other financial institution) of the structured products wants to design products which, according to the EU regulation, have a low risk while still giving large expected return. Translated into mathematics, the issuer wants to have a product $y(X)$ which maximizes its expected return under the physical probability measure and meets the VaR constraint at the same time. The issuer, thus, faces the problem of maximizing the expectation of $y(X)$ over all y ,

$$\max_y E[y(X)] = \max_y \int_{\mathbb{R}_+} y(x) f(x) dx, \quad (3.1)$$

subject to the VaR constraint

$$-\inf\{m|\mathbb{P}(\ln(y(X)) \leq m) > \alpha\} = VaR_\alpha. \quad (3.2)$$

Proposition 3.2.1. *Under the VaR constraint (3.2), there exists an y , such that $\mathbb{E}[y(X)]$ is unbounded from above.*

Proof. Because we assumed that $y(\cdot)$ is nondecreasing, the VaR constraint (3.2) can be rewritten as

$$-\inf\{m|\mathbb{P}(X \leq y^{-1}(\exp(m))) > \alpha\} = VaR_\alpha,$$

which means

$$\mathbb{P}(X \leq y^{-1}(\exp(-VaR_\alpha))) \geq \alpha.$$

We additionally assume that $\exp(-VaR_\alpha) < R_f$, i.e. that the gross return at the VaR is smaller than the risk-free gross rate. This is usually the case: the gross return at VaR is usually below 1 (a loss), while R_f is usually above 1.

Let $z = y^{-1}(\exp(-VaR_\alpha))$. Construct a sequence

$$y_n(x) = \begin{cases} 0, & x \leq z, \\ \exp(-VaR_\alpha), & z < x \leq n, \\ K_n, & x > n. \end{cases} \quad (3.3)$$

Then it holds for every $n > z$ that

$$\mathbb{P}(y_n(X) = 0) = \mathbb{P}(X \leq z) \geq \alpha,$$

$$\mathbb{P}(y_n(X) \leq \exp(-VaR_\alpha)) > \alpha,$$

$$-\inf\{m|\mathbb{P}(y_n(X) \leq \exp(m)) > \alpha\} = VaR_\alpha,$$

thus, the VaR constraint (3.2) is fulfilled.

Due to no-arbitrage, we have under the martingale measure P^*

$$E^*[y_n(X)] = \int_{\mathbb{R}_+} y_n(x) \ell(x) f(x) dx = R_f, \quad (3.4)$$

which is equivalent to

$$\int_0^z 0 \ell(x) f(x) dx + \int_z^n \exp(-VaR_\alpha) \ell(x) f(x) dx + \int_n^\infty K_n \ell(x) f(x) dx = R_f,$$

then we have

$$K_n = \frac{R_f - \exp(-VaR_\alpha) \int_z^n \ell(x) f(x) d(x)}{\int_n^\infty \ell(x) f(x) d(x)}. \quad (3.5)$$

When $n \rightarrow \infty$, the numerator of (3.5) converges to some nonzero constant and the denominator converges to 0. Thus, $\lim_{n \rightarrow \infty} K_n = \infty$.

$$E[y_n(X)] = \int_0^z 0 f(x) dx + \int_z^n \exp(-VaR_\alpha) f(x) dx + \int_n^\infty K_n f(x) dx \quad (3.6)$$

$$\begin{aligned} &= \exp(-VaR_\alpha)(F(n) - F(z)) + \frac{R_f - \exp(-VaR_\alpha) \int_z^n \ell(x) f(x) d(x)}{\int_n^\infty \ell(x) f(x) d(x)} (1 - F(n)) \\ &= \exp(-VaR_\alpha)(F(n) - F(z)) + \left(R_f - \exp(-VaR_\alpha) \int_z^n \ell(x) f(x) d(x) \right) \frac{1 - F(n)}{\int_n^\infty \ell(x) f(x) d(x)}. \end{aligned} \quad (3.7)$$

When $n \rightarrow \infty$, the term $\exp(-VaR_\alpha)(F(n) - F(z))$ and the term

$R_f - \exp(-VaR_\alpha) \int_z^n \ell(x) f(x) d(x)$ in equation (3.7) converge to some nonzero

constants, respectively. The term

$$\frac{1 - F(n)}{\int_n^\infty \ell(x)f(x)d(x)} = \frac{1 - F(n)}{1 - \int_0^n \ell(x)f(x)d(x)}, \quad (3.8)$$

according to L'Hopital, when $n \rightarrow \infty$, equals

$$\frac{-f(n)}{-\ell(n)f(n)} = \frac{1}{\ell(n)} \rightarrow \infty,$$

because we assumed that $\ell(n) \rightarrow 0$ for $n \rightarrow \infty$.

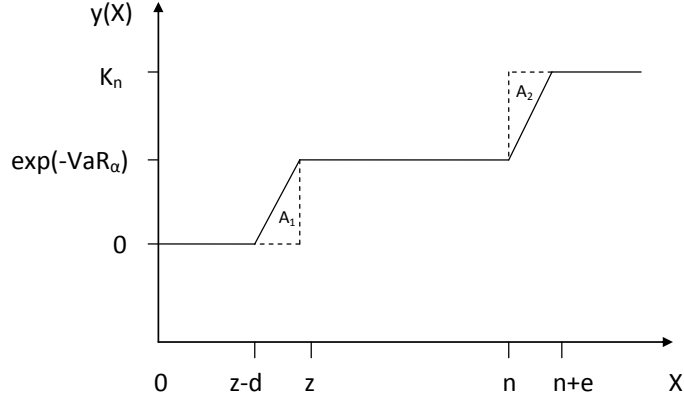
Thus, we have $\lim_{n \rightarrow \infty} E[y_n(X)] = \infty$. \square

Generally, if we see the payoff of the products as a function of the underlying's return, the above proof constructs a type of product that gives investors nothing at the lower tail of the underlying's return distribution. In a large part of the distribution, it gives the investor exactly the log return of the VaR but not more than that. At the upper tail of the distribution, it gives a relatively large return. Furthermore, as we see from the above proof, if this return at the upper tail goes to infinity while its probability goes to zero, the expected return of the whole product (at least theoretically) goes to infinity. This construction meets the VaR constraint and gives (arbitrarily) large expected returns.

We now show that the products offering such a return profile can be constructed with four European call options with the same maturity:

$$y_t = e^{-r_f(T-t)} \left(\frac{\exp(-VaR_\alpha)}{d} (C_t^1 - C_t^2) + \frac{K_n - \exp(-VaR_\alpha)}{e} (C_t^3 - C_t^4) \right). \quad (3.9)$$

The four call options C^1 , C^2 , C^3 and C^4 have strikes $K^1 = z - d$, $K^2 = z$, $K^3 = n$ and $K^4 = n + e$, respectively. C_t^i , $i = 1, 2, 3, 4$, are their values at time t . This means we construct a structured product by longing $\frac{\exp(-VaR_\alpha)}{d}$ units of C^1 , shorting the same number of C^2 , longing $\frac{K_n - \exp(-VaR_\alpha)}{e}$ units of C^3 and shorting again the same amount of C^4 . d and n are parameters of the product which can be adjusted by the issuer. Figure 3.1 gives the payoff diagram of this product.

Figure 3.1: *Payoff diagram*

At maturity time T , the value of this structured product is:

$$\begin{aligned}
 y_T &= \frac{\exp(-VaR_\alpha)}{d}(C_T^1 - C_T^2) + \frac{K_n - \exp(-VaR_\alpha)}{e}(C_T^3 - C_T^4) \quad (3.10) \\
 &= \frac{\exp(-VaR_\alpha)}{d}((X_T - K^1)^+ - (X_T - K^2)^+) + \frac{K_n - \exp(-VaR_\alpha)}{e}((X_T - K^3)^+ - (X_T - K^4)^+) \\
 &= \frac{\exp(-VaR_\alpha)}{d}((X_T - (z-d))^+ - (X_T - z)^+) + \frac{K_n - \exp(-VaR_\alpha)}{e}((X_T - n)^+ - (X_T - (n+e))^+).
 \end{aligned}$$

By doing so, we are able to construct a structured product with a payoff profile:

$$y_n(X) = \begin{cases} 0, & X \leq z-d, \\ \frac{\exp(-VaR_\alpha)}{d}(X - z + d), & z-d < X \leq z, \\ \exp(-VaR_\alpha), & z \leq X \leq n, \\ \exp(-VaR_\alpha) + \frac{K_n - \exp(-VaR_\alpha)}{e}(X - n), & n \leq X \leq n+e, \\ K_n, & X > n+e. \end{cases} \quad (3.11)$$

This product satisfies the VaR constraint (3.2).

Additionally, $y_n(X)$ has to fulfill the no-arbitrage condition (3.4). Thus, the

areas of the triangles A_1 and A_2 in the payoff diagram Figure 3.1 must be equal under P^* :

$$\int_{z-d}^z \frac{\exp(-VaR_\alpha)}{d} (x - z + d) dP_X^*(x) = \int_n^{n+e} \frac{K_n - \exp(-VaR_\alpha)}{e} (x - n - e) dP_X^*(x). \quad (3.12)$$

The expected return of the product is then given by

$$\begin{aligned} E[y(X)] &= 0 \cdot P(X \leq z-d) + \int_{z-d}^z \frac{\exp(-VaR_\alpha)}{d} (x - z + d) f(x) dx + \exp(-VaR_\alpha) P(z \leq X \leq n) \\ &+ \int_n^{n+e} \left(\exp(-VaR_\alpha) + \frac{K_n - \exp(-VaR_\alpha)}{e} (x - n) \right) f(x) dx + K_n P(X > n + e). \end{aligned} \quad (3.13)$$

The first four terms are finite. The last term $K_n P(X > n + e)$ equals

$$K_n (1 - F(n + e)) = \frac{R_f - \exp(-VaR_\alpha) \int_n^n \ell(x) f(x) d(x)}{\int_n^\infty \ell(x) f(x) d(x)} (1 - F(n + e)), \quad (3.14)$$

with K_n derived from the no-arbitrage condition and equation (3.5). Following the same procedure as the proof of Proposition 3.2.1, (3.14) is unbounded, when n goes to infinity, i.e., $\lim_{n \rightarrow \infty} E[y_n(X)] = \infty$. Thus, the expected return of the structured product is unbounded.

If we assume that a behavioral investor follows the (simple and natural) strategy to choose the product with the largest return in his risk class, such products will be highly attractive to him, although they will very likely not reflect his true preference for risk exposure. A sophisticated investor would notice that from the payoff diagram. Risk classifications, however, are designed particularly for unsophisticated, “behavioral” investors.

These products just serve as a theoretical construction for us to demonstrate the problem. They are not important in the market at the moment. However, this kind of structured products might become more and more interesting in the market, because the EU regulation pointed out in this research might drive banks to design products in this direction. A similar type of structured products already exists in the market, namely the so-called *stability notes*. Investors can receive an interest rate higher than the risk-free rate and at maturity they can get their initial capital back, unless some “disruptive event” occurs. A disruptive event can be, e.g., a stock market crash. For example, if the DAX decreases more than 10%, then the product expires and investors get only $\max(0, 100\% - L * [M - 10\%])$ of their capital back, where M is the percentage of how much the DAX decreases. L is the leverage of the product set before. If $L = 10$ and $M = 15\%$, then investors lose half of their capital. If $M = 20\%$, then investors lose all of their capital. Such a product could be in the lowest risk class, according to the EU classification, although a total loss is possible.

A practical example

We now look at some simple numerical examples. Imagine a product of this type constructed on 19 January 2012 with an initial value of 100 EUR and a maturity of one year (52 weeks). Its underlying is the DAX. As risk-free interest rate serves the 12 months Euribor, which was at 1.812% on this day. Based on the historical weekly performance from 2007 to 2011, the 1% quantile of the annual gross return of the underlying is 54.57% (net return -45.43%). By multiplying this return with the price of the DAX on 19 January, 2012, which was at 6416.26, we obtain a target price of $54.57\% \times 6416.26 = 3501.35$. This target price corresponds to z in Section 3.2.1. It is the underlying price level where the corresponding payoff of the structured product is supposed to be $\exp(-VaR_\alpha)$.

Due to the fact that liquidity for DAX options is too low (particularly for strike levels far away from the current DAX level), we instead use four call warrants (Optionsscheine). Warrants are financial derivatives similar to options. One major difference between warrants and options is that the former are issued by private

parties, typically the corporation on which a warrant is based, while the latter are exchange based. We obtained the information on warrants from Scoach, which is a popular trading platform for structured products in German-speaking areas. Real-time quoted prices as well as historical prices of financial securities can be easily found online. The data include issuer, type (call or put), conversion ratio, exercise type (European or American), bid and ask price, etc.

We then follow the approach presented in the last section to construct the structured products with four call warrants. In real markets like Scoach, the strike price is not continuous but discrete. Therefore, we cannot always find warrants with the exact same strikes as suggested by our theory. In case we cannot find a perfect match, we choose the warrants with strikes as close to our “theoretical strikes” as possible. The first call’s strike is chosen to be slightly lower than the target price, the second’s strike is slightly higher than the first call’s strike and very close to the target price. For the other two call warrants, we try different strikes and choose the ones that give the largest average return when used together with the first two warrants. See Table 3.1 for an overview. Theoretically, we would want to have the strikes of the third and fourth call as large as possible to increase the expected return. However, when applied to historical market data (past 5 years’ underlying prices), strikes of 5000 and 5050 for the third and fourth call, respectively, yield the highest average return. One reason for this is probably that call options (warrants) which are far out of the money are overpriced by issuers. Another, maybe more important, reason is that simulating product returns with weekly underlying data from the past 5 years, in accordance with the EU regulation, is probably not a very good approximation to the real expected return as anticipated by the market. Following the methodology prescribed in CESR (2010), we compute the VaR of this product. Table 3.2 shows the performance of this product based on a simulation over the past 5 years with different quantities of each call warrant in it. These quantities of warrants are obtained by solving a system of equations. They have to meet the following conditions: (1) Given a desired volatility level (0.4%, 1%, etc.), the corresponding VaR value should exactly be the return of the product, when only the first and the

	1st Call	2nd Call	3rd Call	4th Call
Strike	3400	3450	5000	5050
Expiry	March 13, 2013	March 13, 2013	March 13, 2013	March 13, 2013
Conversion ratio	100 : 1	100 : 1	100 : 1	100 : 1
Exercise type	American	American	American	American
Mid price	30.87	30.42	16.85	16.45

Table 3.1: *Call warrants used to construct the product.*

	Construction 1	Construction 2	Construction 3	Construction 4	Construction 5
Quantity of 1st Call	201.73	198.93	189.79	168.46	152.8532
Quantity of 2nd Call	-201.73	-198.93	-189.79	-168.46	-152.8532
Quantity of 3rd Call	23.05	26.21	36.48	60.4863	78.04
Quantity of 4th Call	-23.05	-26.21	-36.48	-60.4863	-78.04
Average log-return	10.06%	9.98%	9.71%	8.91%	8.18%
99% VaR	-0.86%	0.54%	5.24%	17.16%	26.88%
EU volatility	0.4%	1%	3%	8%	12%
True volatility	3.87%	4.43%	6.29%	10.98%	17.76%
Risk Class	1	2	3	4	5

Table 3.2: *Performance of the structured products with different compositions, assuming a total investment of 100 EUR. (Negative quantity means short position.)*

second calls are exercised (underlying price is below the strike of the third call); (2) With the budget constrain of 100 EUR, the first and the second calls have the same quantity, the third and the fourth calls also have to have same quantity.

With a 100 EUR budget, the first construction takes 201.73 long positions in the first call warrant, the same short positions in the second call, 23.05 long positions in the third call and the same short positions in the fourth call. By doing so, we achieve an average log-return of 10.06%. According to CESR (2010)'s calculation, the 99% VaR of this product is -0.0086 and the corresponding annual volatility¹ is 0.4%. This product is, thus, classified into risk class 1, i.e., the lowest risk group. The second to the fifth construction use different amounts of warrants and exhibit different risk-return profiles. By changing the quantities of call warrants, we can have the product classified into risk class 2, 3, 4, 5, etc. To consider robustness, we tried different combinations of call warrants with other

¹This is the volatility of a lognormally distributed asset with the same VaR.

strikes and got similar results.²

The trick this product does, is similar to the trick we used in Section 2, namely: in less than 1% of the cases it gives a return smaller than VaR. In some cases returns just equal VaR and in other cases the product gives relatively large returns to make the average return of the whole product as large as possible. This result looks attractive for investors: While the 1 year Euribor on the same day offers an interest rate of 1.812% (a log-return of 1.796%), this product gives investors a log-return of 10.06% with 0.4% “official” volatility whilst being classified into the lowest risk class, according to EU regulations. However, the real risk within this product can be higher. The true volatility based on historical simulations is 3.87%, i.e., almost ten times as high as the “official” volatility. Theoretically, there is a nearly one percent chance that the investor loses all of his investment. (In this example, there is a 1% chance that the DAX is below the target price of 3501.35 in the past 5 years.) By designing the product tailored to the VaR, the tail distribution of the product’s return is ignored. As long as the 1% quantile meets the VaR requirement, other parts of the return distribution can be freely chosen. This leaves space for performance manipulation, especially when financial derivatives are included in the portfolio, which makes it possible to obtain almost any form of return distribution. Again, less sophisticated, behavioral investors will fall into this “trap”.

3.2.2 Alternative risk-return measures

Theory

While previous section clearly demonstrated the strong limitations of the new risk classification scheme based on Value at Risk, the question remains open whether there exists a more appropriate risk measure for structured products.

²We notice that the average log-return of the product does not increase accordingly in the riskier classes. EU regulation (CESR (2010)) simulates log-return of structured products with past 5 years’ weekly underlying prices. The average log-return in Table 3.2 is the average of these simulated log-returns. The use of 5 years’ historical weekly prices, which amounts to less than 300 observations, may not sufficiently reflect the return distribution of the underlying in the future and thus explain the relation between return and risk class in Table 3.2.

A distinct feature of structured products is that, theoretically, any payoff profile is achievable. This leaves space for manipulation as long as only part of, but not the whole payoff distribution, is concerned by the risk measure. Let's take expected shortfall (ES) for example. In the case of a continuous distribution, it is defined as the conditional expectation of the return given that the return is lower than VaR (to be consistent with the definition of VaR in Section 2.1, we define ES with log return here),

$$ES_\alpha = E(\ln(X) | \ln(X) \leq -VaR_\alpha).$$

Although this method considers not only one quantile point as VaR does, but the whole lower tail of the return distribution (the expectation of this part), it still neglects the other part of the return distribution above the lower tail. It is not difficult to design structured products which meet the constraint of ES at the lower tail but still give a high expected return. This kind of products can even also have a certain probability for the investor to lose most of his investment, although this problem is obviously much smaller than with VaR.

The very fact that structured products allow for so much flexibility seems to make it possible to adapt their design to any possible risk classification scheme so that the scheme will underestimate their risk and a handsome expected return with little risk can be promised to prospective investors. In this section we want to discuss whether this is indeed the case.

To this aim, we define at first what we mean with an *appropriate risk–return measure*:

Definition 3.2.2. *An appropriate risk–return measure is a pair of two functions (measuring risk and return, respectively) depending on the product's gains and losses and the market parameters such that:*

- (1) *The risk only depends on the losses of the product.*
- (2) *Risk is strictly monotonic regarding all losses, i.e. if the losses become worse in some situations and not better anywhere, the risk increases.*

- (3) Return is strictly monotonic regarding all gains, i.e. if the gains become better in some situations and not worse anywhere, the return increases.
- (4) The return of a risk-free fixed interest investment corresponds to its interest.
- (5) For a given maximum level of risk, the largest achievable return is bounded.

All of these conditions seem to be intuitive, however, VaR as risk measure and expected return as return measure do not satisfy these conditions: As we have seen in the previous section, condition (5) is violated. Moreover, VaR is not monotonic, because only the payoff that occurs at the 99% probability value plays any role for its computation. Therefore, condition (2) is also violated.

An appropriate risk-return measure would not allow behavioral investors, who simply search for a high return within a certain risk category, to be fooled into taking inappropriate risks as easily. But does there exist any appropriate risk–return measure? To answer this question, we need to recall the concept of martingale measure: This probability measure can be understood as normalized market price for “Arrow-securities”. An Arrow-security at x is a contract that pays out one monetary unit at maturity if and only if the underlying has the value x at maturity. Martingale measures can be estimated from option prices (e.g., Jackwerth and Rubinstein (1996)). It is, therefore, possible to use them for the construction of a risk–return measure – at least in theory, being aware of potential implementation issues.

We define the following risk–return measure:

Definition 3.2.3. Define the risk $\tilde{\sigma}$ and the return $\tilde{\mu}$ of a structured product with payoff function y on a market with martingale measure P^* by

$$\tilde{\sigma} := -E^*[(y(X) - R_f)1_{y(X)<1}] = - \int_{\{x; y(x)<1\}} (y(x) - R_f) dP_X^*(x), \quad (3.15)$$

$$\tilde{\mu} := r_f + E^*[(y(X) - R_f)1_{y(X)>1}] = r_f + \int_{\{x; y(x)>1\}} (y(x) - R_f) dP_X^*(x), \quad (3.16)$$

where 1_A denotes the indicator function of the set A .

We can prove the following theorem:

Theorem 3.2.4. *If $P^* > 0$ a.e. (which is the case if the market is arbitrage-free), then $(\tilde{\sigma}, \tilde{\mu})$ as defined by Definition 3.2.3 is an appropriate risk–return measure in the sense of Definition 3.2.2.*

Proof. Condition (1) holds by definition. (2) and (3) follow from the assumption that $P^* > 0$ a.e. Condition (4) follows from a simple calculation with the expectation, for $y(x) = R_f$:

$$\tilde{\mu} := r_f + E^*[(y(X) - R_f)1_{y(X)>1}] = r_f + E^*[(R_f - R_f)1_{y(X)>1}] = r_f.$$

Finally, we need to prove the crucial condition (5). From the No-Arbitrage Condition we have

$$E^*[y(X)] = R_f. \tag{3.17}$$

Therefore, $\tilde{\mu} - \tilde{\sigma} = r_f$ or $\tilde{\mu} = r_f + \tilde{\sigma}$. In other words: When we prescribe a maximum level of risk $\tilde{\sigma}_{max}$, the maximum return can at most be $r_f + \tilde{\sigma}_{max}$ which provides us with the required bound. \square

From the proof we can also see that using this martingale measure for risk and return, the classical risk–return line based on the CAPM can be converted to a more general setting that includes structured products: all products will be placed on a sloping line in the new $\tilde{\mu}$ – $\tilde{\sigma}$ diagram, like classical assets are placed on a sloping line of the mean–variance diagram.

While the proposed risk–return measure protects against “disguising” of risk, it looks at first glance very unusual: normally, risk and return measures use the physical probability and not the martingale (or risk-neutral) probability. What are the implications of this? The martingale measure can be understood as the prices of Arrow securities. They result from an aggregation of beliefs and preferences of the market participants. In this way, risk preferences are already built

into the risk-return measure. Nevertheless, one might argue that for sophisticated investors these two parameters would not be sufficient to decide on their investments. This is certainly true, however, in this article we discuss behavioral (unsophisticated) investors, since they are the group for which the regulation has been designed. Our result shows that the current regulation poses severe problems for such investors (while sophisticated investors might simply ignore the EU regulated risk measure and use their own measures). The risk-return pair introduced in this section offers *one* possible solution to circumvent this problem. It should be emphasized that we do neither claim uniqueness nor optimality for this measure, but it seems to us a very parsimonious way to achieve the necessary goals.

Applications

We apply the new risk-return measure $\tilde{\sigma}$ and $\tilde{\mu}$ to some real world examples for $y_n(X)$ (equation (3.11)) and compare the results with the VaR-based risk measure. We do not directly estimate martingale measures from market data and then price the two equations (3.15) and (3.16) with the estimated martingale measure. Instead, we use warrants again to construct two sub-portfolios which have the same payoff profile as equations (3.15) and (3.16). Then we find out the cost for the construction of each sub-portfolio. Equation (3.15) can be regarded as a product giving the same payoff as the original product minus the risk-free rate, when the original product's gross return is lower than one, and otherwise it pays nothing. Equation (3.16) is a product having the same payoff as the original product minus the risk-free rate, when the original product's gross return is larger than one, and otherwise it pays nothing. This method is relatively simple and avoids the estimation of the martingale measure.³

In the construction of the two sub-portfolios, some warrant prices are interpolated with cubic splines in case their strikes are unavailable in the market. We use interpolations because this time we need precise strikes. Thus, $\tilde{\sigma}$ is the negative

³A detailed discussion of the methodology for implementing the new risk-return measure, especially the part dealing with the estimation of martingale measure, would be beyond the scope of this research and can be the topic of subsequent research.

	Construction 1	Construction 2	Construction 3	Construction 4	Construction 5
Quantity of 1st Call	201.73	198.93	189.79	168.46	152.8532
Quantity of 2nd Call	-201.73	-198.93	-189.79	-168.46	-152.8532
Quantity of 3rd Call	23.05	26.21	36.48	60.4863	78.04
Quantity of 4th Call	-23.05	-26.21	-36.48	-60.4863	-78.04
Average log-return	10.06%	9.98%	9.71%	8.91%	8.18%
99% VaR	-0.86%	0.54%	5.24%	17.16%	26.88%
EU volatility	0.4%	1%	3%	8%	12%
True volatility	3.87%	4.43%	6.29%	10.98%	17.76%
Risk Class	1	2	3	4	5
$\tilde{\mu}$	10.18%	10.42%	10.87%	11.94%	12.72%
$\tilde{\sigma}$	8.14%	8.38%	8.84%	9.9%	10.68%

Table 3.3: *Performance of the structured products with different compositions, assuming a total investment of 100 EUR. (Negative quantity means short position.)*

cost of the “lower sub-portfolio” relative to the initial value of the whole portfolio (100 EUR). $\tilde{\mu}$ is the risk-free rate plus the cost of the “upper sub-portfolio” relative to the initial value of the whole portfolio. The results are presented in Table 3.3, together with the results of the VaR-based risk measure. We see that $\tilde{\sigma}$ increases as $\tilde{\mu}$ increases, from construction 1 to construction 5 towards riskier classes.

We apply the VaR-based risk measure and our new risk-return measure to two common structured products: capital protected products and discount certificates. The capital protected products we constructed here have the DAX as its underlying. They are constructed with zero-coupon bonds and call warrants. The price of a zero-coupon bond is simply a discount with the Euribor as risk-free rate. Discount certificates here are composed with underlying and short calls. Underlying is again the DAX. All warrant prices are obtained from Scoach, with the 19 January 2012 as construction date. Results are presented in Table 3.4 and Table 3.5. For capital protected products, the risk and return profiles change when the level of protection is changed. A lower level of capital protection leads to more risk and a larger return measure $\tilde{\mu}$. For discount certificates, the risk and return profile changes with the discount rate: The lower the discount rate, the lower the risk, as measured by EU volatility, true volatility or the new risk

	100% Protection	98% Protection	95% Protection	90% Protection
Average log-return	0.94%	0.00%	-1.56%	-4.24%
99% VaR	0	0.0202	0.0513	0.1054
EU volatility	0.77%	1.63%	2.95%	5.23%
True volatility	1.28%	2.70%	4.84%	8.44%
Risk Class	1	2	3	4
$\tilde{\mu}$	2.58%	3.64%	5.17%	7.74%
$\tilde{\sigma}$	0.80%	1.82%	3.48%	5.90%

Table 3.4: *Performance of capital protected products.*

	89.32% Discount	92.53% Discount	95.10% Discount	97.14% Discount
Average log-return	2.61%	1.58%	0.45%	-0.86%
99% VaR	0.3698	0.4050	0.4325	0.4536
EU volatility	16.09%	17.50%	18.59%	19.43%
True volatility	12.63%	14.54%	16.03%	16.93%
Risk Class	6	6	6	6
$\tilde{\mu}$	8.25%	9.16%	9.90%	10.46%
$\tilde{\sigma}$	7.76%	8.69%	9.43%	10%

Table 3.5: *Performance of discount certificates.*

measure $\tilde{\sigma}$. Lower discount rates also correspond with lower return measures $\tilde{\mu}$.⁴

By comparing the results of applying the VaR-based risk measure and our new risk-return measure to the three types of structured products, we make the following observations:

The products $y_n(X)$ of Section 2 have the highest returns, as measured either by average log-return or by our new return measure $\tilde{\mu}$. These products have an average log-return up to 10.06%, while being classified into risk class 1. In contrast, capital protected products have the lowest returns. In the case of 100%

⁴We notice that the differences between $\tilde{\mu}$ and $\tilde{\sigma}$ within each type of the three products are relatively constant, but they differ from type to type of the products. For the products $y_n(X)$, $\tilde{\mu} - \tilde{\sigma}$ is always 2.04%, slightly higher than the risk-free rate (1.812%). For capital protected products, $\tilde{\mu} - \tilde{\sigma}$ is always very close to the risk-free rate. For discount certificates, it is more than 1% below the risk free-rate. Theoretically, the difference between $\tilde{\mu}$ and $\tilde{\sigma}$ should be the risk-free rate. In practice, mispricing for options is not uncommon (e.g., Constantinides et al. (2009)), which explains the deviations. The size of the difference is in line with literature studying the mispricing of structured products.

capital protection, they can be also classified into risk class 1, but deliver only an average log-return of 0.94%. The VaR-based risk measure exhibits a substantial inconsistency here. In risk class 1 (the class with the lowest risk), one product can have a log-return of 10.06%, while another has only 0.94% – obviously, downside risk is not adequately modeled here. If we look at $\tilde{\mu}$ and $\tilde{\sigma}$ of these two products, the difference is well reflected: The product of construction 1 has both a larger $\tilde{\mu}$ and a larger $\tilde{\sigma}$ than the 100% capital protected products. These two products have substantially different risk-return profiles. $y_n(X)$ is much riskier than the 100% capital protected products, judging from the $\tilde{\sigma}$. Classifying them into the same risk class can be seriously misleading for investors. Discount certificates have returns smaller than $y_n(X)$ but higher than capital protected products, as measured both by average log-return and $\tilde{\mu}$. However, they are always classified into risk class 6, a class with relatively high risk.

3.2.3 Conclusion

In this section, we have seen that the new EU regulations for risk classes of structured products give unfortunate incentives to issuers of such products: Risk can be “swept under the carpet” so that products with temptingly high expected returns can be designed, and behavioral investors that follow a simple strategy of maximizing expected returns within a given risk category will be tricked into taking inappropriately high risks. We have proved that it is theoretically possible to construct products with arbitrarily large expected return even in the lowest risk class. In a real life example, we were able to produce an average return of more than 10% in this lowest risk class, a class which is otherwise reserved for bonds of issuers with first class credit rating. All of this demonstrates that a risk measure based on VaR is not appropriate to regulate the market of structured products. Issuers will soon use the resulting loopholes and investors will misjudge the risk trusting the “officially” low risk levels that the issuer can claim.

While it is certainly important to point out weaknesses in regulations, it is much more difficult to suggest a better alternative: Most natural risk classification schemes could suffer from similar problems as the one based on VaR. Nevertheless,

we show that it is, at least theoretically, possible to define a risk–return measure that does not allow for any similar loopholes. In principle, this measure can be computed from option market data, although we admit practical limitations, and that other (better) methods might exist. At the very least, however, the measure provides a “proof of concept” for an alternative regulation that is free of the current regulation’s deficits.

There are further points about the validity of risk classifications – beyond the scope of this article – that should be addressed in future studies. First, issuers might improve the “official” risk category by choosing convenient underlyings. Second, the computation of risk factors based on historical data might influence results depending on the timing within the business cycle or the precise method in which the historical data is used. Both points require further investigations.

3.3 Impact of the underlying on the risk-return profiles of SPs

In this section, we discuss the impacts of the underlying’s return distribution, especially its first four moments, on the risk and return profile of structured products. It is organized as follows: The following subsection presents the theoretical framework of the analysis. Subsection 3.3.2 discusses in a theoretical way the impacts of the underlying moments on the expected return and the VaR of a structured product. The theoretical findings are then tested by Monte Carlo simulation in Subsection 3.3.3. Subsection 3.3.4 presents the results of historical simulation. Subsection 3.3.5 forms the conclusion.

3.3.1 The theoretical framework

Consider two underlyings: underlying 1 and underlying 2. Let their prices X_i , $i = 1, 2$, be random variables with the probability density functions $f_i(x)$ and the cumulative distribution functions $F_i(x)$.

Then, $y(X_1)$ and $y(X_2)$ are two structured products with identical payoff func-

tion, but written on different underlyings. The $(1 - \alpha)$ -VaRs of these two structured products are given by,

$$VaR_{i,\alpha} = -\inf\{m | P(\ln(y(X_i)) \leq m) > \alpha\}, \quad i = 1, 2. \quad (3.18)$$

Among the different ways of defining VaR, we follow the one adopted by the EU regulation (CESR (2010)), where VaR is calculated with log-return of the structured product and with the sign changed.

In order to make the following discussions more consistent, we transform the payoff function $y(x)$ to be based on the log-return of X , namely, $g(\ln(x)) = y(x)$. Then, $g(\cdot) = y(e^{\cdot})$. In the rest of the text, we call $g(\cdot)$ the *payoff algorithm* of the structured product.

Next we will see that a same VaR value does not guarantee the same expected return of structured products with different underlyings. Denote the expected return of the structured product by μ_y , namely, $\mu_y = E[y(x)] = E[g(\ln(x))]$.

Proposition 3.3.1.

$$VaR_{1,\alpha} = VaR_{2,\alpha} \quad (3.19)$$

is **not** a sufficient condition for

$$\mu_{y,1} = \mu_{y,2}, \quad (3.20)$$

where $\mu_{y,i}$ is the expected return of the product $y(X_i)$.

Proof. (3.19) is equivalent to,

$$\ln(y(q_{X_1,\alpha})) = \ln(y(q_{X_2,\alpha})),$$

where $q_{X_i,\alpha}$ is the α -quantile of X_i (i.e. $P(X_i \leq q_{X_i,\alpha}) = \alpha$).

(3.20) is equivalent to,

$$E[y(X_1)] = E[y(X_2)].$$

Assume $y(x)$ is strictly increasing around $q_{X_1,\alpha}$. Since $\ln(y(X))$ is a strictly in-

creasing function, we have further

$$q_{X_1, \alpha} = q_{X_2, \alpha}. \quad (3.21)$$

Let us assume $q_{X_1, \alpha} = q_{X_2, \alpha} = q$, then

$$\mathbb{E}[y(X_1)] = \int_{\mathbb{R}_+} y(x) f_1(x) dx = \int_0^q y(x) f_1(x) dx + \int_q^{+\infty} y(x) f_1(x) dx \quad (3.22)$$

and

$$\mathbb{E}[y(X_2)] = \int_{\mathbb{R}_+} y(x) f_2(x) dx = \int_0^q y(x) f_2(x) dx + \int_q^{+\infty} y(x) f_2(x) dx. \quad (3.23)$$

Since the two terms $\int_0^q y(x) f_1(x) dx$ and $\int_q^{+\infty} y(x) f_1(x) dx$ in (3.22) and the two terms $\int_0^q y(x) f_2(x) dx$ and $\int_q^{+\infty} y(x) f_2(x) dx$ in (3.23) are all nonnegative,

$$\int_{\mathbb{R}_+} y(x) f_1(x) dx = \int_{\mathbb{R}_+} y(x) f_2(x) dx$$

will not always hold for all $f_i(x)$ if we do not have the condition that

$$f_1(x) = f_2(x) \text{ a.e.} \quad (3.24)$$

In this proposition, we have the only condition (3.19), which is equivalent to $q_{X_1, \alpha} = q_{X_2, \alpha} = q$, meaning that

$$\int_0^q f_1(x) dx = \int_0^q f_2(x) dx = \alpha. \quad (3.25)$$

(3.25) is not sufficient for (3.24), so (3.19) is not sufficient for (3.20). \square

The idea of Proposition 3.3.1 may seem not surprising. For a given payoff function of the structured product, there is no one-to-one relationship between the expected return and the VaR. Switching underlying will affect the risk and return profiles of structured products. This motivates a further investigation into the impacts of underlying's return distribution on the structured products.

Consider a fixed payoff function of the structured product, the issuer (a bank or another financial institution) of the product wants to find out an underlying, based on which the product delivers a return as high as possible, subject to a given VaR level. Mathematically, this means, given a payoff function $y(x)$ of the structured product, the issuer faces the problem of maximizing its expected return while meeting the VaR constraint at the same time, i.e. searching for the mean-VaR frontier, by choosing an appropriate underlying X :

$$\max_X \mu_y = \max_X E[y(X)] = \max_X \int_{\mathbb{R}_+} y(x) dP, \tag{3.26}$$

subject to the VaR constraint

$$- \inf\{m | P(\ln(y(X)) \leq m) > \alpha\} \leq \overline{VaR}_\alpha. \tag{3.27}$$

Let $L = \ln(X)$ and $E[L] = \mu$. First, we expand the VaR of the product with its underlying's moments.

The $(1 - \alpha)$ -VaR of $y(x)$ is given by,

$$VaR_\alpha = - \inf\{m | P(\ln(y(X)) \leq m) > \alpha\}. \tag{3.28}$$

Assume y is strictly increasing around $q_{X,\alpha}$, the α -quantile of X , then (3.28) is equivalent to,

$$\begin{aligned} VaR_\alpha &= - \inf\{m | P(X \leq y^{-1}(e^m)) > \alpha\} \\ &= - \inf\{m | P(\ln(X) \leq \ln(y^{-1}(e^m))) > \alpha\}, \end{aligned} \tag{3.29}$$

because $\ln(x)$ is strictly increasing.

Let the α -quantile of L be q_α (i.e. $P(L \leq q_\alpha) = P(\ln(X) \leq q_\alpha) = \alpha$), then

$$VaR_\alpha = -\ln(y(\exp(q_\alpha))) = -\ln(g(q_\alpha)), \quad (3.30)$$

VaR_α is decreasing in q_α , apparently.

Based on Cornish-Fisher expansion (Cornish and Fisher (1937), Fisher and Cornish (1960) and Hill and Davis (1968)), the α -quantile of a non-normal random variable can be approximated with its first four moments and the standard normal quantile,

$$q_\alpha = \mu + \sigma \left(p_\alpha + \frac{p_\alpha^2 - 1}{6} \gamma_1 + \frac{p_\alpha^3 - 3p_\alpha}{24} \gamma_2 - \frac{2p_\alpha^3 - 5p_\alpha}{36} \gamma_1^2 \right), \quad (3.31)$$

where μ , σ , γ_1 , γ_2 and q_α are expectation, standard deviation, skewness, excess kurtosis and α -quantile of $\ln(X)$, respectively. p_α is α -quantile of a standard normal distribution. Then, the $(1 - \alpha)$ -VaR of the structured product $y(X)$ is given by,

$$\begin{aligned} VaR_\alpha &= -\ln(g(q_\alpha)) \\ &= -\ln \left(g \left(\mu + \sigma \left(p_\alpha + \frac{p_\alpha^2 - 1}{6} \gamma_1 + \frac{p_\alpha^3 - 3p_\alpha}{24} \gamma_2 - \frac{2p_\alpha^3 - 5p_\alpha}{36} \gamma_1^2 \right) \right) \right). \end{aligned} \quad (3.32)$$

Discussions have been raised on the validity and the accuracy of Cornish-Fisher expansion and there are also other quantile approximation methods, see, e.g. Wallace (1958). The reasons we choose Cornish-Fisher expansion to approximate the quantile of underlying's log-return are: Firstly, because it is one of the earliest methods for quantile approximation, it is also one of the most well-known methods (e.g. Gabrielsen et al. (2012) recently employs Cornish-Fisher expansion to forecast VaR with time varying moments); Secondly, Cornish-Fisher expansion requires that the limiting distribution of the approximated random variable is normal distribution, which is theoretically the case of the log-return of financial assets, according to the Central Limit Theorem; Thirdly, this article aims to study

the direction of the impacts of underlying moments on the risk-return profiles of structured products. Our primary concern is the sign before each moment in (3.32). The accuracy of the approximation is thus relatively in a secondary place.

Next, we expand the expected return of the product. Let us further assume that the payoff function of the structured product is a piecewise linear function with the form,

$$y(X) = \begin{cases} a_1X + b_1, & \text{if } X \in A_1, \\ a_2X + b_2, & \text{if } X \in A_2, \\ \dots & \\ a_nX + b_n, & \text{if } X \in A_n, \end{cases} \quad (3.33)$$

where $a_i \geq 0$, $b_i \in \mathbb{R}$, for $i = 1, 2, \dots, m$, $\bigcup_{i=1}^n A_i = \mathbb{R}_+$ and at least for one j , $a_j > 0$.

The vast majority of structured products will have payoff functions of this form. In fact, almost all the official categories of structured products currently listed at European Structured Investment Products Association (2012) can be described with a payoff function in form of Equation (3.33). Exceptions are, e.g. twin-win certificates, which have a decreasing payoff part, i.e. $a_i < 0$ for some i , and this part is defined only on a finite interval of X .

Rewrite (3.33) with the payoff algorithm g and $\ln(X)$,

$$y(X) = g(\ln(X)) = g(L) = \begin{cases} a_1e^{\ln(X)} + b_1, & \text{if } X \in A_1, \\ a_2e^{\ln(X)} + b_2, & \text{if } X \in A_2, \\ \dots & \\ a_ne^{\ln(X)} + b_n, & \text{if } X \in A_n, \end{cases} \quad (3.34)$$

$$= \begin{cases} a_1e^L + b_1, & \text{if } L \in B_1, \\ a_2e^L + b_2, & \text{if } L \in B_2, \\ \dots & \\ a_ne^L + b_n, & \text{if } L \in B_n. \end{cases} \quad (3.35)$$

The expected return of $y(X)$ is then given by,

$$\begin{aligned}
\mu_y &= \int_{\mathbb{R}} g(L)dP = \int_{B_1} (a_1e^L + b_1)dP + \int_{B_2} (a_2e^L + b_2)dP + \cdots + \int_{B_n} (a_n e^L + b_n)dP \\
&= a_1 \int_{B_1} e^L dP + a_2 \int_{B_2} e^L dP + \cdots + a_n \int_{B_n} e^L dP \\
&\quad + b_1P(B_1) + b_2P(B_2) + \cdots + b_nP(B_n).
\end{aligned} \tag{3.36}$$

Because $a_i, b_i, P(B_i)$ and $\int_{B_i} e^L dP$ are all nonnegative, there exist $\bar{a} \in [\min(a_i), \max(a_i)]$ and $\bar{b} \in [\min(b_i), \max(b_i)]$, such that

$$a_1 \int_{B_1} e^L dP + a_2 \int_{B_2} e^L dP + \cdots + a_n \int_{B_n} e^L dP = \bar{a} \sum_i \int_{B_i} e^L dP$$

and

$$b_1P(B_1) + b_2P(B_2) + \cdots + b_nP(B_n) = \bar{b} \sum_i P(B_i).$$

Thus, (3.36) can be written as,

$$\mu_y = \bar{a} \sum_i \int_{B_i} e^L dP + \bar{b} \sum_i P(B_i) = \bar{a} \int_{\mathbb{R}} e^L dP + \bar{b}, \tag{3.37}$$

namely,

$$\mu_y = \bar{a}E[e^L] + \bar{b}. \tag{3.38}$$

Let us expand e^L at μ (the expectation of L) with Taylor series,

$$e^L = e^\mu + e^\mu(L-\mu) + \frac{e^\mu}{2!}(L-\mu)^2 + \frac{e^\mu}{3!}(L-\mu)^3 + \frac{e^\mu}{4!}(L-\mu)^4 + \sum_{i=5}^{\infty} \frac{e^\mu}{i!}(L-\mu)^i. \tag{3.39}$$

Take expectation of both sides of (3.39),

$$E[e^L] = e^\mu + \frac{e^\mu}{2!}\sigma^2 + \frac{e^\mu}{3!}\mu_3 + \frac{e^\mu}{4!}\mu_4 + O(\mu_5), \quad (3.40)$$

where σ^2 is the variance and μ_i is the i -th central moment of L , respectively.

Namely,

$$E[e^L] = e^\mu + \frac{e^\mu}{2!}\sigma^2 + \frac{e^\mu}{3!}\sigma^3\gamma_1 + \frac{e^\mu}{4!}\sigma^4(\gamma_2 + 3) + O(\mu_5), \quad (3.41)$$

where γ_1 and γ_2 are the skewness and the excess kurtosis of L , respectively. Let $\gamma_1 \geq -\frac{3}{\sigma}$, then, c.p., $E[e^L]$ is increasing in μ in (3.41). For example, if $\sigma = 0.3$, $\gamma_1 \geq -\frac{3}{\sigma}$ means the skewness of L is no smaller than -10 .

Equation (3.38) becomes then,

$$\mu_y = \bar{a}e^\mu + \frac{\bar{a}e^\mu}{2!}\sigma^2 + \frac{\bar{a}e^\mu}{3!}\sigma^3\gamma_1 + \frac{\bar{a}e^\mu}{4!}\sigma^4(\gamma_2 + 3) + \bar{a}O(\mu_5) + \bar{b}, \quad (3.42)$$

with positive \bar{a} and \bar{b} .

Via Equation (3.42), we are able to expand the expected return of a structured product with its underlying's moments.

3.3.2 The impacts of underlying moments

By considering Equation (3.42) and Equation (3.32) together, we are able to discuss the impacts of each moment of the underlying on the expected return and the VaR of a structured product, simultaneously. Before we proceed, let us summarize the important assumptions we have made,

(A1) The payoff function $y(\cdot)$ follows the form of (3.33);

(A2) The payoff function $y(\cdot)$ is strictly increasing around $q_{X,\alpha}$, the α -quantile of X ;

(A3) The skewness γ_1 of the underlying log-return $\ln(X)$, is no smaller than $-\frac{3}{\sigma}$, where σ is the underlying volatility.

Proposition 3.3.2. *If $\ln(X)$ follows a normal distribution (i.e. X is log-normal distributed), and the payoff function $y(X)$ of the structured product satisfies (A1) and (A2), then under the mean-VaR framework, underlyings with large expected log-return is preferred, the preference for its variance is ambiguous.*

Proof. When $\ln(X)$ follows normal distribution, i.e. $\ln(X) \sim N(\mu, \sigma^2)$,

$$q_\alpha = \mu + \sigma p_\alpha, \quad (3.43)$$

where p_α denotes the α -quantile of a standard normal distribution. Equation (3.30) becomes,

$$VaR_\alpha = -\ln(g(\mu + \sigma p_\alpha)). \quad (3.44)$$

Since the level α for VaR is always at the left tail of the distribution (e.g., α is 1% in the EU regulation CESR (2010)), p_α in (3.44) is negative. VaR_α is thus decreasing in μ and increasing in σ .

Furthermore, (3.42) becomes

$$\mu_y = \bar{a}e^\mu + \frac{\bar{a}e^\mu}{2!}\sigma^2 + \frac{3\bar{a}e^\mu}{4!}\sigma^4 + \bar{a}O(\mu_5) + \bar{b}, \quad (3.45)$$

because $\gamma_1 = 0$ and $\gamma_2 = 0$ for normal distributed $\ln(X)$.

Observe (3.44) and (3.45) together: Because we have $\bar{a} \geq 0$, The first term $\bar{a}e^\mu$, the second term $\frac{\bar{a}e^\mu}{2!}\sigma^2$ and the third term $\frac{3\bar{a}e^\mu}{4!}\sigma^4$ in (3.45) are all increasing in μ . Larger μ will thus increase μ_y , the expected return of the product. μ_y is obviously increasing in σ^2 , too. Larger underlying variance will increase expected return of the product, it will however, increase the VaR of the product in (3.44) at the same time. \square

In the CAPM world, the market portfolio is considered to be (log-)normal distributed and mean-variance efficient. Proposition 3.3.2 suggests that in the CAPM world, choosing the market portfolio as the underlying of structured products does not necessarily improve their risk and return profiles. Expected return

is increasing both in the mean and in the variance of underlying's log-return, however, the VaR will be decreasing in the mean and increasing in the variance of the underlying at the same time. Small underlying volatility will decrease both the return and the risk of the structured product.

Proposition 3.3.3. *If $\ln(X)$ is distributed with zero skewness (e.g. symmetrically distributed), and the payoff function $y(X)$ of the structured product satisfies (A1) and (A2), then under the mean-VaR framework, underlyings with large expected log-return will be preferred, the preferences for the variance and the kurtosis are both ambiguous.*

Proof. In this case, (3.42) becomes

$$\mu_y = \bar{a}e^\mu + \frac{\bar{a}e^\mu}{2!}\sigma^2 + \frac{\bar{a}e^\mu}{4!}\sigma^4(\gamma_2 + 3) + \bar{a}O(\mu_5) + \bar{b}. \quad (3.46)$$

And (3.32) becomes,

$$VaR_\alpha = -\ln \left(g \left(\mu + \sigma \left(p_\alpha + \frac{p_\alpha^3 - 3p_\alpha}{24} \gamma_2 \right) \right) \right). \quad (3.47)$$

The term $p_\alpha + \frac{p_\alpha^3 - 3p_\alpha}{24} \gamma_2$ and the term $\frac{p_\alpha^3 - 3p_\alpha}{24}$ in (3.47) are both negative, because p_α is negative and γ_2 is nonnegative. Consequently, the $(1 - \alpha)$ -VaR of the structured product VaR_α will be decreasing in μ , increasing in σ and in γ_2 .

The expected return μ_y of the product is obviously increasing in μ . μ_y will also be increasing in σ^2 and in γ_2 , since $\gamma_2 \geq 0$. Because VaR_α is decreasing in μ and increasing in σ and in γ_2 , only large expectation from the underlying's log-return is preferred. The preferences for the variance and the kurtosis of the underlying in this case are both conflicting: Larger variance (kurtosis) will increase the expected return of the product, but increase the VaR at the same time. □

Let the excess kurtosis γ_2 in Proposition (3.3.3) be zero, (3.46) becomes (3.45) and (3.47) becomes (3.44). We will have a corollary with the same result as Proposition (3.3.2).

Corollary 3.3.4. *If $\ln(X)$ follows a mesokurtic distribution (i.e. $\ln(X)$ has zero excess kurtosis) with zero skewness, and the payoff function $y(X)$ of the structured product satisfies (A1) and (A2), then under the mean-VaR framework, underlyings with large expected log-return is preferred, the preference for its variance is ambiguous.*

Finally, let us discuss the general case of the underlying distribution.

Proposition 3.3.5. *If the payoff function $y(X)$ of the structured product satisfies (A1) and (A2) and the underlying satisfies (A3), then under the mean-VaR framework, underlyings with large expected log-return will be preferred. Large positive skewness is also preferred. The preference for the underlying variance is ambiguous, the preference for underlying kurtosis is also ambiguous.*

Proof. Let us directly look at (3.42) and (3.32).

For VaR_α , it is clearly decreasing in μ . The impact of σ is ambiguous, because term

$$p_\alpha + \frac{p_\alpha^2 - 1}{6}\gamma_1 + \frac{p_\alpha^3 - 3p_\alpha}{24}\gamma_2 - \frac{2p_\alpha^3 - 5p_\alpha}{36}\gamma_1^2 \quad (3.48)$$

can be both positive and negative, depending on the combination of γ_1 and γ_2 .

As for γ_1 , (3.48) is a quadratic function of γ_1 . The minimum is achieved at $\gamma_1 = \frac{3p_\alpha^2 - 3}{2p_\alpha^3 - 5p_\alpha}$, which is negative for $p_\alpha < -1.581$ ($\alpha < 0.057$), a typical level for VaR. Thus, when $\gamma_1 \leq \frac{3p_\alpha^2 - 3}{2p_\alpha^3 - 5p_\alpha}$, the VaR is increasing in γ_1 ; when $\gamma_1 > \frac{3p_\alpha^2 - 3}{2p_\alpha^3 - 5p_\alpha}$, the VaR is decreasing in γ_1 . It is safe to say that large positive skewness will reduce the VaR.

As for γ_2 , it is obvious that VaR_α is increasing in γ_2 . Because $\sigma > 0$ and the term $\frac{p_\alpha^3 - 3p_\alpha}{24} < 0$ for $\alpha < 0.042$ ($p_\alpha < -1.732$), which is typical for VaR.

In (3.42), the impact of μ on μ_y is obviously positive. μ_y is increasing both in γ_1 and in γ_2 . Because $\sigma > 0$, μ_y is also increasing in σ .

Putting VaR together: Keeping other moments fixed, underlyings with large expected log-return will be preferred. Large positive skewness will be preferred, too. The impact of underlying variance is ambiguous. Increasing the underlying kurtosis will increase the expected return of the product and however, increase the VaR of the structured product, too. \square

Let the excess kurtosis γ_2 in Proposition (3.3.5) be zero, we will have the following corollary.

Corollary 3.3.6. *If $\ln(X)$ follows a mesokurtic distribution (i.e. $\ln(X)$ has zero excess kurtosis), and the payoff function $y(X)$ of the structured product satisfies (A1) and (A2) and the underlying satisfies (A3), then under the mean-VaR framework, underlyings with large expected log-return will be preferred. Large positive skewness is also preferred. The preferences for the underlying variance is ambiguous.*

Let us summarize the above propositions and corollaries. If the payoff function $y(X)$ of the structured product satisfies (3.33), then other moments being fixed, its expected return is increasing in the expectation, in the variance, in the skewness and in the kurtosis of the underlying's log-return. Its VaR is increasing in the kurtosis and decreasing in the expectation of the underlying's log-return. The impacts of the variance and the skewness are in general ambiguous. In other words, under the mean-VaR framework:

1. Expectation μ

Other moments being fixed, large expected log-return of the underlying is always preferred.

2. Variance σ^2

The preferences for the variance of the underlying's log-return is ambiguous.

3. Skewness γ_1

Large positive skewness is preferred.

4. Kurtosis γ_2

The preference for the kurtosis of the underlying's log-return is always ambiguous. Large kurtosis increases both the expected return and the VaR of the product.

The results of this section indicate that mean-variance efficient underlyings, in contrast to the intuition, do not necessarily improve the risk-return profiles of structured products, because on the one hand, small variance may decrease the VaR, dependent on the specific product setup; on the other hand, it will decrease the return of the product as well. The importance of kurtosis risk in the context of portfolio with derivatives, is also confirmed by the results. Ignoring the kurtosis of the underlying whose log-return is leptokurtically distributed, will lead to underestimations for the VaR of the structured product.

3.3.3 Monte Carlo simulation

Because it is not always possible to find out different real-world underlyings with one moment being different and other three moments being similar, it will be more feasible to test the theoretical results in the previous section with simulations. In this section, random numbers are generated from a given distribution. These random numbers as log-returns of the underlying are used to calculate the underlying prices at the maturity of the structured product and consequently, the payoffs of the structured product can be simulated. Based on the simulated payoffs, we will have the simulated expected return (the mean of the simulated return) and the VaR of the product. The distributions used are normal distribution, t -distribution and NIG-distribution. The structured products considered are tracker certificates, discount certificates and capped outperformance certificates.

Normal distribution $N(\mu, \sigma^2)$ is the distribution underlying the Black-Scholes model (Black and Scholes (1973)) and the CAPM. If a random variable $L \sim N(\mu, \sigma^2)$, then the expectation, the variance, the skewness and the excess kurtosis of L are μ , σ^2 , 0 and 0, respectively. Although it is widely used, literature has suggested that financial asset's log-return does not necessarily follow a normal distribution. Asymmetry and "fat tail" are not uncommon in financial markets (Tsay (2002)). Alternatives to normal distribution are, for example, t -distribution and NIG-distribution.

If a random variable T follows a t -distribution with η degrees of freedom, then

its probability density function is given by,

$$f(t) = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\eta\pi}\Gamma\left(\frac{\eta}{2}\right)} \left(1 + \frac{t^2}{2}\right)^{-\frac{\eta+1}{2}}, \quad (3.49)$$

where Γ is the gamma function. When $\eta > 4$, the expectation, the variance, the skewness and the excess kurtosis of T are 0, $\frac{\eta}{\eta-2}$, 0 and $\frac{6}{\eta-4}$, respectively. A transformed version of t -distribution is often used. If L follows a transformed t -distribution with η degrees of freedom, a location parameter μ and a scale parameter σ , then

$$L = \mu + \sqrt{\frac{\eta-2}{\eta}} T \sigma, \quad (3.50)$$

where, T has a probability density function of (3.49). In comparison to a normal distribution, L in (3.50) will have positive excess kurtosis and can better capture the “fat tails” of the financial asset’s return.

The normal-inverse Gaussian (NIG) distribution is a subclass of the generalised hyperbolic distribution. If a random variable G follows a NIG-distribution with parameters μ_{NIG} , δ , α and β , then its probability density function is given by,

$$f(g) = \frac{\alpha \delta K_1\left(\alpha \sqrt{\delta^2 + (g - \mu_{\text{NIG}})^2}\right)}{\pi \sqrt{\delta^2 + (g - \mu_{\text{NIG}})^2}} e^{\delta \gamma + \beta(g - \mu_{\text{NIG}})}, \quad (3.51)$$

where K_1 is the modified Bessel function of the third kind with index 1, $\gamma = \sqrt{\alpha^2 - \beta^2}$. We will use a transformed variant in the simulation,

$$L = \mu + \frac{G - (\mu_{\text{NIG}} + \delta\beta/\gamma)}{\delta\alpha^2/\gamma^3} \sigma. \quad (3.52)$$

This distribution is meant to capture both the skewness and “fat tail” from financial asset’s log-return.

Tracker certificates are one of the participation products. They simply track the performance of the underlying assets. Holding a tracker certificate has basically the same payoff as holding the underlying itself. We consider this product in the simulation as an example for products with very simple payoff functions. They

are usually constructed with zero-strike calls (LEPO). For more detail of tracker certificates, see Blümke (2009). The payoff function of a tracker certificate is simply $y(X) = X$, and thus its payoff algorithm is $g(L) = g(\ln(X)) = e^{\ln(X)} = e^L$.

Discount certificates are one of the yield enhancement products. As introduced in Chapter 1, on the one hand, they offer the buyer shares of an underlying at a price lower than its current price. The buyer, on the other hand, has to accept a fixed maximum return (the cap). At maturity, if the underlying price is lower than the cap, the buyer will receive one share of the underlying per discount certificate; otherwise, the buyer will receive a cash settlement equivalent to the cap. Discount certificates are usually constructed by holding the underlying and selling call options with strike being the cap. The payoff function of a discount certificate can be described by,

$$y(X) = X - (X - K)^+ = \begin{cases} K, & \text{if } X > K, \\ X, & \text{if } 0 < X \leq K, \end{cases} \quad (3.53)$$

where K is the cap. Its payoff algorithm is thus,

$$g(L) = g(\ln(X)) = \begin{cases} K, & \text{if } \ln(X) > \ln(K), \\ e^{\ln(X)}, & \text{if } \ln(X) \leq \ln(K), \end{cases} \quad (3.54)$$

$$= \begin{cases} K, & \text{if } L > \ln(K), \\ e^L, & \text{if } L \leq \ln(K). \end{cases} \quad (3.55)$$

Capped outperformance certificates (also called turbo certificates) are another type of participation products. They allow for a disproportionate participation in the gains of the underlying, at any level above the strike price. In return, the buyer's profit is limited (capped) on the upside. They are usually constructed with the underlying, a long at-the-money call and two short out-of-the-money

calls. Their payoff function can be described by,

$$y(X) = X + (X - K_1)^+ - 2(X - K_2)^+ = \begin{cases} 2K_2 - K_1, & \text{if } X > K_2, \\ 2X - K_1, & \text{if } K_1 < X \leq K_2, \\ X, & \text{if } 0 < X \leq K_1, \end{cases} \quad (3.56)$$

for $K_2 > K_1$, where K_1 is the strike of the first long call, which is usually set to be the spot price of the underlying, K_2 is the strikes of the two short calls determined by the premium paid for the first call. The payoff algorithm of a capped outperformance certificate is thus given by,

$$g(L) = g(\ln(X)) = \begin{cases} 2K_2 - 1, & \text{if } \ln(X) > \ln(K_2), \\ 2e^{\ln(X)} - 1, & \text{if } \ln(1) < \ln(X) \leq \ln(K_2), \\ e^{\ln(X)}, & \text{if } \ln(X) \leq \ln(1), \end{cases} \quad (3.57)$$

$$= \begin{cases} 2K_2 - 1, & \text{if } L > \ln(K_2), \\ 2e^L - 1, & \text{if } 0 < L \leq \ln(K_2), \\ e^L, & \text{if } L \leq 0, \end{cases} \quad (3.58)$$

because we have normalized the spot price to be 1, $K_1 = X^0 = 1$.

Next, we simulate the underlying price at the maturity of the structured product with the three distributions and plug them into the payoff functions of the three products discussed above. In each case, the yearly risk free interest rate is assumed to be 0.6%. The maturity of all products is assumed to be 1 year. The confidence level for the VaR is set to be 99% ($\alpha = 0.01$), the same as the CESR (2010). Option prices are obtained also from the simulation, namely, the discounted mean of the simulated payoffs at maturity. The strike K_2 of the second call in capped outperformance certificates are determined by Black-Scholes option prices. Underlying price at time 0 are assumed to be 1,000. The cap of the discount certificate is 1,500, 1.5 times of the innitial underlying price.

1. Simulation results with normal distribution

Figure 3.2, 3.3 and 3.4 present the simulation results of normal distribution for

tracker certificates, discount certificates and capped outperformance certificates, respectively. The parameter μ ranges from -0.2 to 0.2 , σ is from 0.05 to 0.35 . The simulation times are $100,000$. The mean return and the VaR of the products are plotted against the mean and volatility of the simulated underlying log-return.

For the mean returns, all three products reveal similar shapes. All three products exhibit clear upward trends of the mean return with respect to increasing underlying mean μ . From the plots, the three products' mean returns also increase moderately, when the underlying volatility σ increases.

For the VaR, all three products have almost the same shape in the figures, too. There are downward trends of the VaR with respect to an increasing μ and upward trends respect to an increasing σ . These results are in line with the theoretical findings in previous section: If the underlying's log-return follows normal distribution $N(\mu, \sigma^2)$, then the expected return of the product is increasing in μ and in σ . The VaR of the product is decreasing in μ and increasing in σ .

2. Simulation results with t -distribution

Figure 3.5, 3.6 and 3.7 present the simulation results of t -distribution. The upper parts of the figures plot the mean return and the VaR against the mean and the volatility of the simulated underlying log-return, respectively. The degrees of freedom of the t -distribution is 6 , corresponding to a kurtosis of 3 . The parameter μ ranges from -0.2 to 0.2 , σ is from 0.05 to 0.35 . The simulation times are $100,000$. All three products reveal similar shapes: Product returns are increasing in μ and in σ . Products' VaRs are decreasing in μ and increasing in σ .

The lower parts of the figures plot the mean return and the VaR against the excess kurtosis γ_2 of the simulated underlying log-return with different volatilities (30% , 32.5% , 35% , 37.5%) and $\mu = 0.1$, η ranging from 5 to 9 , respectively. For all products, both the return and the VaR reveal slightly upward trends with an increasing kurtosis. Besides, at a fixed kurtosis, larger volatility increases both the product return and VaR.

The results also confirm Proposition 3.3.3 that for a underlying whose log-return is distributed with zero skewness, a large μ is always preferred and the

impacts of volatility and kurtosis are conflicting: They both increase the product return and the VaR at the same time.

3. Simulation results with NIG-distribution

Figure 3.8, 3.9 and 3.10 present the simulation results of NIG-distribution. The upper parts of the figures plot the mean return and the VaR against the mean and the volatility of the simulated underlying log-return, respectively. The parameters of the NIG-distribution are $\mu_{\text{NIG}} = 0.1$, $\delta = 3$, $\alpha = 50$ and $\beta = -2$. The simulation times are 100,000. All three products reveal similar shapes: Product returns are increasing clearly in μ and moderately in σ . Products' VaRs are decreasing in μ and increasing in σ .

The lower parts of the figures plot the mean return and the VaR against the excess kurtosis γ_2 and skewness γ_1 of the simulated underlying log-return, respectively. The parameters of the NIG-distribution are $\mu = 0.1$, $\sigma = 0.3$, $\mu_{\text{NIG}} = 0$, $\delta = 1$, α ranging from 1 to 2 and β ranging from -0.5 to 0.5. Different from previous cases, where we can discuss the impact of one moment while keeping the other moments fixed; the skewness and the excess kurtosis of the NIG-distribution simulation appear to be related with each other in this case. The excess kurtosis γ_2 seems to be a quadratic function of the skewness γ_1 . It is thus difficult to identify the impact from each of them.

For the skewness γ_1 , there appear to be moderate upward trends of product return with respect to increasing γ_1 and clear upward trends of VaR with respect to decreasing γ_1 for all the three products (without γ_2 being fixed). For the excess kurtosis γ_2 , product return appears to be increasing with γ_2 , when γ_1 is large (positive); when γ_1 is small (negative), product return appears to be decreasing in γ_2 . However, these trends against γ_2 is affected by γ_1 at the same time. We are unable to tell when γ_1 is small (negative), whether the decreasing product return is a consequence of a decreasing γ_1 or a consequence of an increasing γ_2 . For VaR, it is the same. The VaRs appear to be increasing with γ_2 , when γ_1 is small (negative); when γ_1 is large (positive), products' VaR appear to be decreasing in γ_2 . However, we are unable to tell when γ_1 is large (positive), whether the decreasing VaR is a

Country	Austria	Finland	France	Germany	Ireland	Italy	Netherlands
Stock index	ATX	OMX Helsinki 25	CAC 40	DAX	ISEQ Overall	FTSE MIB	AEX
Mean	-0.0207	0.0436	0.0066	0.0846	0.0184	-0.0712	0.0172
Vola	0.3124	0.2704	0.2019	0.1986	0.3048	0.2494	0.2428
Skewness	-0.8503	-0.8287	-0.7119	-0.9776	-1.8469	-0.5596	-1.2310
Ex. Kurtosis	0.2444	0.1727	-0.1374	0.2503	3.5372	-0.0196	1.6501

Table 3.6: *Descriptive statistics for the log-return of major European stock market indices, from 3 March 2008 to 24 February 2014.*

consequence of an increasing γ_1 or a consequence of an increasing γ_2 .

The results are in line with the parts of Proposition 3.3.5 concerning the impacts of expectation, variance and skewness. Since we are unable to distinguish the effect of the excess kurtosis from that of the skewness in the simulation, the impacts of excess kurtosis on the expected return and on the VaR of the product are unclear in this case; although its impacts are confirmed in the previous case of t -distribution simulation.⁵

3.3.4 Historical simulation

After seeing the results of Monte Carlo simulation in Section 3.3.3, we also checked our theoretical findings with real-world data. We consider seven major stock market indices in the eurozone and simulate the performances of the three above-mentioned structured products with weekly historical stock index levels from 2008 to 2014. See Table 3.6 for the descriptive statistics of the stock market indices we used. The maturities of the products are again assumed to be 1 year. Option prices are obtained with Black-Scholes formula. For the risk-free interest rate, we take the average 12 months Euribor of the same year as the construction date. For the volatility, we calculate the realized volatility of each index over the next 52 weeks after the construction date. The VaR level is again 99%. The cap of the discount certificate is 1.5 times of the index levels on the construction date.

One day in every week from 3 March 2008 to 25 February 2013, we construct

⁵We have also carried out simulations with skewed t -distribution (Hansen (1994)). The results are similar as NIG-distribution: γ_2 appears to be a quadratic function of γ_1 and we are unable to distinguish the effect of the excess kurtosis from that of the skewness.

the three structured products. Then we plug the index levels 52 weeks later from this day into the payoff function of the products to obtain products' returns. Based on these historically simulated returns, we calculate the VaR of the products. The average return and the VaR of each of the three products for the seven European stock indices are presented in Table 3.7. Products written on different underlyings exhibit different risk-return profiles, as measured by average return and VaR. In this case, it is impossible to perfectly compare the results between different underlyings, because we cannot compare one moment while controlling other three moments. However, we can still make the following observations:

First, Germany's DAX appears to be the best performing underlying for all three structured products. Products written on DAX have always the highest average return and the lowest VaR. In contrast, Italy's FTSE MIB delivers much worse results. For all the three products, FTSE MIB has always lower average returns but higher VaRs than products written on DAX. This can be explained by the fact that DAX in this period has substantially the highest mean of the log-return among the seven underlyings, while FTSE MIB has the lowest mean.

Second, let us look at Austria (ATX) and Finland (OMX Helsinki 25). The mean of their log-returns differ substantially. However, the differences between their volatilities and between their skewness are small. The difference between their kurtosis is relatively small, too, compared to that between their means. Finland's stock index, which has a higher mean than Austria does, delivers better risk-return profiles for structured products than Austria's stock index does. This again confirms the positive impact from the mean of the underlying log-return.

Third, for Austria (ATX) and France (CAC 40), the mean of their indices are very close to each other and the skewness of CAC 40 is higher than that of ATX. According to our theoretical results, products based on CAC 40 should have (slightly) better risk-return profiles than products based on ATX. This can be confirmed by the historically simulated results in Table 3.7, where products written on CAC 40 have similar or a little higher average returns, but lower VaRs than products written on ATX. Although differences between their volatilities and between their kurtosis are not small, these two moments' impacts on the risk-

Country	Austria	Finland	France	Germany	Ireland	Italy	Netherlands
Stock index	ATX	OMX Helsinki 25	CAC 40	DAX	ISEQ Overall	FTSE MIB	AEX
Tracker certificates							
Average return	1.0240	1.0804	1.0262	1.1085	1.0589	0.9592	1.0449
99%-VaR	0.8468	0.7133	0.5044	0.4445	1.0531	0.7057	0.6939
Discount certificates							
Average return	1.0387	1.0887	1.0373	1.1205	1.0716	0.9756	1.0534
99%-VaR	-0.0379	-0.085	-0.0366	-0.1138	-0.0691	0.0247	-0.052
Capped. out. cer.							
Average return	0.3847	0.4425	0.5487	0.5629	0.3165	0.4007	0.4585
99%-VaR	0.9554	0.8154	0.6002	0.5747	1.1504	0.9145	0.7797

Table 3.7: *Historically simulated average gross return and 99%-VaR of structured products (tracker certificates, discount certificates and capped outperformance certificates), written on major European stock market indices. Simulation with historical indices levels from 3 March 2008 to 24 February 2014.*

return profiles of structured products are ambiguous, according to our theoretical findings.

3.3.5 Conclusion

In this section, we have tried to discuss the impacts of underlying's moments on the expected return and the VaR of structured products – in other words, the preferences for the underlying's moments under the mean-VaR framework. We first see that the risk-return profile of a structured product can theoretically be affected by different underlyings. Then, we expand the expected return of a structured product with Taylor series and expand the VaR of the product with Cornish-Fisher approach. This allows us to study the impacts of underlying's first four moments on the expected return and the VaR of a product, simultaneously. Theoretical results are derived for the cases where the underlying's log-return follows normal distribution, zero-skewness distribution, mesokurtic distribution or a general distribution with nonzero skewness and nonzero excess kurtosis.

Under the mean-VaR framework, the findings show that for the majority of structured products: other moments being fixed, underlyings with large expected log-return are always preferred. The preference for the volatility of the underlying is ambiguous. Large positive skewness is also preferred. The impacts of

its kurtosis on the expected return and the VaR of the product are ambiguous – large kurtosis increases both the expected return and the VaR (risk) at the same time. The results indicate that mean-variance efficient underlyings (the market portfolio if in the CAPM world), do not necessarily improve the risk-return profile of a structured product. Because small variance may decrease the VaR on the one hand, it will on the other hand decrease the return of the product as well. The importance of kurtosis risk in the context of portfolio with derivatives is also confirmed by the results. Ignoring the kurtosis of the underlying whose log-return is leptokurtically distributed, will lead to underestimations for the VaR of the structured product.

The theoretical results are tested with Monte Carlo simulations. We consider the cases of normal distribution, t -distribution and NIG-distribution. Structured products used in the simulation are tracker certificates, discount certificates and capped outperformance certificates. Simulation results are not at odds with the theoretical findings.

A limitation of the research is that the discussions are based on one period models. Although it is true that buyers of structured products usually implement a buy-and-hold strategy and there is basically no trading before maturity, this one period framework is not capable of considering the path-dependent products, e.g. products with barrier option component or variance swaps, etc. Further study can extend the framework of the research to a multi-period one.

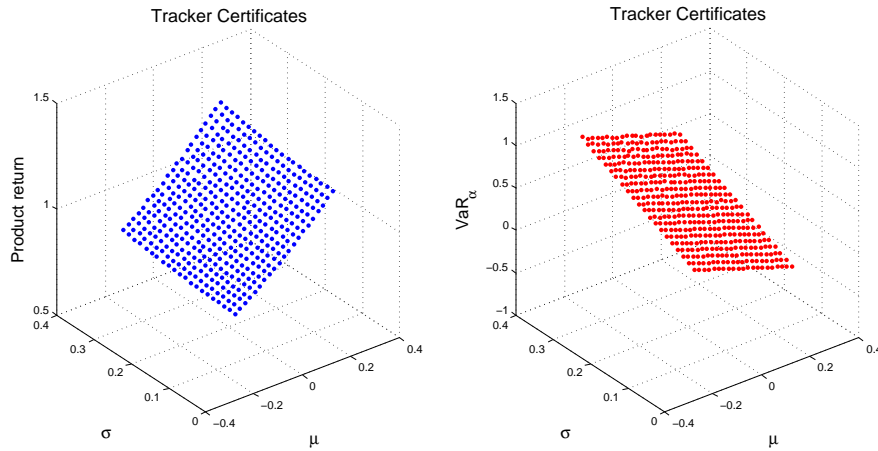


Figure 3.2: *Simulated mean return (left) and VaR (right) of tracker certificates with normal distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation.*

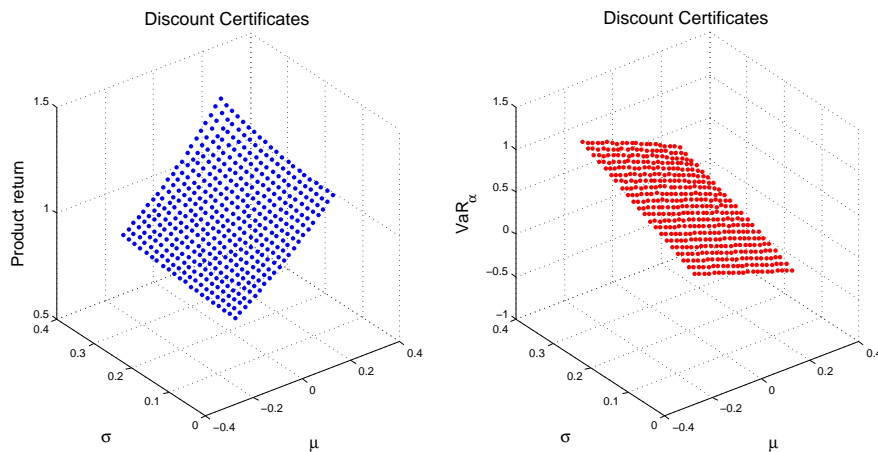


Figure 3.3: *Simulated mean return (left) and VaR (right) of discount certificates with normal distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation.*

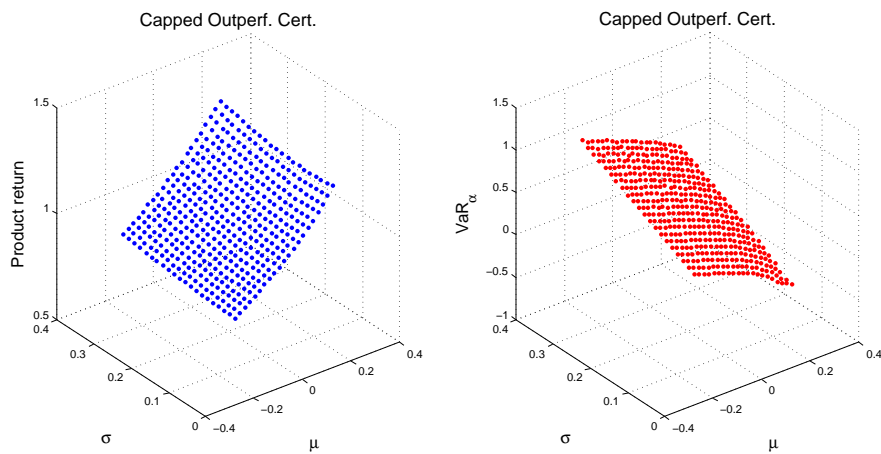


Figure 3.4: Simulated mean return (left) and VaR (right) of capped outperformance certificates with normal distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation.

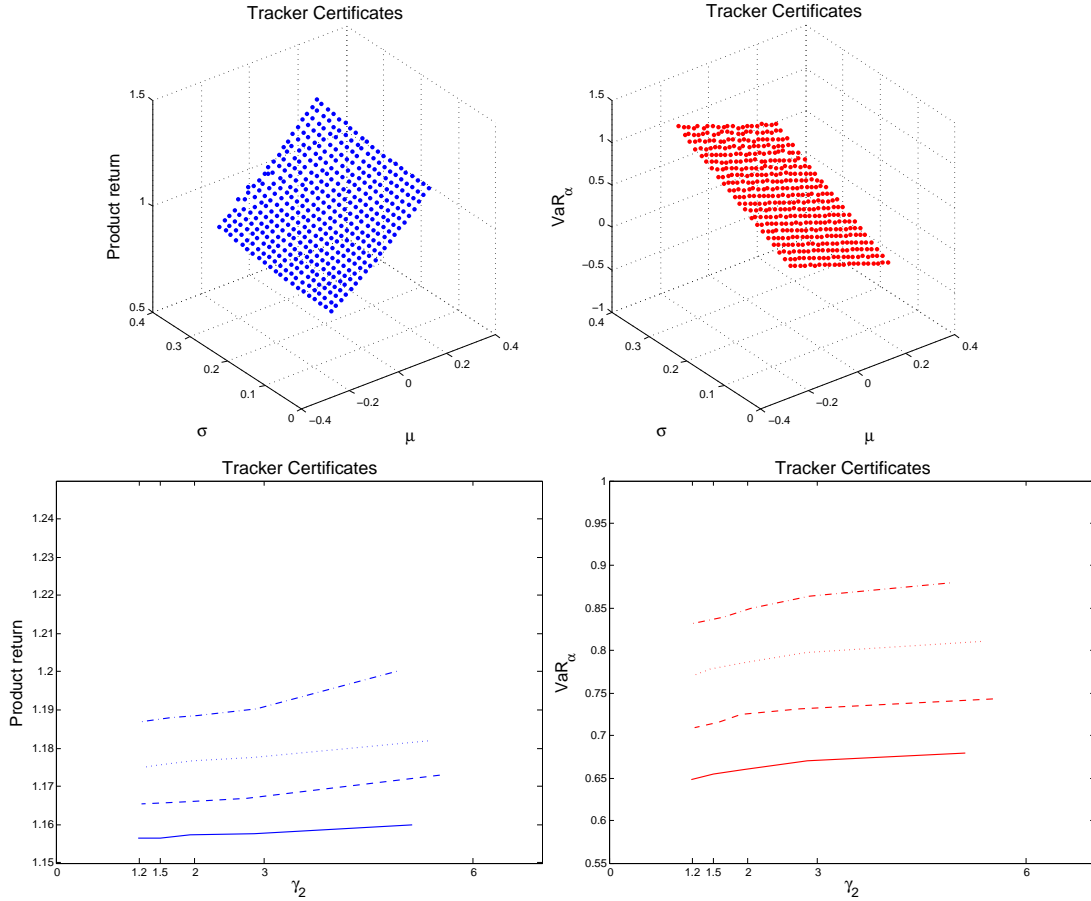


Figure 3.5: Top: Simulated mean return (left) and VaR (right) of tracker certificates with t -distributed (6 degrees of freedom) underlying log-return. The x -axis is the mean of the simulated underlying log-return. The y -axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of tracker certificates with t -distributed underlying log-return. The x -axis is the excess kurtosis of the simulated underlying log-return. The solid line, the dashed line, the dotted line and the “-.” line stand for a volatility of 30%, 32.5%, 35% and 37.5%, respectively. 1,000,000 times simulation.

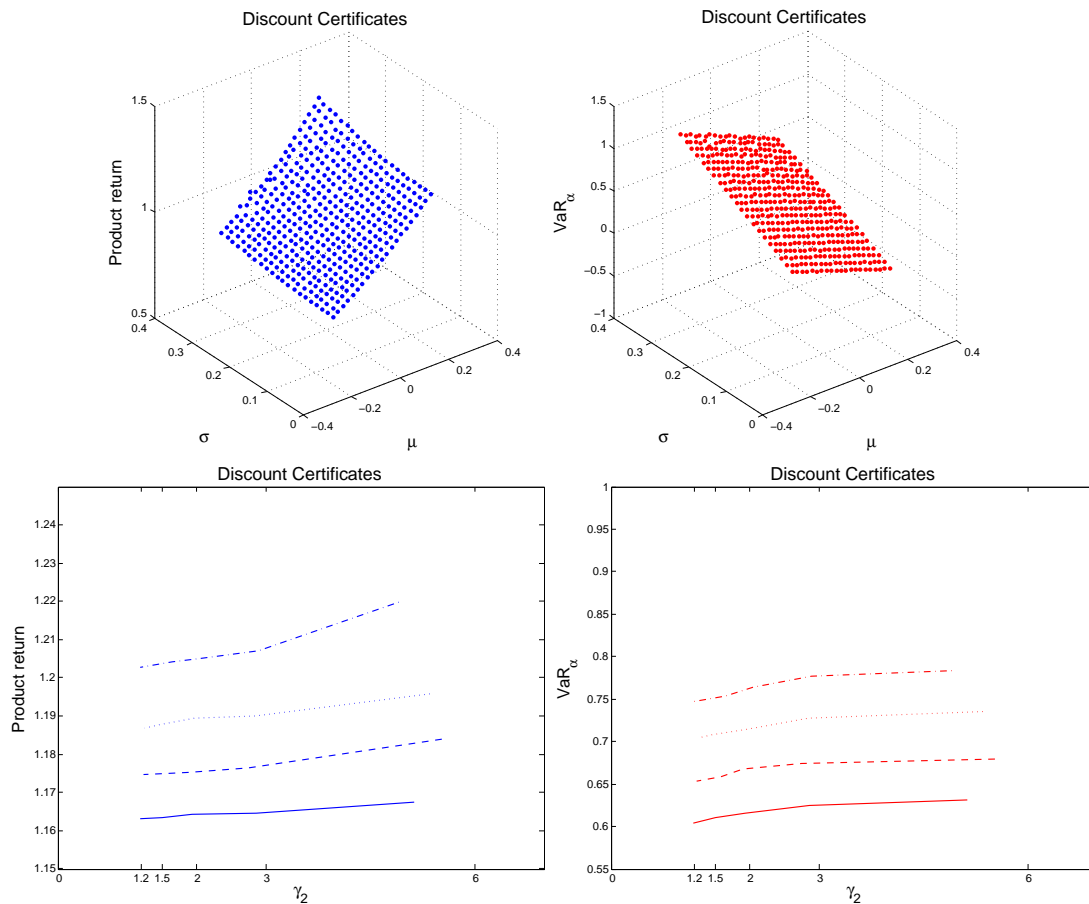


Figure 3.6: Top: Simulated mean return (left) and VaR (right) of discount certificates with t -distributed (6 degrees of freedom) underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of discount certificates with t -distributed underlying log-return. The x-axis is the excess kurtosis of the simulated underlying log-return. The solid line, the dashed line, the dotted line and the “-.” line stand for a volatility of 30%, 32.5%, 35% and 37.5%, respectively. 1,000,000 times simulation.

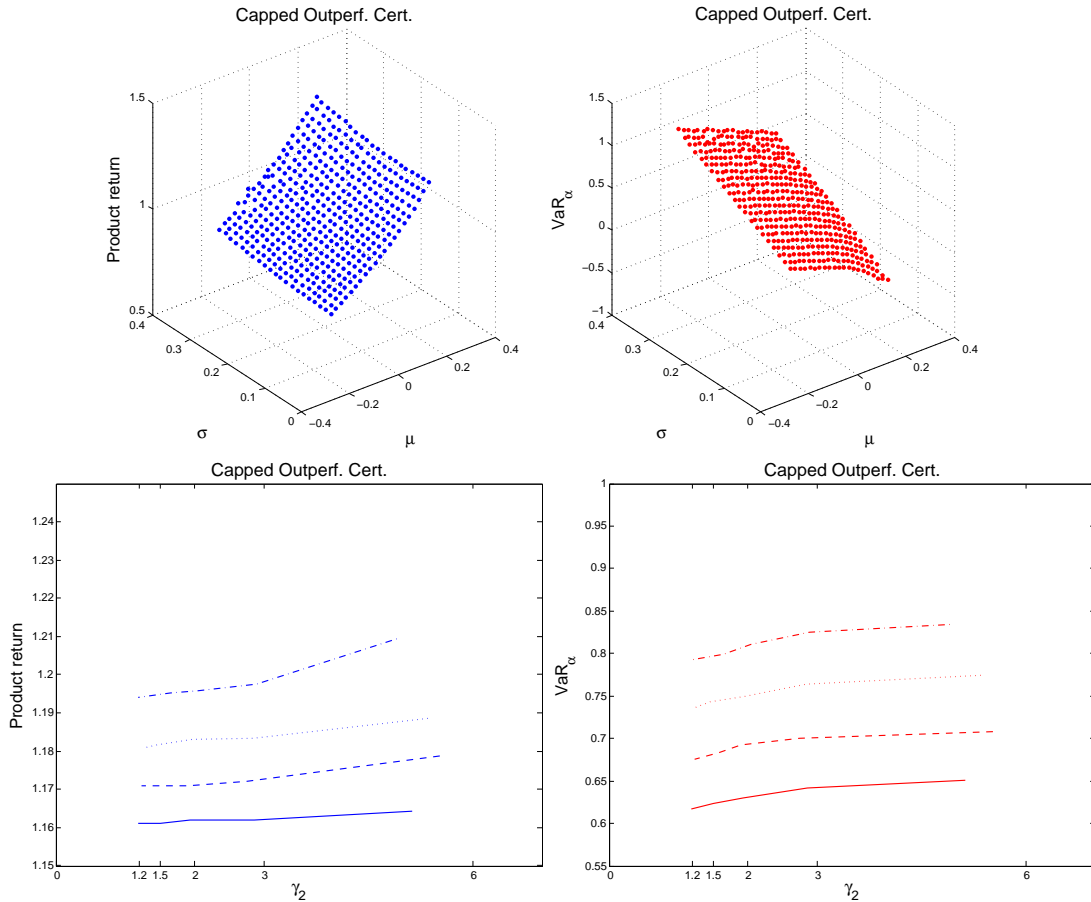


Figure 3.7: Top: Simulated mean return (left) and VaR (right) of capped outperformance certificates with t -distributed (6 degrees of freedom) underlying log-return. The x -axis is the mean of the simulated underlying log-return. The y -axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of capped outperformance certificates with t -distributed underlying log-return. The x -axis is the excess kurtosis of the simulated underlying log-return. The solid line, the dashed line, the dotted line and the “-.” line stand for a volatility of 30%, 32.5%, 35% and 37.5%, respectively. 1,000,000 times simulation.

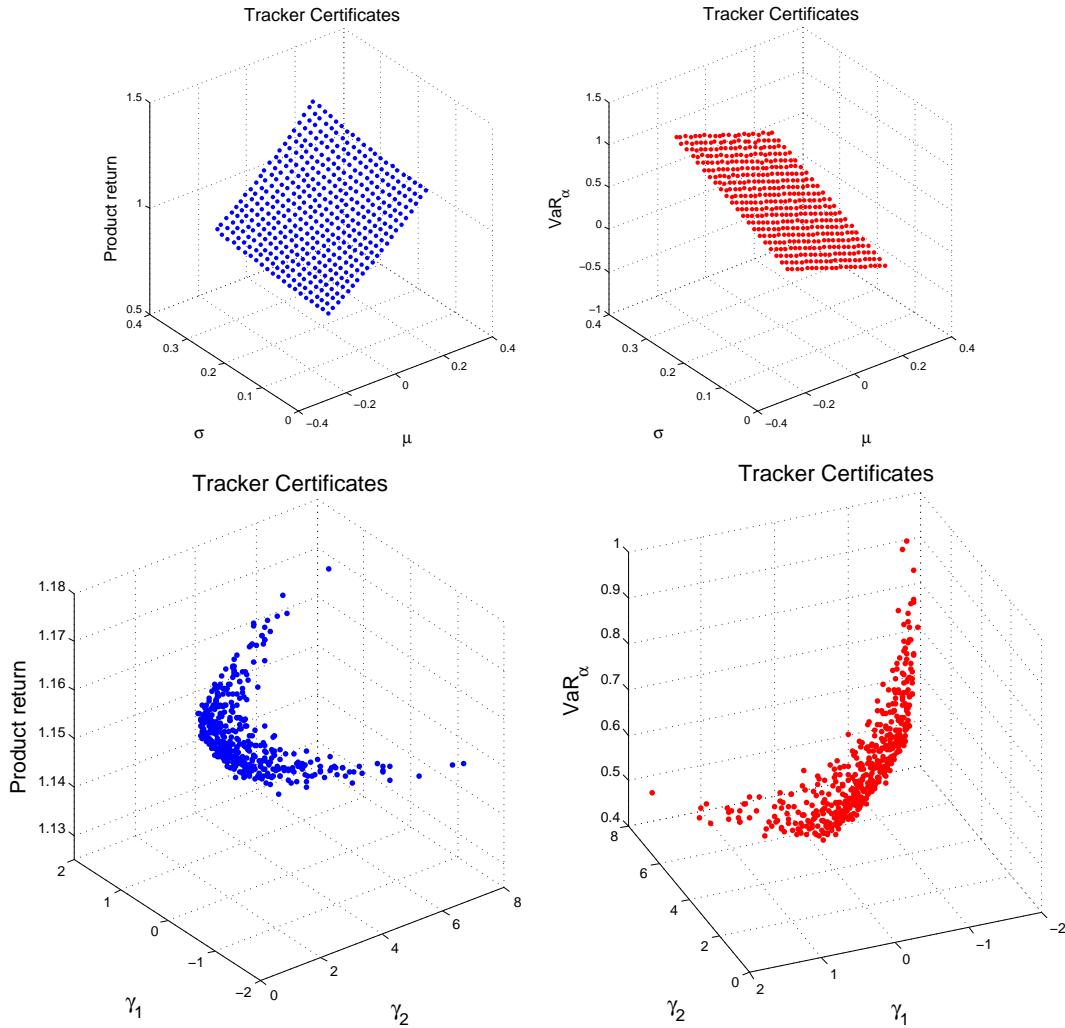


Figure 3.8: Top: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis and the y-axis are the excess kurtosis (γ_2) and the skewness (γ_1) of the simulated underlying log-return, respectively. The VaR plot is rotated for a better view angle. 100,000 times simulation.

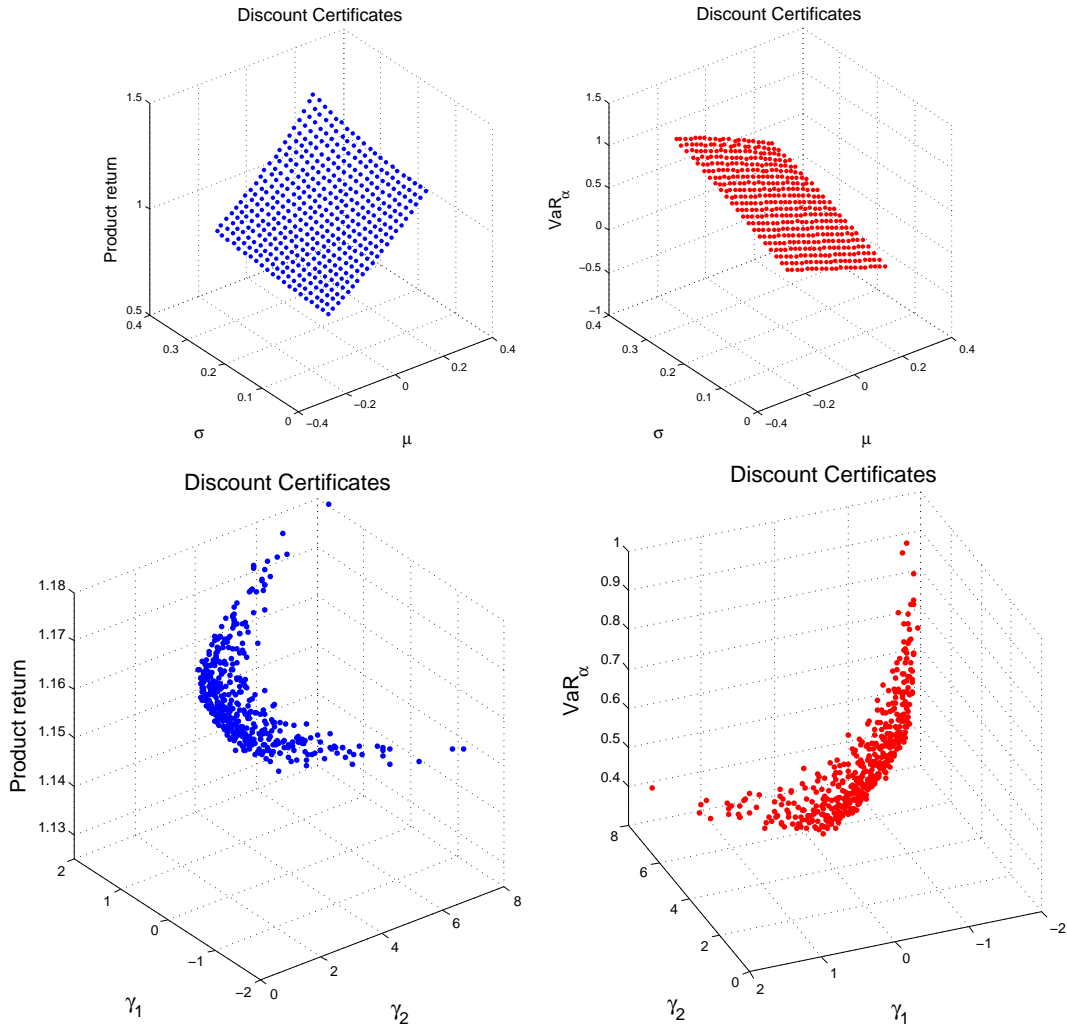


Figure 3.9: Top: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis and the y-axis are the excess kurtosis (γ_2) and the skewness (γ_1) of the simulated underlying log-return, respectively. The VaR plot is rotated for a better view angle. 100,000 times simulation.

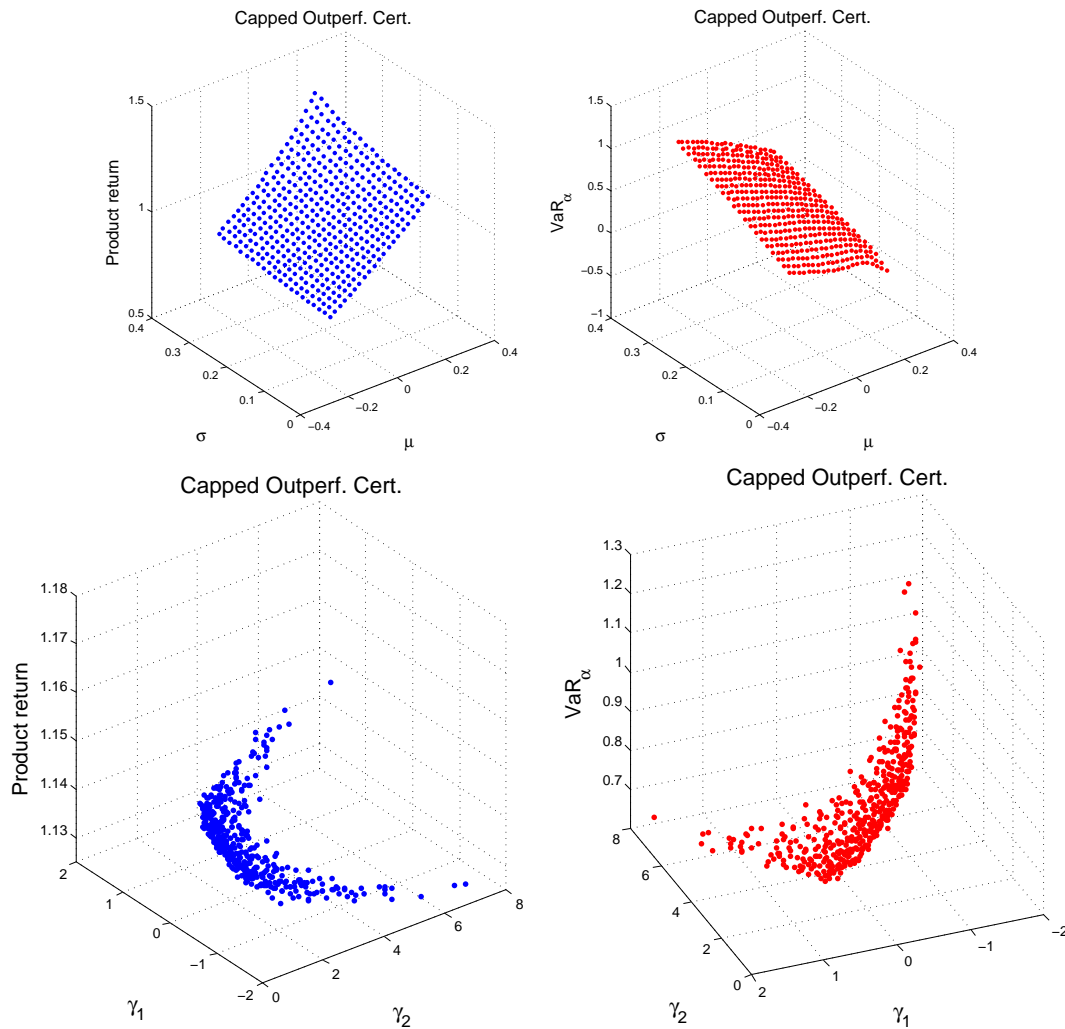


Figure 3.10: Top: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis is the mean of the simulated underlying log-return. The y-axis is the volatility of the simulated underlying log-return. 100,000 times simulation. Bottom: Simulated mean return (left) and VaR (right) of tracker certificates with NIG-distributed underlying log-return. The x-axis and the y-axis are the excess kurtosis (γ_2) and the skewness (γ_1) of the simulated underlying log-return, respectively. The VaR plot is rotated for a better view angle. 100,000 times simulation.

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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Trier

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