# Determining the Structure of Diversification

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# Abstract

The classic Capital Asset Pricing Model and the portfolio theory suggest that investors hold the market portfolio to diversify idiosyncratic risks. The theory predicts that expected return of assets is positive and that reacts linearly on the overall market. However, in reality, we observe that investors often do not have perfectly diversified portfolios. Empirical studies find that new factors influence the deviation from the theoretical optimal investment.

In the first part of this work (Chapter 2) we study such an example, namely the influence of maximum daily returns on subsequent returns. Here we follow ideas of Bali et al. (2011). The goal is to find cross-sectional relations between extremely positive returns and expected average returns. We take account a larger number of markets worldwide. Bali et al. (2011) report with respect to the U.S. market a robust negative relation between MAX (the maximum daily return) and the expected return in the subsequent time. We extent substantially their database to a number of other countries, and also take more recent data into account (until end of 2009). From that we conclude that the relation between MAX and expected returns is not consistent in all countries. Moreover, we test the robustness of the results of Bali et al. (2011) in two time-periods using the same data from CRSP. The results show that the effect of extremely positive returns is not stable over time. Indeed we find a negative cross-sectional relation between the extremely positive returns and the average returns for the first half of the time series, however, we do not find significant effects for the second half. The main results of this chapter serve as a basis for an unpublished working paper Yuan and Rieger (2014b).

While in Chapter 2 we have studied factors that prevent optimal diversification, we consider in Chapter 3 and 4 situations where the optimal structure of diversification was previously unknown, namely diversification of options (or structured financial products). Financial derivatives are important additional investment form with respect to diversification. Not only common call and put options, but also structured products enable investors to pursue a multitude of investment strategies to improve the risk-return profile. Since derivatives become more and more important, diversification of portfolios with dimension of derivatives is of particularly practical relevance.

We investigate the optimal diversification strategies in connection with underlying stocks for classical rational investors with constant relative risk aversion (CRRA). In particular, we apply Monte Carlo method based on the Black-Scholes model and the Heston model for stochastic volatility to model the stock market processes and the pricing of the derivatives. Afterwards, we compare the benchmark portfolio which consists of derivatives on single assets with derivatives on the index of these assets. First we compute the utility improvement of an investment in the risk-free assets and plain-vanilla options for CRRA investors in various scenarios. Furthermore, we extend our analysis to several kinds of structured products, in particular capital protected notes (CPNs), discount certificates (DCs) and bonus certificates (BCs). We find that the decision of an investor between these two diversification strategies leads to remarkable differences. The difference in the utility improvement is influenced by risk-preferences of investors, stock prices and the properties of the derivatives in the portfolio. The results will be presented in Chapter 3 and are the basis for a yet unpublished working paper Yuan and Rieger (2014a).

To check furthermore whether underlyings of structured products influence decisions of investors, we discuss explicitly the utility gain of a stock-based product and an index-based product for an investor whose preferences are described by cumulative prospect theory (CPT) (Chapter 4, compare to Yuan (2014)). The goal is that to investigate the dependence of structured products on their underlying where we put emphasis on the difference between index-products and single-stock-products, in particular with respect to loss-aversion and mental accounting. We consider capital protected notes and discount certificates as examples, and model the stock prices and the index of these stocks via Monte Carlo simulations in the Black-Scholes framework. The results point out that market conditions, particularly the expected returns and volatility of the stocks play a crucial role in determining the preferences of investors for stock-based CPNs and index-based CPNs. A median CPT investor prefers the index-based CPNs if the expected return is higher and the volatility is lower, while he prefers the stock-based CPNs in the other situation. We also show that index-based DCs are robustly more attractive as compared to stock-based DCs for CPT investors.

**Key words:** extreme positive returns, cross-sectional returns, factor-model, idiosyncratic volatility, diversification, options, structured products, expected utility, cumulative prospect theory, underlying stocks

# Zusammenfassung

Das klassische Capital Asset Pricing Model und die Portfolio-Theorie suggerieren, dass Investoren ein Marktportfolio von Aktien halten, um idiosynkratische Risiken zu diversifizieren. Die Theorie sagt voraus, dass die erwartete Rendite der Wertpapiere positiv und linear auf die Bewegung des Gesamtmarktes reagiert. Allerdings lässt sich in der Realität beobachten, dass Investoren häufig nicht perfekt diversifizierte Portfolios halten. Empirische Studien finden immer wieder neue Faktoren, die diese Abweichungen von der theoretisch optimalen Anlage beeinflussen.

Im ersten Teil dieser Arbeit (Kapitel 2) untersuchen wir ein solches Beispiel, nämlich den Einfluss von maximalen Tagesrenditen auf nachfolgende Renditen. Wir folgen dabei der Idee von Bali et al. (2011). Das Ziel ist es, die Querschnittsbeziehung zwischen extrem positiven Renditen und erwarteten durchschnittlichen Renditen unter Einbezug einer großeren Anzahl von Märkten weltweit zu erforschen. Bali et al. (2011) berichten hierbei bezogen auf den US-Markt über eine robuste negative Beziehung zwischen dem MAX (die maximale tägliche Rendite) und den erwarteten Rendite im nachfolgenden Zeitraum. Wir erweitern deutlich ihre Datenbasis auf eine Reihe von anderen Ländern und nehmen auch Daten der jüngsten Vergangenheit in die Analyse auf (bis Ende 2009). Dabei stellen wir fest, dass die Beziehung zwischen dem MAX und den erwarteten Renditen nicht in allen Ländern konsistent ist. Darüber hinaus testen wir die Robustheit der Ergebnisse von Bali et al (2011) über zwei Teilperioden mit dem gleichen Datenbestand aus CRSP. Die Ergebnisse zeigen, dass die Wirkung von extrem positiven Renditen über die Zeit nicht stabil ist.

Wir können die negative Querschnittsbeziehung zwischen den extrem positiven Renditen und den Durchschnittsrenditen für die erste Hälfte des Beobachtungszeitraums bestätigen, jedoch finden wir keinen signifikanten Beweis für sie in der zweiten Hälfte. Die Hauptergebnisse dieses Kapitels dienen als Basis für ein noch unveröffentlichtes Working Paper Yuan und Rieger (2014b)

Während in Kapitel 2 Faktoren untersucht wurden, die optimale Diversifikation verhindern, betrachten wir in Kapitel 3 und 4, Situationen, in denen die Struktur einer optimalen Diversifikation bislang unbekannt war, nämlich bei der Diversifikation von Optionen (oder Strukturierten Finanzprodukten). Finanzderivate sind wichtige ergänzende Anlageformen in Bezug auf Diversifikation. Nicht nur herkömmliche Call- und Put-Optionen, sondern auch Strukturierte Produkte ermöglichen Anlegern eine Vielzahl von Anlagestrategien, um das Rendite-Risiko-Profil zu verbessern. Da Derivate zunehmend an Bedeutung gewinnen, ist Diversifikation von Portfolios mit Finanzderivaten von besonderer praktischer Relevanz.

Wir untersuchen die optimalen Diversifikationsstrategien im Zusammenhang mit zugrunde liegenden Aktien für klassische rationale Investoren bei konstanter relativer Risikoaversion (CRRA). Insbesondere wenden wir hier Monte-Carlo-Verfahren basierend auf dem Black-Scholes-Modell und dem Heston Modell für stochastische Volatilität an, um die Prozesse der zugrunde liegenden Aktien sowie die Bewertung der Derivate zu modellieren. Anschließend vergleichen wir ein Portfolio, welches aus Derivaten auf einzelne Aktien besteht, mit einem Derivat auf den Index dieser Aktien. Zunächst bewerten wir die Nutzenverbesserung einer Investition in risikofreie Anlagen und Plain-Vanilla-Optionen für einen CRRA Investor in verschiedenen Szenarien. Des Weiteren erweitern wir unsere Analyse auf bestimmte Arten von Strukturierten Produkten: Kapitalschutz-Produkte (CPNs), Discount-Zertifikate (DCs) und Bonus-Zertifikate (BCs). Wir stellen fest, dass sich die Entscheidung von Investoren zwischen diesen beiden Diversifikationsstrategien merkbar unterscheidet. Die Differenz der Nutzenverbesserung kann durch Risikopräferenzen von Investoren, Aktienkurse und den Eigenschaften von Derivaten im Portfolio getrieben werden. Diese Ergebnisse werden in Kapitel 3 präsentiert und bilden die Grundlage für ein noch unverffentlichtes Working Paper Yuan und Rieger (2014a)

Um weiter zu prüfen, ob Basiswerte von Strukturierten Produkten Entscheidungen der Anlegern möglicherweise beeinflussen, diskutieren wir nun explizit den Nutzengewinn eines aktienbasierten Produktes und eines indexbasierten Produktes für einen Investor, dessen Präferenzen durch die kumulative Prospect Theory (CPT) beschrieben werden (Kapitel 4, vergleiche Yuan (2014)). Das Ziel ist es, die Abhängigkeit der Strukturierten Produkte zu ihren zugrunde liegenden Basiswerten zu erforschen, wobei hier vor allem die Attraktivität eines Indexproduktes im Vergleich zu einem Produkt, welches als Basiswert eine der im Index enthaltenen Aktien besitzt, für Anleger mit Verlustaversion und mentaler Buchhaltung untersucht wird. Wir ziehen Kapitalschutz-Produkte (CPNs) und Discount-Zertifikate (DCs) als Beispiele heran und simulieren die zugrunde liegenden Aktienpreise und den Index dieser einzelnen Aktien über Monte-Carlo-Simulationen im Rahmen des Black-Scholes-Modells. Die Ergebnisse deuten an, dass die Marktbedingungen, unter anderem, erwartete Renditen und Volatilitäten der Aktien, eine entscheidende Rolle bei der Bestimmung der Präferenz der Investoren für aktienbasierte CPNs oder indexbasierten CPNs spielen. Ein Median CPT Investor bevorzugt die indexbasierten CPNs im Falle höherer erwarteter Renditen und niedrigere Volatilitäten, während er die aktienbasierte CPNs vice versa vorzieht. Es wird auch gezeigt, dass die indexbasierte DCs robust attraktiver im Vergleich zu den aktienbasierten DCs für die CPT Investoren sind.

**Stichwörter:** extrem positive Rendite, Querschnittsrendite, Faktormodell, idiosynkratische Volatilität, Diversifikation, Optionen, Strukturierte Produkte, erwarteter Nutzengewinn, kumulative Prospect Theory, zugrunde liegende Aktien

# Chapter 1

# Introduction

The standard theory of mean-variance portfolio choice suggests that investors hold diversified portfolios to reduce or eliminate unsystematic risks. However, many investors, especially retail investors, are observed holding under-diversified portfolios which is inconsistent with the classic framework. The desire for under-diversified portfolios may be explained by psychological reasons. As a matter of fact, many actual behavior characteristics are linked to investment decisions, such as investors' propensity to gambling, preference for a certain group of stocks, framed decisions as gains and losses, and so on. These behavioral biases are shown to systematically increase the subjective attractiveness of many complex financial products, which have been tremendously grown and developed in recent years. Among others, the broadly known structured products offer retail investors access to financial derivatives which could not be reached previously, hence making them a useful complement tool compared to traditional asset class for portfolios diversification. Nevertheless, the significant success of structured products is intriguing in that it is difficult to explain the demand of a rational investor for such a product. In this light, this dissertation explores several key issues arising from diversification in investors' portfolios, in particular in the choice of structured products.

More specifically, I firstly focus on stock markets and investigate the relation between lottery-like returns and subsequent cross-sectional returns in different countries. Many studies have observed that investors include a disproportional number of lottery-type stocks with low prices and high idiosyncratic volatility in their portfolios in the hope of obtaining higher returns in the future, even though the chances of this are extremely low. Consequently, this gambling preference by those investors causes an overpricing of stocks with lottery features. Evidence from the U.S. market indicates the negative significance of extreme positive returns in the cross-sectional pricing of stocks. Inspired by previous results from the U.S. market in Bali et al. (2011), the first article considers this anomaly in a much wider range of countries, including Canada, France, Germany, the U.K., China and Japan. Furthermore, it also examines the robustness of the results in the U.S. market by concentrating on a more recent time period.

In the second study, I analyze the diversification choices of financial derivatives for expected utility theory investors given two investment strategies: a combination of derivatives each based on an individual stock, and a derivative based on the index composed of these individual stocks. As financial derivatives play an invaluable role in improving investors' utility in terms of better diversifying their portfolios, diversification within derivatives is of practical importance for both institutional and retail investors. However, very few papers have explicitly considered the diversification strategy within financial derivatives with respect to their underlying assets. To this end, the second article investigates the optimal diversification strategy of options as well as structured products.

Following the second study, the third one presents numerical and empirical evidence examining whether the underlying stocks of structured products influence investors' choices in a behavioral finance context. Often, structured products are targeted at retail investors who lack financial sophistication and are subject to numerous cognitive or behavioral biases. On the other hand, for an issuing bank considering designing a new product, an appropriate reference asset is one crucial factor in success of the product – by no means a straightforward task. Motivated by these two points, this article takes the cumulative prospect theory as a starting point and describes favorable structured products in terms of two alternative underlying assets: individual stocks and an index composed of these stocks.

# Chapter 2

# Maxing out: the puzzling influence of past maximum returns on future asset prices in a cross-country analysis

## 2.1 Introduction

> "*THOSE who occasionally bet on horse races like the Kentucky Derby or the Grand National have a tendency to favour 100-1 outsiders. Their motivation may be the desire for a big win to justify the act of gambling at all.*[1]

The cross-sectional variation in expected returns has been debated for decades. Many real investment behaviors observed in the complicated financial markets like the foreword mentioned go far beyond the theoretical models and reflect an investment mania that seems insane, in that the standard rational asset pricing models are not able to explain the empirical facts of the aggregate stock market nor individual trading behaviors. Then what drives a usually risk-averse investor to invest in stocks with high risk? If investors hold many of these stocks in their portfolios, would this imperfect diversification eventually affect the equilibrium asset prices on average? Such anomalies have recently been found, e.g. regarding volatility and skewness (Harvey and Siddique (2000), Ang et al. (2006, 2009), and Barberis and Huang (2008)).

Very recently, Bali et al. (2011) have found a statistically and economically significant negative relation between the maximum daily return over the previous month (MAX) and the cross-section of expected stock returns. The constructed new variable, MAX, capturing the extremely positive daily returns, is to used to investigate the predictability of the maximum daily stock return in stock pricing. The motivation is that investors frequently hold underdiversified portfolios and more often have a preference for lottery-like assets that usually have a small probability of large payoffs, e.g. low-priced stocks with high idiosyncratic volatility[2]. While it is no

[1]Reported in The Economist (May 31, 2011). Liquidity and lottery tickets–Why investors overpay for certain assets

[2]See Odean (1999), Campbell et al. (2001), Mitton and Vorkink (2007), Goetzmann and Kumar (2007).

surprise that stocks with extremely positive daily returns tend to be small and have high idiosyncratic volatility and skewness as defined for lottery-type stocks in Kumar (2009), the result is robust after controlling for a variety of these and other firm-level variables and anomalies. Particularly Bali et al. (2011) show that including MAX reverses the negative relation between returns and idiosyncratic volatility reported in Ang et al. (2006, 2009). The result is interpreted as consistent with cumulative prospect theory (Tversky and Kahneman (1992) and Barberis and Huang (2008)) as well as the optimal beliefs framework of Brunnermeier et al. (2007).

In this chapter, we reexamine Bali et al. (2011) to clarify the existence and significance of a relation between extremely positive returns and expected returns across different countries as well as for various sample periods. We expand the scope with data on other stock markets from Datastream for seven countries (Germany, the U.K., China, Canada, France, Japan and the U.S.). Overall, we find that the influence of the MAX factor is not the same across countries and times. While the Chinese stock market shows a similar pattern to that observed by Bali et al. (2011), in which high exposure to extremely positive returns tends to produce low expected returns, our results in Canada, the U.K. and the U.S. indicate a statistically positive cross-sectional relation between extreme positive returns and average returns. Moreover, we fail to find any statistically significant relation between extremely positive returns and the subsequent average returns in the French, Japanese and German stock markets.

Given a relatively larger sample in the U.S. market and its conflicting result, we also look at idiosyncratic volatility of stocks that cannot be easily disentangled from features of lottery-like stocks. Merton (1987) underlines that, in an information-segmented market, stocks with larger firm-specific variances require high average returns to compensate investors for holding imperfectly diversified portfolios. Similarly, Barberis and Huang (2001) point out that stocks with higher idiosyncratic volatility have higher expected returns. It is intuitively clear that high MAX stocks also have a propensity for higher idiosyncratic volatility. In fact, the positive MAX-return relation disappears once we control for idiosyncratic volatility. The bivariate sorts and cross-sectional regressions reveal a robustly significant positive relation between idiosyncratic volatility and expected stock returns based on our data set.

We further test the robustness of Bali et al. (2011) on a shorter time period, while the original results were based on the full period from July 1962 to December 2005. We use CRSP data on NYSE, AMEX, NASDAQ stocks, but split the whole period during 1963 to 2010 into two subperiods. The result is similar to the finding of Bali et al. (2011) for the period from July 1963 to December 1989, however, we find no evidence of a significantly negative relation between the maximum daily return of the previous month and the cross-section of expected returns for the period from January 1990 to December 2010.

Given the results on different countries as well as recent time periods, our results suggest that the negative relation between the maximum daily return over the previous month and expected stock returns reported by Bali et al. (2011) is not consistent, but changes over time. Therefore, it is conceivable that macroeconomic conditions or changes in the investor population might play a critical role in determining the average returns affected by the performance of lottery-type stocks. The investors in Bali et al. (2011) are assumed to be not well-diversified and have a preference for lottery-type stocks, i.e. stocks that have a small probability of a large payoff. Investors who are attracted toward those stocks with high MAX accept lower expected returns given a chance of a large gain. Thus, ultimately, the preferences of those MAX-seeking investors could influence the cross-sectional expected stock returns. Inspired by this interpretation, we conjecture that the proportion of MAX-seeking investors in the overall market may relate to the relation between extremely positive stock returns and expected returns. More explicitly, the MAX puzzle should be strongest in a market where retail investors with preference for MAX dominate. Interestingly, Han and Kumar (2008) confirm that stocks with high proportion of retail investors tend to earn lower future returns, especially if they are speculative stocks. The lottery-type stocks, i.e. stocks with high MAX, contain exactly those features that could be attractive to retail investors, which supports the conclusion in Kumar (2009). More impressively, Han and Kumar (2013) examine the characteristics and pricing of stocks that are actively traded by speculative retail investors, and conclude that speculative retail trading affects stock prices. Collectively, we presume that stocks in a relatively younger stock market, such as China or earlier time periods in the U.S., are dominated by more retail investors, hence exhibit a stronger negative MAX premium, while in well-developed markets, such as the U.S. nowadays, the negative premium is much weaker, hence this relation evaporates.

The remainder of this chapter is organized as follows. Section 2.2 describes the data and construction of variables. Section 2.3 presents the main empirical results in the seven countries, and discusses in detail the MAX effect using the U.S. data. Section 2.4 examines the robustness of Bali et al. (2011)'s results. Section 2.5 concludes.

## 2.2 Data and Variables

The data sample comprises data on firms from seven markets for the period from January 1994 to December 2009. All the data sets are obtained from Datastream. Specifically, the individual stock returns we select are the Datastream data for the U.S., S&P/TSX Composite for Canada, the FTSE-All shares for the U.K. market, the HDAX for Germany, Shanghai A-Shares for China, the Datastream data for France and the Datastream data for Japan.

In accordance with Bali et al. (2011) we measure extremely positive returns as the maximum daily stock returns over one month.

$$MAX_{i,t} = \max(R_{i,d}) \qquad d = 1, \ldots, D_t, \tag{2.1}$$

where $R_{i,d}$ is the return on stock i on day d and $D_t$ is the number of trading days in month $t$. Meanwhile we also take other economic explanatory components into consideration. We measure the systematic risk with the market beta in line with the CAPM, which is estimated as the slope in the regression of individual stocks' return on the value-weighted index market returns. For each month, we calculate the monthly market beta using daily returns within the month, and run time-series regression within the month on excess market returns. The estimated slope coefficient is the market beta for each month. Moreover, we obtain market capitalization and book-to-market ratio at the end of each month, which are all available in Datastream. We use monthly returns to calculate proxies for intermediate-term momentum and short-term reversals, as a control for the effects of past returns. Specifically, momentum is defined as the cumulative return over the previous 11 months from $t-12$ to $t-2$. The reversal variable is the stock return over the previous month. Additionally, Amihud (2002) suggests a positive relation between illiquidity and cross-sectional returns. Following this idea, we measure the illiquidity by the ratio of the absolute monthly return and its trading volume in value. We ignore missing values, so that a firm is eliminated if the relevant information is missing for a particular variable.

Table 2.1 reports an overview of statistics for the stock returns and other firm characteristic data across the pooled samples. We use data from January 1994 to December 2009, in total 192 months. We present the time-series averages of the monthly values for monthly return, monthly maximum daily return, market capitalization and book-to-market ratio. Firm size is the market value of equity and is measured by the natural logarithm.

Firstly, we see that the sample is relatively large in the U.S., Japan, and China, where more than 800 firms are included. The number of firms in Germany is the smallest, containing only 108 firms. Secondly, there is a moderate variation in monthly MAX across different countries. The average MAX ranges from 7.34% in Canada to 3.9% in the U.K. Correspondingly, the Canadian market tends to have the highest return at 2.29% per month, whereas the monthly return in Japan is the lowest at 0.59%. Lastly, for the fundamental characteristics of firms, the average logarithmic firm size is the highest in Japan at 11.8, and lowest in the U.K. at 5.8. Canada has the highest book-to-market ratio (0.81), compared to China which has the lowest book-to-market ratio (0.34).

Table 2.1: Summary statistics of the international data

| Countries | Number of firms | Number of months | Monthly return | Monthly MAX | Size | Book-to-market |
|---|---|---|---|---|---|---|
| U.S. | 990 | 192 | 1.61% | 5.11% | 8.024 | 0.460 |
| Japan | 992 | 192 | 0.59% | 5.06% | 11.776 | 0.807 |
| China | 845 | 192 | 2.04% | 6.27% | 7.668 | 0.344 |
| U.K. | 612 | 192 | 1.21% | 3.90% | 5.838 | 0.746 |
| Canada | 234 | 192 | 2.29% | 7.34% | 6.360 | 0.850 |
| Germany | 108 | 192 | 1.26% | 5.14% | 7.832 | 0.608 |
| France | 246 | 192 | 1.44% | 5.17% | 6.483 | 0.710 |

This table summarizes the pooled descriptive statistics of stocks for the seven countries during the period January 1994 to December 2009. Monthly return is the monthly raw return, and monthly MAX is the maximum daily return over one month. Size is the natural logarithm of market value of equity at the end of each month, and book-to-market ratio has been directly obtained from Datastream. For each country, variables are the time series average of the monthly values.

## 2.3 Results

In this section we discuss the results of the analyses across the seven countries. We start with a portfolio-level analysis, then we move on to cross-sectional regressions at the firm level.

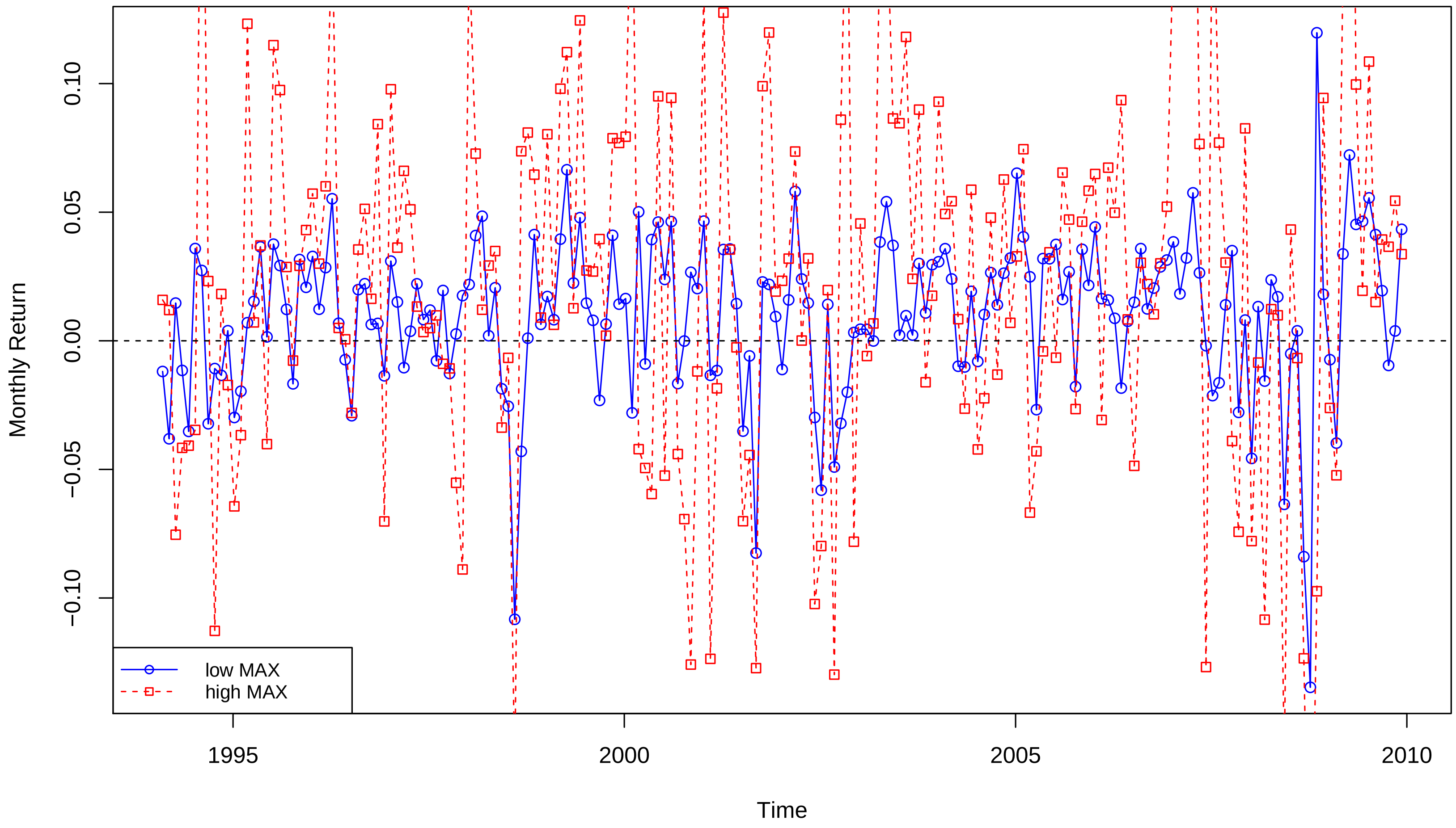
### 2.3.1 Portfolio Analysis

Before we check the characteristics of extremely positive returns on the individual country's level, we first examine it in a global way. We form decile portfolios including all 4028 monthly stock returns across countries ranked on MAX rebalanced every month. Portfolio 1 (low MAX) contains stocks with the lowest MAX in the previous month and Portfolio 10 (high MAX) includes stocks with the highest MAX in the previous month. Table 2.2 presents the equally weighted returns of the decile portfolios. As we compare the ten equally weighted portfolios, a striking feature different from the results of Bali et al. (2011) is that a high MAX stocks portfolio (decile 10) tends to generate higher returns (2.53% per month on average) compared to other percentile portfolios, particularly compared to a low MAX stocks portfolio (decile 1). The average return difference is 1.50% with a significant t-statistic of 2.63 (not reported in the Table). Moreover, the high MAX portfolio exhibits substantially higher monthly volatility, at 8.18%, than that of the low MAX portfolio at 3.13%. We also find that portfolio 9 and 10 with relatively higher MAX stocks present positive skewness, while the other 8 portfolios show negative skewness.

In addition, in Figure 2.1 we plot the time-series average portfolio returns across the seven countries by comparing the high MAX (top 10% MAX percentile) and the low MAX (bottom 10% MAX percentile) portfolios. It is clearly visible that the high MAX portfolio is more volatile than the low MAX portfolio across the seven countries.

Next, we move to a univariate sorting method to test the performance of the stocks that earned the highest daily return over one month at the country level. Due to relatively small data samples on some countries like Germany and France, in each month, we sort stocks based on the maximum daily return within the previous month into *quintile* portfolios for individual countries respectively. That is, all stocks of each country are allocated into five portfolios based on MAX over the past month. Portfolio 1 is the portfolio of stocks with the lowest 20% MAX, and portfolio 5 is

Figure 2.1: Cross-section of average monthly returns of Low MAX and High MAX portfolios




The figure shows the time-series averages of the highest MAX portfolio (decile 10) and the lowest MAX portfolio (decile 1), respectively, across the seven markets. The sample period is Jan. 1994 - Dec. 2009.

Table 2.2: Distribution of monthly returns for stocks across countries

| Percentiles | Mean | Median | Std.Dev. | Skewness |
|---|---|---|---|---|
| Low | 1.03% | 1.46% | 3.13% | -96.15% |
| 2 | 0.88% | 1.25% | 3.30% | -123.24% |
| 3 | 0.87% | 1.24% | 3.45% | -157.40% |
| 4 | 1.01% | 1.36% | 3.69% | -137.00% |
| 5 | 1.05% | 1.38% | 4.03% | -107.76% |
| 6 | 1.09% | 0.94% | 4.22% | -111.09% |
| 7 | 1.24% | 1.35% | 4.93% | -53.15% |
| 8 | 1.47% | 1.63% | 5.24% | -65.13% |
| 9 | 1.69% | 1.49% | 6.72% | 36.25% |
| High | 2.53% | 2.41% | 8.18% | 38.13% |

Ten portfolios are formed each month from January 1994 to December 2009 across all of the seven countries based on the maximum daily return over previous month. Low MAX (high MAX) represents the portfolio returns of the lowest (highest) maximum daily return over the previous month. The table reports the time-series descriptive statistics for 4028 monthly returns.

the portfolio of stocks with the highest 20% MAX. The time-series average MAX and monthly returns of equal-weighted portfolios and value-weighted portfolios are reported in Table 2.3.

Table 2.3 suggests that the relation between MAX and the cross-section of expected returns tends to vary across different countries. For stocks from the U.S. and Canada, both the value-weighted and equal-weighted quintile portfolios show that the average return differentials between quintile 5 and 1 are positive and statistically significant. For example, as shown in Panel A, the average value-weighted return differential between quintile 5 and 1 is 2.84% per month for Canada with a t-statistic of 3.83. The value-weighted average return difference for the U.S. is 1.37%

Table 2.3: Value-weighted and equal-weighted portfolios sorted by exposure to MAX

| MAX | France | | Canada | | China | | Germany | | U.K. | | U.S. | | Japan | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Value-weighted returns on MAX portfolios | | | | | | | | | | | | | | |
| Decile | MAX | VW | MAX | VW | MAX | VW | MAX | VW | MAX | VW | MAX | VW | MAX | VW |
| 1 | 1.68 | 1.67 | 1.83 | 1.69 | 3.49 | 1.46 | 2.21 | 1.09 | 1.02 | 1.23 | 2.04 | 1.25 | 2.15 | 0.79 |
| 2 | 3.09 | 1.43 | 3.20 | 1.51 | 4.86 | 2.18 | 3.40 | 1.48 | 2.10 | 1.04 | 3.13 | 1.43 | 3.41 | 0.42 |
| 3 | 4.20 | 1.18 | 4.55 | 1.51 | 5.88 | 1.69 | 4.34 | 1.23 | 3.10 | 1.20 | 4.17 | 1.56 | 4.44 | 0.89 |
| 4 | 5.66 | 0.99 | 6.88 | 1.92 | 7.21 | 1.71 | 5.64 | 0.75 | 4.47 | 1.26 | 5.68 | 1.73 | 5.79 | 0.63 |
| 5 | 11.23 | 1.61 | 20.20 | 4.52 | 9.93 | 1.07 | 10.06 | 1.51 | 8.82 | 1.71 | 10.55 | 2.62 | 9.53 | 0.88 |
| 5-1 | | -0.07 | | 2.84 | | -0.38 | | 0.41 | | 0.48 | | 1.37 | | 0.09 |
| t-statistics | | (-0.11) | | (3.83) | | (-0.84) | | (0.76) | | (1.16) | | (2.63) | | (0.22) |
| Panel B: Equal-weighted returns on MAX portfolios | | | | | | | | | | | | | | |
| Decile | MAX | EW | MAX | EW | MAX | EW | MAX | EW | MAX | EW | MAX | EW | MAX | EW |
| 1 | 1.68 | 1.22 | 1.83 | 1.93 | 3.49 | 2.32 | 2.21 | 1.26 | 1.02 | 0.84 | 2.04 | 1.29 | 2.15 | 0.45 |
| 2 | 3.09 | 1.95 | 3.20 | 1.36 | 4.86 | 2.49 | 3.40 | 1.36 | 2.10 | 0.98 | 3.13 | 1.28 | 3.41 | 0.46 |
| 3 | 4.20 | 1.20 | 4.55 | 1.49 | 5.88 | 2.24 | 4.34 | 1.12 | 3.10 | 1.05 | 4.17 | 1.40 | 4.44 | 0.51 |
| 4 | 5.66 | 1.11 | 6.88 | 1.74 | 7.21 | 1.98 | 5.64 | 0.97 | 4.47 | 1.22 | 5.68 | 1.64 | 5.79 | 0.38 |
| 5 | 11.23 | 1.45 | 20.20 | 4.71 | 9.93 | 1.51 | 10.06 | 1.46 | 8.82 | 1.74 | 10.55 | 2.36 | 9.53 | 0.78 |
| 5-1 | | 0.23 | | 2.78 | | -0.81 | | 0.21 | | 0.90 | | 1.07 | | 0.33 |
| t-statistics | | (0.59) | | (3.24) | | (-2.80) | | (0.31) | | (2.76) | | (2.11) | | (1.04) |

We form *value-weighted* (in Panel A) and *equal-weighted* (in Panel B) quintile portfolios every month, using the maximum daily return data over the previous month from January 1994 to December 2009. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) maximum daily returns over the past one month, and average returns are given in percentage terms. Row (5-1) refers to the difference in monthly returns between portfolio 5 and portfolio 1. Newey and West (1987) adjusted t-statistics are given in the parentheses.

per month with a t-statistic of 2.63. The 5-1 difference of equal-weighted portfolios in U.K. is 0.90% per month with a corresponding t-statistic of 2.76, while the difference in average returns on value-weighted quintile portfolios 5 and 1 is much smaller in magnitude and statistically insignificant at 0.48% per month with a t-statistic of 1.16. On the other hand, for China we observe that equal-weighted average raw return difference is negative at −0.81% with a t-statistic of −2.80, suggesting that high MAX stocks produce lower expected returns in the following month. Nevertheless, the value-weighted average return difference is positive at 0.03%, albeit statistically insignificant. This is likely a result of the size effect. Moreover, there is no evidence for a significant link between MAX and expected returns for stocks in France, Germany or Japan, as the value-weighted and the equal-weighted 5-1 portfolio difference is small in magnitude and also statistically insignificant.

### 2.3.2 Cross-Sectional Fama-MacBeth Regressions

We have so far provided portfolio sorts based on MAX to interpret the relation between MAX and expected returns. Now we implement the Fama and MacBeth (1973) regressions by imposing a functional form on the relation between MAX and future returns given the presence of other firm characteristics. The reasons for that are:

1. Portfolio level analysis throws away a large amount of information in the cross-section via aggregation (Bali et al. (2011));
2. An analysis based on portfolio might exaggerate the relationship between the returns and the explanatory variables. (Lo and Mackinlay (1990));
3. Some countries have smaller samples as datasets, such as Germany in comparison to the U.S., therefore, results of these countries deduced from the constructed quintile portfolios based on MAX are not sufficient and even might be biased.

The basic equation that we estimate is:

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t}MAX_{i,t} + \lambda_{2,t}BETA_{i,t} + \lambda_{3,t}SIZE_{i,t} + \lambda_{4,t}BM_{i,t} + \lambda_{5,t}MOM_{i,t} + \lambda_{6,t}REV_{i,t} + \lambda_{7,t}ILLIQ_{i,t} + \varepsilon_{i,t+1}, \quad (2.2)$$

where $R_{i,t+1}$ is the realized return on stock $i$ in month $t+1$. The predictive cross-sectional regressions are run on the one-month lagged values of maximum daily return (MAX), market beta (BETA), log market capitalization (SIZE), log book-to-market ratio (BM), momentum (MOM), short-term reversal (REV) and illiquidity (ILLIQ). Monthly cross-sectional regressions are run as the above econometric specification for all of the stocks in the seven markets individually. We then calculate the premium estimates $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$, $\lambda_6$, $\lambda_7$, respectively, as the time-series average of the 191-month (from January 1994 to December 2009) slope coefficients. Statistical significance is determined by the Newey and West (1987) adjusted t-statistics.

Table 2.4 reports the results of regressions. The results of the regressions also manifest similar predictive patterns of MAX as those we obtain from the portfolio analysis. Looking individually at the univariate regressions in Panel A, we find that MAX in China is negatively and significantly related to the cross-section of expected returns with an average slope coefficient of -0.175 ($t = -4.684$), which indicates that average firm returns decrease as MAX becomes more extreme. For countries like France and Germany, the coefficient on MAX is negative but much smaller in magnitude than in China, and also statistically insignificant. In contrast, the results for Canada, the U.K. and the U.S. suggest that high maximum daily return stocks tend to generate high expected returns. The loadings of MAX in the univariate regressions are 0.184, 0.089 and 0.117 for Canada, the U.K. and the U.S., respectively, all of which are strongly significant. The slope in Japan is positive, albeit statistically insignificant.

In order to control for other potential economic explanations, we include various firm characteristics as well as other risk components in the regressions, as shown in Panel B. Clearly, the MAX effect in China is even stronger in terms of magnitude

Table 2.4: Cross-sectional return regressions in the seven countries

| | France | Canada | China | Germany | UK | US | Japan |
|---|---|---|---|---|---|---|---|
| Panel A: Univariate Regressions | | | | | | | |
| MAX | -0.055 | 0.184 | -0.175 | -0.007 | 0.089 | 0.117 | 0.034 |
| | (-1.592) | (3.771) | (-4.684) | (-0.070) | (2.345) | (2.911) | (0.781) |
| Panel B: Multivariate Regressions | | | | | | | |
| MAX | -0.098 | 0.151 | -0.255 | 0.042 | 0.127 | 0.075 | 0.029 |
| | (-2.756) | (2.220) | (-6.591) | (0.595) | (3.212) | (2.648) | (1.119) |
| BETA | 0.245 | 0.118 | 0.973 | -0.293 | -0.226 | -0.050 | -0.075 |
| | (1.031) | (0.720) | (2.836) | (-0.969) | (-1.477) | (-0.244) | (-0.489) |
| BM | 0.403 | 1.322 | 0.934 | 0.875 | 0.511 | 0.493 | 0.715 |
| | (1.414) | (3.976) | (4.723) | (3.380) | (4.243) | (4.306) | (3.902) |
| SIZE | -0.195 | -0.276 | -0.396 | -0.063 | 0.043 | -0.445 | -0.206 |
| | (-2.459) | (-2.322) | (-1.871) | (-0.574) | (1.625) | (-6.620) | (-2.365) |
| MOM | 0.011 | 0.016 | 0.006 | 0.017 | 0.013 | 0.005 | 0.007 |
| | (2.090) | (3.608) | (1.398) | (2.661) | (2.286) | (0.876) | (1.349) |
| REV | 0.012 | 0.008 | -0.026 | 0.009 | 0.016 | 0.005 | 0.010 |
| | (0.984) | (0.550) | (-2.466) | (0.407) | (0.932) | (0.487) | (0.977) |
| ILLIQ | 0.016 | 0.016 | 0.177 | -0.054 | 0.272 | 0.322 | 0.667 |
| | (0.955) | (0.955) | (0.866) | (-1.739) | (0.213) | (1.095) | (0.039) |

The table reports Fama-MacBeth (1973) regressions in Eq. 2.2 for stocks in the seven countries. Each month from January 1994 to December 2009 we run firm-level cross-sectional regressions of the return on the lagged variables in the previous month. The slopes are the time-series average of coefficients of monthly regressions, and the Newey and West (1987) adjusted t-statistics are reported in the parentheses.

when all other variables are included in the regressions. The coefficient is $-0.255$ ($t = -6.951$). Similarly, the coefficient of MAX in France reduces to $-0.098$ and becomes statistically significant ($t = -2.756$). Furthermore, the positive relation between MAX and expected returns prevails for the full specification with MAX and the six control variables in Canada, the U.K. and the U.S., in which we find an average slope of 0.151 ($t = 2.22$), 0.127 ($t = 3.21$), and 0.075 ($t = 2.65$), respectively. For Germany and Japan, the MAX coefficients are 0.042 and 0.029 and remain insignificant as in Panel A.

In summary, unlike the findings in Bali et al. (2011), we do not find uniformly strong evidence for an economically and statistically significant negative relation between extremely positive returns and expected returns for the period from January 1994 to December 2009 across the seven countries. Except for the result in China which is consistent, we find contradicting results for Canada, the U.K. and the U.S., in which the coefficients on MAX are all positive and statistically significant. In addition, the MAX effect is not robustly strong observed in Germany, France and Japan.

### 2.3.3 A More Detailed Look at the U.S. Market

Surprisingly, as shown in Table 2.4, MAX is strongly positively associated with future stock returns in the U.S., Canada, and the U.K.. In particular, the result of the U.S. is the direct opposite of that of Bali et al. (2011), which demonstrated a negative and significant relation between MAX and expected returns. Therefore in this section, we take a detailed look at the effect of MAX with the U.S. data, where a relatively large number of firms allows for greater power in investigating the cross-sectional determinants of the effect.

The full specification with MAX and other control variables shows that MAX has a positive impact on the cross-section of expected returns. One may argue that the MAX's predictive ability on subsequent returns is due to its proxy for some other well-known effects. Given the characteristics of the high MAX stocks, our first conjecture is that the MAX effect could be closely associated with the size effect (e.g. Banz (1981) and Fama and French (1992)). As the size effect indicates, small firms have higher expected returns than large firms. Naturally, high MAX stocks are likely to be small stocks, which would potentially dominate the positive relation between MAX and future returns in our sample on the U.S. data. The size effect, however, is already mostly controlled for in the Fama-MacBeth regressions in the previous section.

Another possibility is that, as numerous studies have pointed out, idiosyncratic risk is positively related to the cross-sectional expected returns (Levy (1978), Merton (1987), Malkiel and Xu (2002), Jiang and Lee (2006) and Fu (2009)). Idiosyncratic risk is the risk that is unique to an individual firm and is independent from the aggregate market. Special events of certain firms, like record-breaking events, achieving extremely positive daily returns, are intuitively and firmly linked to stocks' idiosyncratic volatility. Investors, therefore, would demand a premium for holding stocks with high idiosyncratic risk. Following Bali et al. (2011), to estimate idiosyncratic volatility for an individual stock, we assume a single factor generating process and measure the firm-level idiosyncratic volatility using the following model:

$$R_{i,d} - r_{f,d} = \alpha_i + \beta_i (R_{m,d} - r_{f,d}) + \varepsilon_{i,d}, \tag{2.3}$$

where $\varepsilon_{i,d}$ is the idiosyncratic return on day $d$. The idiosyncratic volatility of stock $i$ in month $t$ is then determined as the standard deviation of the residuals:

$$IVOL_{i,t} = \sqrt{var(\varepsilon_{i,d})}. \tag{2.4}$$

In particular, the correlation between cross-sectional average of MAX and idiosyncratic volatility is remarkably high with the correlation coefficient of 93%, as one can see visually in Figure 2.2. Obviously, stocks with high (low) MAX are frequently those stocks with high (low) idiosyncratic volatility (henceforth IVOL).

Thus as a robustness check we control the potential poxy variables IVOL while examining the MAX effect.

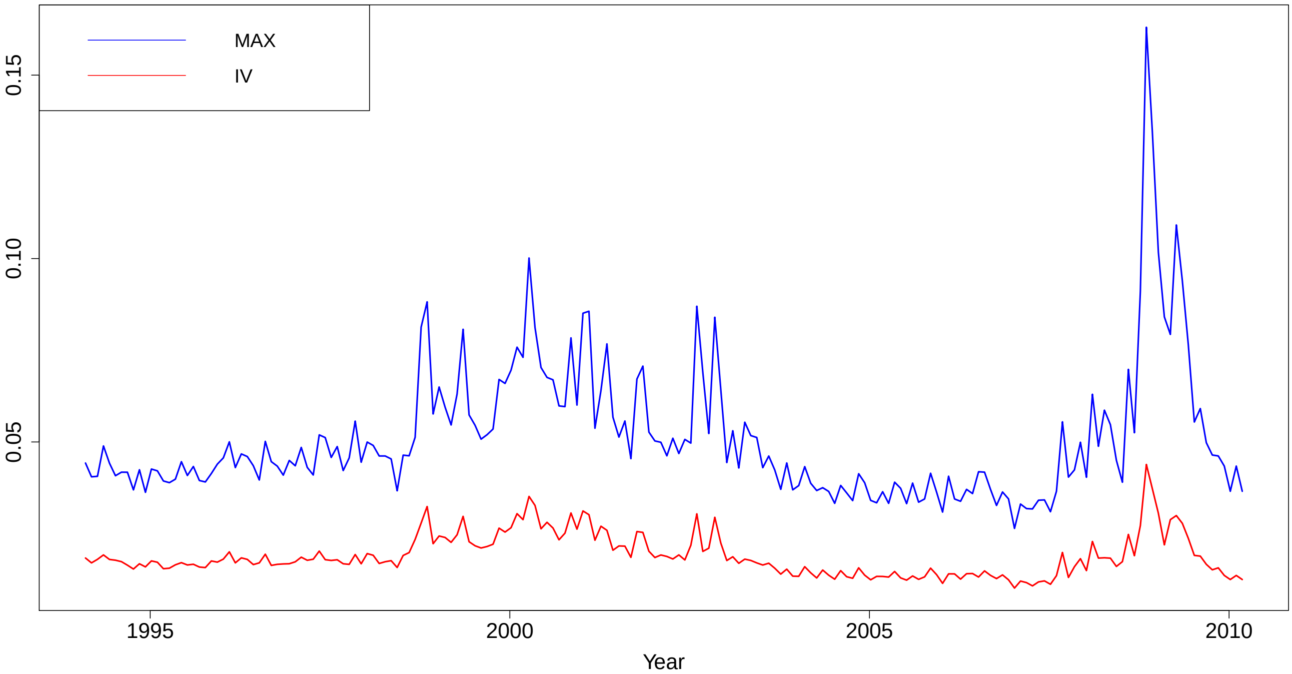



Figure 2.2: Cross-sectional average of MAX and IVOL

In order to control for the two possible explanations of the MAX phenomenon, size and idiosyncratic volatility, we firstly conduct bivariate portfolio sorts. Specifically, we create quintile portfolios each month from January 1994 to December 2009 ranked on firm size and idiosyncratic volatility of the previous month respectively, and then within each quintile we sort stocks on their maximum daily returns. For the sake of saving space, we do not report all $(5 \times 5)$ portfolios, but present average returns across the spectrum of the control variables for the 5 quintile portfolios with dispersion in MAX. As a result, quintile 1 (5) consists of the 20% lowest (highest) MAX stocks, and each quintile is formed with variation in MAX, but with similar levels of control variables (size and IVOL). The return difference is the time-series average returns between the high MAX portfolios and the low MAX portfolios. Table 2.5 reports the average equal-weighted and value-weighted returns over control of size and IVOL respectively, and their associated Newey-West adjusted t-statistics. The value-weighted quintile portfolio is value weighted using market capitalization at the end of the preceding month.

Table 2.5: Portfolio of stocks sorted by MAX after screening for Size and IVOL

| | *Control for Size* | | *Control for IVOL* | |
|---|---|---|---|---|
| Quintile | EW | VW | EW | VW |
| low MAX | 2.22 | 2.19 | 1.72 | 1.04 |
| 2 | 1.86 | 1.78 | 1.57 | 1.00 |
| 3 | 1.69 | 1.70 | 1.50 | 0.96 |
| 4 | 1.71 | 1.66 | 1.48 | 1.02 |
| high MAX | 2.01 | 1.95 | 1.68 | 1.02 |
| | | | | |
| difference(5-1) | -0.20 | -0.24 | -0.04 | -0.02 |
| | (-0.72) | (-0.82) | (-0.28) | (-0.11) |

The above table shows equal-weighted (EW) and value-weighted (VW) quintile portfolios formed every month from January 1994 to December 2009. We first sort stocks each month based on size or idiosyncratic volatility, then within each quintile we sort stocks on the maximum daily return (MAX). The table reports average returns across the 5 size/IVOL quintile portfolios. Return difference is the difference in average monthly returns between quintile 5 and 1. Newey and West (1987) adjusted t-statistics are reported in the parentheses controlling for heteroscedasticity and autocorrelation.

Somewhat surprisingly, it is shown in Table 2.5 that the positive effect of MAX disappears for both screening processes. Instead, the 5-1 differences for equal- and value-weighted portfolios are negative but insignificant. More specifically, after controlling for the firm size, the equal-weighted average return difference is $-0.20\%$ per month with a t-statistic of $-0.72$, and the value-weighted average return difference is $-0.24\%$ with a t-statistic of $-0.82$. When controlling for IVOL, the equal-weighted and value-weighted spreads between the high MAX and the low MAX quintile are almost zero and of no statistical significance. Hence, the bivariate sorts screening for size and IVOL reduce the cross-sectional variation in MAX, implying that both firm size and IVOL largely interact with MAX. Therefore, we conclude that the positive MAX effect with the U.S. data is not robust, and controlling for other risk factors such as size and IVOL is critical to the significance of the result.

Furthermore, Table 2.6 presents the raw value-weighted and equal-weighted returns of univariate sorts and bivariate sorts of IVOL after controlling for firm size and MAX. Applying a similar procedure to that as in Table 2.5, for the bivariate sorts, we first form 5 portfolios by sorting stocks each month on size or MAX, and then we again sort stocks based on IVOL into 5 quintile portfolios within each control quintile, which results in 25 ($5 \times 5$) IVOL portfolios conditioned on size or MAX. For brevity, we report the average portfolio returns with dispersion of IVOL across the control variables.

We find that while both raw value-weighted and equal-weighted return differences between the high IVOL portfolio and the low IVOL portfolio are positive, only the difference in equal-weighted portfolios shows statistical significance, with a t-statistic of 2.86. Particularly, the value-weighted average return of quintile 5 is 1.28% per month, which is much lower than the equal-weighted average return of quintile 5 (2.62% per month). This is not surprising though: quintile 5 contains the smallest 20% of the stocks sorted by IVOL, so on the other hand, it also represents a smaller proportion of the market capitalization.[3] As we know that small stocks have higher average returns than large stocks, this is the reason that the value-weighted return of quintile 5 is much lower than the equal-weighted return of the same quintile.

After controlling for the size in the 4th and 5th column, the equal-weighted and

[3]Quintile 5 contains 20% of the stocks, however, it only accounts for 8.97% of the overall market value (not reported in the tables). As comparison, quintile 1 to 4 contributes 30.36%, 25.41%, 20.59%, 14.68% the market share respectively.

value-weighted return differences between the high IVOL and the low IVOL quintile are 0.87% and 0.96% per month respectively, both with highly significant t-statistics. When we look at the last two columns, where portfolios are controlled by MAX, the positive effect of IVOL persists at 0.45% and 1.18% spreads per month for value-weighted and equal-weighted portfolio formation respectively. These results are by all means more impressive than those sorts based on MAX. Particularly noteworthy is the finding that using the bivariate sorts does not dramatically reduce the higher returns of quintile 5 based on idiosyncratic volatility.

Table 2.6: Univariate IVOL sorts and bivariate IVOL sorts after screening for Size and IVOL

| | VW | EW | VW | EW | VW | EW |
|---|---|---|---|---|---|---|
| Quintile | | | *Control for Size* | | *Control for MAX* | |
| 1(Low IV) | 0.97 | 1.12 | 1.23 | 1.24 | 0.83 | 1.11 |
| 2 | 0.97 | 1.20 | 1.29 | 1.33 | 0.98 | 1.28 |
| 3 | 0.73 | 1.36 | 1.43 | 1.46 | 1.07 | 1.46 |
| 4 | 1.00 | 1.70 | 1.70 | 1.75 | 1.00 | 1.81 |
| 5(High IV) | 1.28 | 2.62 | 2.10 | 2.19 | 1.28 | 2.29 |
| difference(5-1) | 0.31 | 1.50 | 0.87 | 0.96 | 0.45 | 1.18 |
| | (0.57) | (2.86) | (3.20) | (3.69) | (2.17) | (6.60) |

The above table shows value-weighted (VW) and equal-weighted (EW) quintile portfolios formed every month from January 1994 to December 2009. We first use univariate sorts each month based on idiosyncratic volatility (the second and third column), then bivariate sort portfolios by adding size or the maximum daily return (MAX) as control variable (4-7th column). The table reports average raw returns, returns controlled for size and MAX in monthly percentage terms. Return difference is the difference in average monthly returns between quintile 5 and 1. Newey and West (1987) adjusted t-statistics are reported in the parentheses control for heteroscedasticity and autocorrelation.

Next we investigate the relation between average returns and IVOL by applying the Fama and MacBeth (1973) cross-sectional regressions. We extend the Equation 2.2 to incorporate idiosyncratic volatility. Table 2.7 reports the time-series averages of estimated coefficients from various specifications. In addition to the variables mentioned before, we include dummy high MAX and low MAX in the regressions. Dummy high (low) MAX takes the value one if a stock appears in the highest (lowest) MAX quintile over the previous month and zero otherwise. The idea is that if a stock's previous MAX is relatively high (low), it will potentially persist over the next month.

Among models 1, 2, 3, and 5 shown in Table 2.7, where MAX is included in the regression in the absence of IVOL, the coefficient estimate is significant and positive, which suggests that stocks with higher MAX earn higher returns in the following month. Both dummy high MAX and low MAX have little influence on the results. Augmenting the regression with firm's size does not help: the positive relation between MAX and expected returns remains. When IVOL is added to the regression in model 4, the average slope coefficient on MAX reverses its sign to negative with the estimate of $-0.057$, yet is only marginally significant with a t-statistic of $-1.80$. In contrast, the average slope coefficient on IVOL is 0.642 and statistically significant with a t-statistic of 3.88, which is in line with the result from the portfolio analysis: once IVOL is controlled for, the positive effect of MAX vanishes. Moreover, when we consider MAX, size and IVOL in one regression (model 8), while the power of the IVOL and size is significant and consistent with the former results, the MAX coefficient is small and has an insignificant t-statistic of $-1.32$. Similarly, model 9 and 10 further show that the coefficient of IVOL is robust after controlling for other well-known economic explanations, whereas the explanatory power of MAX is marginal and statistically insignificant.

In summary, the result is intriguing. Based on the double-sorting of portfolios and also cross-sectional regressions, we see that idiosyncratic volatility wins against

Table 2.7: Cross-sectional regressions with MAX, IVOL and other control variables

| Model | MAX | Dummy high MAX | Dummy low MAX | IVOL | BETA | SIZE | BM | MOM | REV | ILLIQ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.117 | | | | | | | | | |
| | (2.91) | | | | | | | | | |
| 2 | 0.107 | 0.002 | | | | | | | | |
| | (3.12) | (0.70) | | | | | | | | |
| 3 | 0.123 | | 0.001 | | | | | | | |
| | (3.21) | | (0.91) | | | | | | | |
| 4 | -0.057 | | | 0.642 | | | | | | |
| | (-1.80) | | | (3.88) | | | | | | |
| 5 | 0.080 | | | | | -0.496 | | | | |
| | (2.04) | | | | | (-5.92) | | | | |
| 6 | | | | 0.477 | | | | | | |
| | | | | (3.29) | | | | | | |
| 7 | | | | 0.328 | | -0.453 | | | | |
| | | | | (2.30) | | (-5.50) | | | | |
| 8 | -0.040 | | | 0.442 | | -0.450 | | | | |
| | (-1.32) | | | (2.86) | | (-5.65) | | | | |
| 9 | | | | 0.313 | -0.009 | -0.409 | 0.514 | 0.005 | 0.007 | 0.233 |
| | | | | (2.88) | (-0.05) | (-6.61) | (4.39) | (0.94) | (0.89) | (0.77) |
| 10 | -0.032 | | | 0.397 | -0.005 | -0.406 | 0.513 | 0.005 | 0.007 | 0.232 |
| | (-0.92) | | | (2.36) | (-0.03) | (-6.61) | (4.36) | (0.97) | (0.82) | (0.75) |

Each month from January 1994 to December 2009 we regress the return on the lagged MAX, IVOL and other control variables over the past month. Two dummy variables are also included: if the stock appears in the highest (lowest) MAX quintile over the previous month, respectively, it takes value 1 and zero otherwise. The table reports the time-series average of Fama-MacBeth (1973) coeffcients of various cross-sectional regressions. Newey and West (1987) adjusted t-statistics are reported in the parentheses.

MAX in explaining the cross-sectional variation of average returns. Although MAX alone exhibits significantly positive explanatory power in forecasting the subsequent monthly returns, this cross-sectional relation is insignificant and even negative in the presence of idiosyncratic volatility. Hence these findings indicate that the positive relation between MAX and future stock returns in the U.S. market, if any, is not robust after controlling for idiosyncratic volatility as estimated by daily stock returns in the previous month. On the other hand, the positive relation between IVOL and expected returns is large in magnitude and statistically significant in various regression specifications. This evidence implies that MAX seems to be a proxy for IVOL, and it is the IVOL which drives the positive relation in determining the cross-section of expected returns.

## 2.4 Robustness test of Bali et al. (2011) for a recent sample period

We have so far shown that the positive relation between MAX and expected return is not robust, which is, however, not consistent with the findings documented by Bali et al. (2011), in which they find a statistically and economically significant relation between MAX and future returns. Of particular interest is the fact that their results are robust to other control variables, including idiosyncratic volatility. An immediate concern would be that the aforementioned results are based on a dramatically different database from that of Bali et al. (2011). Specifically, we use a data set concentrating on a relatively recent time span of January 1994 to December 2009 including only 990 stocks from Datastream for the U.S. data, whereas they focus on all NYSE/AMEX/NASDAQ stocks incorporating more than 20,000 stocks from the Center for Research in Security Prices (CRSP) covering a period from July 1962 to December 2005.

To check the robustness for recent periods of the findings in Bali et al. (2011), in this section, we further investigate the issue also using a sample of NYSE, AMEX and NASDAQ stocks from CRSP. However, instead of using the full sample period from July 1962 to December 2010, we divide it into two subperiods, namely, from July 1962 to December 1989 and from January 1990 to December 2010, while all variables are defined the same as in Bali et al. (2011). More specifically, our data include daily and monthly returns of NYSE, AMEX, and NASDAQ financial and nonfinancial firms from July 1962 to December 2010. Additionally, we use data from Compustat for the book value of individual firms. The purpose is to clarify the existence and significance of a relation between MAX and the cross-section of expected stock returns on a more recent period, which is perhaps the most relevant period.

Similarly, we firstly examine returns of portfolios formed on the sorting of MAX. The procedure of the portfolio-based approach is easy to implement and produces a straightforward result. For each month in the two subperiods, we sort stocks based on the maximum daily return over the preceding month to form 10 portfolios with an equal number of stocks. Portfolio 1 contains the lowest 10% stocks of MAX and portfolio 10 consists of the 10% stocks that have the highest MAX. Table 2.8 presents the value-weighted and equal-weighted returns of percentile portfolios in the following month. It also presents the magnitude and statistical significance of the intercepts (Fama-French-Carhart four-factor alphas) from the regressions of the value-weighted and equal-weighted portfolio returns on a constant, the excess market return, the SMB, the HML, and the MOM factors[4]

As shown in Panel A of Table 2.8, where we compose portfolios each month during the post-1990 period, the average maximum daily return of stocks within a month increases sharply from less than 2% in the lowest decile to 31% in the highest decile. Especially, the MAX is 14.6% in decile 9, while it soars to 31.01% in decile 10,

[4]See Fama and French (1993) and Carhart (1997). SMB, HML and MOM refer to small minus big, high minus low, winner minus loser, respectively. Factors are described in details in Kenneth French's data library.

Table 2.8: Portfolios sorted by MAX

| | VW Portfolios | | EW Portfolios | | |
|---|---|---|---|---|---|
| Decile | Return | Four-Factor alpha | Return | Four-Factor alpha | Average MAX |
| *Panel A: 1990.1-2010.12* | | | | | |
| 1 (Low) | 0.93 | 0.13 | 1.15 | 0.37 | 1.85 |
| 2 | 1.00 | 0.18 | 1.26 | 0.36 | 3.08 |
| 3 | 1.06 | 0.22 | 1.35 | 0.40 | 3.98 |
| 4 | 1.12 | 0.21 | 1.35 | 0.41 | 4.90 |
| 5 | 1.15 | 0.23 | 1.31 | 0.34 | 5.91 |
| 6 | 1.15 | 0.11 | 1.30 | 0.36 | 7.12 |
| 7 | 0.93 | 0.02 | 1.23 | 0.34 | 8.66 |
| 8 | 1.00 | -0.06 | 1.25 | 0.38 | 10.84 |
| 9 | 0.41 | -0.61 | 1.22 | 0.36 | 14.60 |
| 10 (High) | 0.36 | -0.77 | 1.20 | 0.41 | 31.01 |
| 10–1 | -0.57 | -0.89 | 0.05 | 0.04 | |
| | (-0.78) | (-1.83) | (0.08) | (0.11) | |
| *Panel B: 1962.7-1989.12* | | | | | |
| Low | 0.93 | 0.07 | 1.33 | 0.28 | 1.44 |
| 2 | 0.98 | 0.09 | 1.49 | 0.38 | 2.33 |
| 3 | 0.92 | -0.04 | 1.54 | 0.39 | 3.01 |
| 4 | 1.16 | 0.21 | 1.55 | 0.34 | 3.69 |
| 5 | 1.08 | 0.05 | 1.50 | 0.23 | 4.45 |
| 6 | 1.22 | 0.16 | 1.50 | 0.19 | 5.34 |
| 7 | 0.98 | -0.15 | 1.37 | 0.01 | 6.46 |
| 8 | 0.98 | -0.33 | 1.29 | -0.14 | 7.99 |
| 9 | 0.62 | -0.71 | 0.94 | -0.57 | 10.52 |
| High | 0.29 | -1.21 | 0.35 | -1.25 | 19.49 |
| 10–1 | -0.64 | -1.28 | -0.98 | -1.53 | |
| | (-1.65) | (-4.81) | (-2.73) | (-8.53) | |

Each month we sort stocks based on the maximum daily return over the past one month (MAX). Value-weighted and equal-weighted decile portfolios are formed in two subperiods: in Panel A from January 1990 to December 2010 (252 months) and in Panel B from July 1963 to December 1989 (318 months). Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) MAX. The table reports the value-weighted (VW) and equal-weighted (EW) average monthly returns and the four-factor Fama-French-Carhart alphas on the value-weighted and equal-weighted protfolios, and the corresponding average MAX. Row $10 - 1$ refers to the difference in monthly returns between portfolio 10 and 1. Newey-West (1987) adjusted t-statistics are in parentheses. All the returns are in percentage terms.

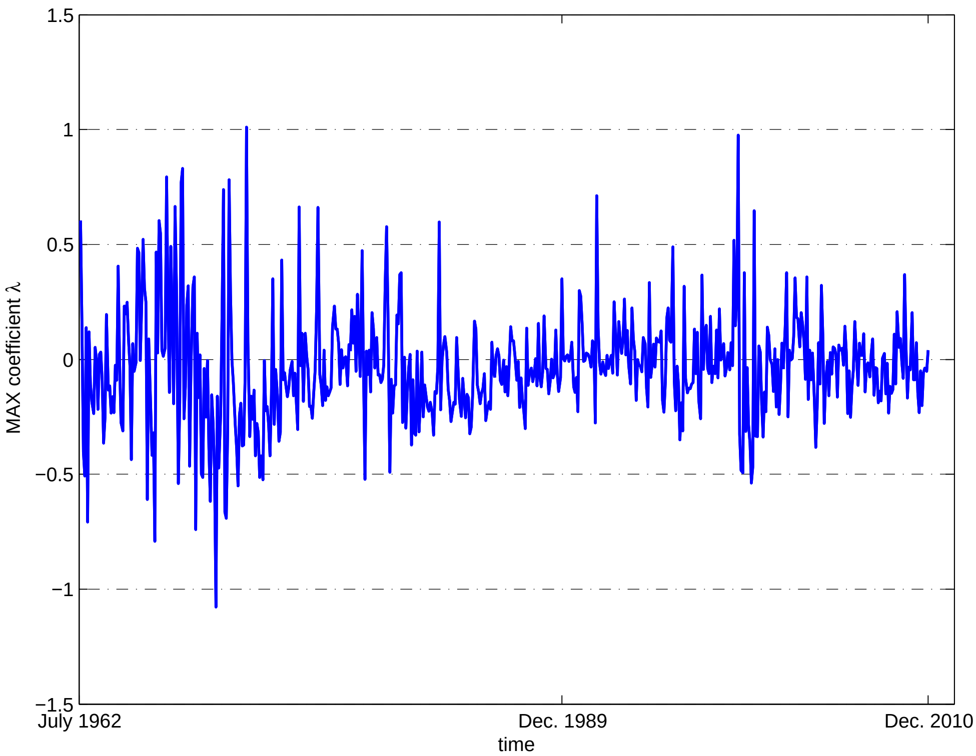
in the last column. On the other hand, more importantly, we do not find significant indications of a negative MAX effect. The value-weighted portfolio return differences between decile 10 and 1 is $-0.57\%$ per month with an insignificant Newey-West t-statistic of $-0.78$. The intercept of the time-series regressions of value-weighted excess returns on the four-factor in the column of four-factor-alpha is 0.13% for the lowest MAX decile and $-0.77\%$ for the highest MAX decile. A hedging portfolio longing decile 10 and shorting decile 1 yields a monthly return of $-0.89\%$, yet not strongly statistically significant. The equal-weighted portfolio returns display a strikingly different picture that is by no means in support of the results revealed in Bali et al. (2011). Portfolio returns from decile 1 to decile 10 are flat with no clear trend. The return spread between the highest MAX and lowest MAX is small in magnitude at 0.05% per month, and cannot be statistically distinguished from zero. The four-factor alphas also exhibit a similar pattern showing the alpha difference of 0.04% with little statistical significance.

As a comparison we provide the portfolio analysis on the pre-1990 period back to July 1962 in Panel B. It is clear that, despite the insignificant results in Panel A, the overall pattern in Panel B coincides with Table 1 of Bali et al. (2011). More explicitly, in their paper deciles 1-8 have approximately the same levels of return, whereas the returns of deciles 9 and 10 are significantly lower. The same holds in our results. Furthermore, except for the second column, where we obtained a less economically and statistically significant result for the value-weighted return difference, all the other results are conceivable and expected, with highly statistical significance. The average value-weighted portfolio difference between decile 10 and 1 is $-0.64\%$ per month with a t-statistic of -1.65. It is worth noting that in Table 1 of the original paper, the value-weighted high MAX portfolio has abysmally low return ($-0.02\%$ per month), whereas the counterpart in our result is remarkably higher (0.29%). Nevertheless, the value-weighted alpha difference is $-1.28\%$ with a t-statistic of $-4.18$, which is similar to the results in the original paper ($-1.18\%$

with a t-statistic of −4.71). As with the equal-weighted portfolios, the monthly difference for decile returns and alpha is −0.98% and −1.53%, respectively, both of which are highly significant.

Collectively, as preliminary evidence, the above results indicate that the average raw and risk-adjusted return differences between stocks in the lowest and highest MAX deciles are much less impressive and weaker than that documented in Bali et al. (2011), once we rely on a sample period from January 1990 to December 2010. Nevertheless, the negative average return differentials, in any case, are notably stronger in the pre-1990 period than in the post-1990 period. Our conjecture is that if the main finding of Bali et al. (2011) exists, it is the strong negative relation in the early decades that dominates.

Figure 2.3: Monthly MAX coefficients of the Fama-MacBeth regressions




To assure the findings from the portfolio analysis, we further examine the cross-sectional relation between MAX and expected returns at the firm level by using

Fama-MacBeth regressions. Firstly, the cross-section of one-month-ahead returns are regressed against only MAX, and Figure 2.3 illustrates the time-series Fama-MacBeth regression coefficients of MAX from July 1962 to December 2010. Furthermore, we include idiosyncratic volatility along with the same set of controls that are used in Table 2.7 into the regressions, as introducing MAX sheds light on the puzzle of idiosyncratic volatility according to Bali et al. (2011)[5].

Table 2.9 presents results of the cross-sectional regressions with MAX, IVOL and other six independent control variables. Idiosyncratic volatility is estimated within-month daily returns following Ang et al. (2006, 2009) with respect to the three-factor Fama and French (1993) regression:

$$R_i - r_f = \alpha_i + \beta_i(R_m - r_f) + s_i SMB + h_i HML + \varepsilon_i. \tag{2.5}$$

The idiosyncratic volatility for stock $i$ is measured as the standard deviation of the residuals $\varepsilon_i$. The other variables are calculated as described in previous sections.

The models 1-4 in panel A are regression results conducted in the period from January 1990 to December 2010. Consistent with the results of our portfolio analysis, we do not find a robust negative relation between MAX and stock expected returns. The average coefficient of MAX in a univariate regression (model 1) is almost zero, $-0.007$, with a t-statistic of $-0.53$. When we run the regression with only IVOL as in model 2, the average slope of IVOL is 0.011, but it is not statistically significant ($t = 0.20$). Model 3 generates a negative coefficient ($-0.083$) for MAX and a positive coefficient (0.357) for IVOL, both of which are statistically significant. This evidence is in line with the results in Table 10 of Bali et al. (2011). However, neither a significant effect of MAX nor IVOL shows up when we run the multivariate regression with full specification in model 4. The MAX coefficient is $-0.009$ with a t-steatitic of $-0.53$ and the coefficient on idiosyncratic volatility is 0.055 with a t-statistic of 0.57.

[5]In the original paper, their results show that inclusion of MAX variable reverses the anomalous negative relation between idiosyncratic volatility and returns in Ang et al. (2006).

Table 2.9: Cross-sectional regressions with MAX, IVOL and other control variables

| | MAX | IVOL | BETA | SIZE | BM | MOM | REV | ILLIQ |
|---|---|---|---|---|---|---|---|---|
| *Panel A: 1990.1-2010.12* | | | | | | | | |
| Model 1 | -0.007 | | | | | | | |
| | (-0.53) | | | | | | | |
| Model 2 | | 0.011 | | | | | | |
| | | (0.20) | | | | | | |
| Model3 | -0.083 | 0.269 | | | | | | |
| | (-4.53) | (2.61) | | | | | | |
| Model 4 | -0.009 | 0.055 | -0.046 | -0.081 | 0.001 | 0.055 | -0.040 | 0.027 |
| | (-0.53) | (0.57) | (-1.12) | (-2.53) | (0.53) | (0.17) | (-7.89) | (4.61) |
| *Panel B: 1962.7-1989.12* | | | | | | | | |
| Model 5 | -0.057 | | | | | | | |
| | (-2.84) | | | | | | | |
| Model 6 | | -0.104 | | | | | | |
| | | (-1.39) | | | | | | |
| Model 7 | -0.170 | 0.357 | | | | | | |
| | (-8.16) | (3.01) | | | | | | |
| Model 8 | -0.026 | -0.040 | 0.027 | -0.107 | 0.011 | 0.938 | -0.067 | 0.020 |
| | (-1.70) | (-0.39) | (0.82) | (-3.13) | (2.49) | (5.04) | (-9.72) | (2.72) |

Each month we regress the cross-section of monthly stock returns onto MAX, IVOL and other control variables over the past month. The regressions are split into two subperiods: in Panel A the sample period is 1990/01–2010/12 and in Panel B the sample period is 1963/07–1989/12. The table reports the time-series averages of the cross-sectional regression slope coefficients and their associated Newey and West (1987) adjusted t-statistics in the parentheses.

Compared to the results in panel A, the models 5, 6, 7 in panel B are qualitatively similar to those in Table 10 of Bali et al. (2011): MAX is negatively and significantly related to the cross-section of expected returns with an average slope coefficient of $-0.057$ ($t = -2.84$) in model 5. The coefficient of IVOL is $-0.104$ ($t = -1.39$). In the presence of MAX and IVOL at the same time, the IVOL coefficient is 0.357 with a t-statistic of 3.01 in model 7, whereas the coefficient becomes $-0.40$ with a t-statistic of $-0.39$ after extending the regression to include the six other control variables in model 8. The estimate of MAX is $-0.170$ with an extreme t-statistic ($t = -8.16$) in model 7, and it changes to $-0.026$ in model 8 with a weakly significant t-statistic ($t = -1.70$).

The results from the portfolio-level and firm-level analysis suggest that the MAX factor has not been able to produce an economically or statistically significant effect in the recent decades. On the other hand, MAX is far more important in explaining the variation of cross-sectional stock returns in early periods, for example, until 1989. Therefore, our conclusion is that the MAX effect along the lines of Bali et al. (2011), if anything, is driven mostly by its strong impact in early periods. This risk factor deteriorates and tends to diminish over time.

Intuitively, stocks with high MAX are naturally idiosyncratic, and vice versa, stocks with high volatility often exhibit extreme returns. Han and Kumar (2008) provide evidence that retail investors exhibit disproportionate preferences for high idiosyncratic volatility stocks, which reflects their propensity to speculate or gamble. As opposed to retail investors, institutional investors typically hold well-diversified portfolios and are less likely to respond to "noise" information that leads to deviation from the market[6]. Given the distinct trading difference between retail and institutional investors, we suggest that the MAX-return relation depends crucially on the extent of MAX-seeking behavior among retail investors. Hence with a reducing proportion of MAX-seeking investors in the aggregate market, the negative

[6] See e.g. Cohen et al. (2002).

premium of MAX is mitigating and less significant. As a matter of fact, Karceski (2002) and Baker et al. (2011) note that after 1983, institutional investors have become progressively more numerous. Based on the above observations, we may therefore attribute the insignificant MAX effect in the period of the recent 20 years to the decreasing proportion of MAX-seeking investors in the aggregate market.

## 2.5 Conclusion

We investigated the impact of extremely positive returns and examined the cross-sectional relation between maximum daily return and expected stock returns across different countries. The aim was to verify whether the robust negative relation identified by Bali et al. (2011) also holds for other stock markets. The results suggest that the findings on the U.S. market can neither be easily generalized to other countries, nor to more recent time periods in the U.S..

More specifically, the empirical analysis shows a statistically negative relation between one-month lagged extremely positive returns and expected stock returns in the Chinese stock market. This result is robust to controls for other well-known risk elements. Expected returns on the stocks that showed extremely positive returns in the previous month are lower in the subsequent month, which implies that investors are likely to pay more for those stocks and accept a lower average return. Secondly, we find that there is a positive link between the extremely positive returns and the average returns in terms of their ability to explain the cross-section of the expected returns in the Canadian, U.K. and U.S. markets. As we find the exact opposite result to that of Bali et al. (2011) for the U.S. stock market including all stocks in NYSE/AMEX/NASDAQ, we run an additional test to control for the idiosyncratic volatility of individual stocks on the sample data set from Datastream. After the inclusion of idiosyncratic volatility, however, there is no significant positive relation between MAX and expected returns, but MAX seems to be a proxy for idiosyncratic volatility. Lastly, we do not find any significant results with respect to the stock markets in Germany, Japan and France.

Additionally, to understand why we obtain conflicting evidence on the link between MAX and the cross-sectional expected returns, we investigated the CRSP data set as in Bali et al. (2011), and tested the robustness of their results on two sample subperiods. The pattern can be replicated for the period of July 1962 to December 1989, whereas it is not robustly negative for the period from January 1990 to December 2010. This makes it challenging to interpret the MAX anomaly in pricing the cross-sectional expected returns. On the other hand, the results in China indeed also present a similar pattern to the findings in the early periods of the U.S.. Thus we may conjecture that the lower efficiency of the earlier U.S. stock market as well as the current Chinese market, in which a higher proportion of retail investors demands the MAX-stocks, so that they are willing to overpay for those stocks, lowers the average expected cross-sectional returns. As the market develops and become more mature, intuitional investors become dominant, and speculative preferences reduce their importance, which results in the weak MAX-impact on stock prices, as reflected in the recent decades. In fact, this conjecture is supported by Han and Kumar (2013), who demonstrate that stocks with high retail trading proportion (RTP) have strong lottery features, and these stocks earn stronger negative premium or are located in regions where people exhibit stronger gambling propensity.

In a nutshell, our results suggest that the relation between high MAX and low returns is not stable over time. Market-specific stories are likely to play various roles in determining the prices of stocks. In particular, behavioral asset pricing factors might affect markets differently over time and space as markets change. While markets become more rational with more sophisticated participants, stock returns as well as the overall pattern of the aggregate market can be potentially affected, which may cast doubt on the robustness of relevant risk factors in the empirical performance with asset-pricing models.

# Chapter 3

# Optimal Diversification with Options and Structured Products

## 3.1 Introduction

Diversification has played a crucial role in both asset pricing and risk management for years. Generally speaking, the primary objective of diversification is to protect the overall portfolio in case any single holding suffers a complete meltdown. The mean-variance model of Markowitz (1952) facilitates investors to achieve this goal by optimizing portfolios on the mean-variance efficient frontier. This classic framework suggests that the optimal investment portfolio can be constructed as a combination of the market portfolio and risk-free asset. Thus, there is no role for financial derivatives in this framework, which in real life, however, are very important supplementary investment vehicles with respect to hedging, optimization and speculation. It has also been demonstrated that adding derivatives such as plain vanilla options into portfolios can improve the market efficiency and enhance investors' utility (e.g., Liu and Pan (2003), Jones (2006), Driessen and Maenhout (2007) and Branger et al. (2008)). This systematical alternation in the broad market augments investors' usual portfolios of stocks and bonds with options and other complex financial instruments. It is therefore surprising that very few papers explicitly consider the different possibilities of diversification with financial derivatives.

While in principle adding financial derivatives into portfolios plays an invaluable role in improving investors' risk-return trade off, there are two approaches to construct diversification strategies involving financial derivatives with respect to their underlying assets: combining derivatives on single stocks, and a derivative linked to the index consisting of those stocks. We compare these two slightly different strategies and their resulting utility for investors measured with their respective certainty equivalent rates. Given expected utility specifications, in this chapter, we analyze the two strategies from the perspective of an individual investor who has access to options, and the risk-free asset and maximizes his expected utility in the Black-Scholes as well as the stochastic volatility setup. Moreover, we extend the analysis to a related new class of derivatives – structured financial products.

While conventional call and put options have been traded for many years, the financial structured products have gained popularity among especially retail investors in recent years. Just like options, they are alternative investment vehicles to direct financial investments. More than that, they consist of at least two components, primarily combining stocks or bonds with options, generating a tailored payoff profile that otherwise may not be available in the market, at least for retail investors. In Europe, structured products are particularly popular. According to the German Derivatives Association (Deutscher Derivate Verband), for example, the outstanding volume of these products reached a value of 90.6 billion Euros by the end of first quarter of 2014, indicating a stable and significant market capitalization even after the financial shock in 2007.

Despite the substantial popularity of structured products, their successes cannot be explained easily. Branger and Breuer (2008) and Henderson and Pearson (2011) find that it is difficult to rationalize the purchase of structured products in the context of a plausible normative model of rational investors. Hens and Rieger (2014) compare the Markowitz-style (two funds) investment to structured products and conclude that the improvement of structured products is too small to be useful for classic rational investors. On the other hand, beyond the rational mean-variance theory, it is noted that the popularity of certain structured products is consistent with behavioral portfolio theory, and there is substantial evidence that behavioral biases such as loss aversion, probability weighting, and misestimation, are often associated with features of structured products.(Breuer and Perst (2007), Rieger (2012), Helberger (2012), Das and Statman (2013))

In another strand of literature, academic research has empirically focused on the issuing side in the context of pricing. Typically, studies on the pricing of structured products suggest a positive premium above their theoretical fair price. Wilkens et al. (2003) and Stoimenov and Wilkens (2005) examine the market for structured products in Germany and find mispricing in favor of the issuing institution. This phenomenon is not unique. Burth et al. (2001) and Wallmeier and Diethelm (2009) document a pricing discrepancy for the Swiss market. Outside of Europe, Benet et al. (2006) and Henderson and Pearson (2011) examine the valuation of structured products in the U.S. and report significantly higher prices in the market than estimates of the products' fair values. In recent research, Das and Statman (2013) find that the benefits of capital protected notes, one of the most important structured products, are not dissipated as perceived by investors' mental account until overpricing reaches 17.8%.[1]

As shown above, previous studies mostly concentrate either on the behavioral explanations or the mispricing of structured products. In this chapter, we analyze the issue of integrating and diversifying options and structured products into a portfolio in terms of underlying assets. We do not aim at finding the overall optimal strategy, but rather restrict our analysis to the utility comparison of diversification strategies with only pure derivatives. Options are important components for structured products. Gaining insights into methods of diversification for options therefore helps us to understand the diversification with structured products. A first fundamental question we study is which diversification strategy consisting of options is better for investors with constant relative risk aversion (CRRA). We use simulations where options are priced theoretically, and consider portfolios of stock-options and index-options respectively. Particularly, since the index level is just the weighted sum of individual stock prices, we do not model the index exogenously, but derive the behavior of the index from the joint behavior of the component individual stocks. By applying two different return generating processes with and without stochastic volatility (Black and Scholes (1973) and Heston (1993)), we calculate the utility improvement of the investment in risk-free asset and plain vanilla options for a CRRA investor in various scenarios. Moreover, we extend our analysis to three

[1]Empirical studies typically find a premium of overpricing of about 0.5% for simple products on large markets, but average values are as high as 8% in other situations. Some single products can have even larger mispricing.

types of structured products: capital protected notes (CPNs), discount certificates (DCs) and bonus certificates (BCs).

By comparing the terminal expected utility of the two diversification approaches in various scenarios, we find first of all that, given the benchmark parameters, investors with moderate risk-aversion always prefer the index strategy for call options and CPNs, while they gain higher utility improvement from the single-stock combinations for the derivative investment of put, DCs and BCs. Secondly, the sensitivity analysis shows that parameters play a crucial role in determining the optimal choice for investors. Specifically, varying risk-aversion, the expected return and the volatility of individual stocks, the strike price of options and the options' positions in the portfolio, respectively, has an important impact on the superiority of option diversification strategy. In particular, increasing the weight of the call in the portfolio leads to a better performance of the stock-based call combinations, which indirectly implies that, in the presence of higher participation rate (or lower protection level), CPNs linked to individual stocks can be more appealing for a CRRA investor. Furthermore, our main results hold for both the Black-Scholes model and the Heston stochastic volatility model. Interestingly, our numerical results with the base-case parameters on the CPNs and the DCs are somehow in line with the empirical research documented by Henderson and Pearson (2007), in which they point out the stylized fact that structured products linked to equity indices usually have convex payoff functions, while those linked to individual stocks mostly have concave payoff functions.

To our best knowledge, this article is the first one addressing this diversification issue theoretically based on a numerical comparison of the expected utilities for derivatives related to underlying assets. From an economic point of view, our findings provide insights into how to optimally and precisely choose derivatives in the presence of two alternative underlying assets in order to meet CRRA investors' preferences, and more importantly, they shed light on financial applications of the optimal diversification strategy for structured products

This chapter proceeds as follows: In section 3.2, we present the conceptual framework for modeling the underlying individual stocks and the index level, as well as measuring expected utility for a CRRA investor. Section 3.3 describes the structured products that we use in the analysis. In section 3.4, we introduce fundamental settings for the Monte Carlo simulation. Section 3.5 discusses the simulation results for options and structured products. We then conclude in section 3.6 in the end.

## 3.2 Model Setup

### 3.2.1 Black Scholes Framework

Our main focus is on options and structured products, on individual stocks as well as on the index composed of these stocks. Thus, a crucial setting in the analysis is to determine the return-generation process for each stock and the index obeying the aggregation restriction, which is defined as a weighted sum of the stock prices (Branger and Schlag (2004)).

Within the classical Black and Scholes (1973) option pricing framework, we assume the price of an individual underlying stock $S_t$ follows the geometric Brownian motion, where parameters are set to be identical across all $N$ stocks in the index. We assume that the market is frictionless, and there are no taxes, and no transaction costs. Moreover, we assume a constant, continuously compounded risk-free interest rate $r$. The price process of each stock $S_t^i$ for $i = 1, \ldots, N$ under the physical measure $P$ is then given by:

$$dS_t^i = \mu S_t^i dt + \sigma S_t^i dW_t^i, \tag{3.1}$$

where $\mu$ is the expected return, $\sigma$ is the constant volatility, and the term $dW^i$ represents the Wiener process that follows the evolution of a normal distribution.

### 3.2.2 Heston Model

We also use the model proposed by Heston (1993), which adds stochastic volatility to the Black Scholes framework. Moreover, it is able to capture non-lognormal distributions of stock prices and generate volatility smile surfaces observed frequently in reality. The dynamics of the individual stock's price $S_t^i$ under the physical measure $P$ follow the equation:

$$dS_t^i = \mu S_t^i dt + \sqrt{\upsilon_t} S_t^i dW_t^1, \quad (3.2)$$

$$d\upsilon_t = \kappa(\theta - \upsilon_t)dt + \eta\sqrt{\upsilon_t}\left(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2\right), \quad (3.3)$$

where $\upsilon_t$ is the instantaneous variance of the spot price and follows a mean-reversion stochastic process. $W_t^1$ and $W_t^2$ are independent Wiener processes. The underlying stock price and the volatility are correlated with $\rho$. $\kappa$ represents the mean-reversion rate, $\theta$ stands for the long-run mean, and $\eta$ is the volatility of the volatility.

### 3.2.3 The Index

Having modelled the behavior of the individual stocks, we define an index of these stocks, which is essentially determined by the stochastic processes of single stocks and dependence of each pair of stocks. Given the weight of each stock $\{w_i\}$, the index is just a weighted sum of the composing stocks. A general formula for the index level $I_t$ is given by

$$I_t = \sum_{i=1}^{N} w_t^i S_t^i,$$

where $S_t^i$ denotes the price of stock $i$ at time $t$.

In the analysis, we assume an equal constant correlation between any two stocks with a coefficient $\rho_s$, i.e.

$$dW_t^i dW_t^j = \rho_s dt, \ i \neq j. \quad (3.4)$$

The correlation between pairs of stocks is of high importance for the index. Firstly, the impact of the correlation measuring the shape of smile for index options and individual options is distinctively different[2]. Secondly, higher correlation among individual stocks increases the market index volatility, and hence lowers diversification benefits for the index.

Suppose we have a correlation matrix denoted by $C$, and every two stocks have the identical correlation coefficient $\rho_s$.

$$C = \begin{pmatrix} \rho_{1,1} & \rho_{1,2} & \cdots & \rho_{1,n} \\ \rho_{2,1} & \rho_{2,2} & \cdots & \rho_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n,1} & \rho_{n,2} & \cdots & \rho_{n,n} \end{pmatrix} = \begin{pmatrix} 1 & \rho_s & \cdots & \rho_s \\ \rho_s & 1 & \cdots & \rho_s \\ \vdots & \vdots & \ddots & \vdots \\ \rho_s & \rho_s & \cdots & 1 \end{pmatrix}$$

The correlation matrix $C$ is symmetric and positive-definite. By performing the Cholesky Decomposition we transform uncorrelated random variables $\epsilon'$ into correlated random variables $\epsilon$, so that we can simulate the correlated stock prices, where

$$\epsilon = M^T \epsilon',$$

and $M$ is the matrix that satisfies

$$M^T M = C$$

The objective is to compare the two diversification strategies for option as well as for structured products based on different underlying stocks. In order to achieve this goal, we restrict our analysis to the setup described above and do not consider the case of jumps in the stock price and other complex settings, since we want this first study on this subject to be as parsimonious as possible. Moreover, we take the price processes as given and do not consider equilibrium implications of diversification strategies.

### 3.2.4 Expected Utility Theory

The expected utility theory (EUT) has been the standard model of decision making under risk for descriptive as well as normative purposes. Standard finance is based

[2]See Branger and Schlag (2004) for example.

on the assumption that investors behave rationally, are strictly risk-averse, and make investment decisions that optimize their expected utilities. In this article, we consider an investor with constant relative risk aversion (CRRA), i.e. his risk attitude is proportional to his wealth and the percentage allocation assigned to the risky assets remains constant for different levels of his overall wealth (Gollier (2004)). One representative utility function in this subclass is the power utility function:

$$U(W) = \begin{cases} \frac{1}{1-\gamma} W^{1-\gamma} & \text{for } \gamma \neq 1 \\ \ln W & \text{for } \gamma = 1, \end{cases}$$

where $\gamma$ is the constant relative risk aversion coefficient and $W$ is the wealth of the investor. Since we are focusing on retail investors, we further impose two restrictions. First of all, the investor follows a buy-and-hold strategy for the holding period and cannot trade during the term to maturity. Secondly, short selling is not permitted, neither on the money market nor the derivative account.

The investor starts with a positive wealth $W_0$, and has the opportunity to invest in a money market account and options. Therefore, At maturity $T$ his terminal wealth from investing in the portfolio of options and risk-free assets is given by

$$W_T = W_0 \left[ e^{rT}(1 - \psi_0) + \psi_0 \frac{C_T}{C_0} \right],$$

where $\psi_0$ denotes the portfolio weight of the option with price $C_0$ at time 0 and payoff $C_T$ at maturity $T$. Furthermore, we assume the $\psi_0$ is equal for the two diversification strategies for the sake of comparison[3]. As financial application, we also calculate the utility of the investment in structured products. We will provide a more detailed description for structured products in the later section.

The investor derives utility from the terminal wealth $W_T$ of the investment strategies and his expected utility at maturity is

$$E\left[U(W_T)\right]. \tag{3.5}$$

[3]Since our aim is the comparison of derivatives on different underlying stocks ceteris paribus, we do not search for the optimal fraction of the wealth in derivatives nor the exposure of the optimal portfolio to different risk factors for the retail investor.

For the sake of clarity and directness, we ultimately translate the expected utility of various payoffs into the certainty equivalent rate, i.e. the risk-free rate given the same utility. Then we compare differences of certainty equivalent rates for the two derivative strategies on single stocks and the index.

## 3.3 Structured Products as Investment Portfolios

Structured products are tailored to meet investors' special return-risk profiles and market expectations, which may not be attained otherwise through traditional financial assets that are readily available to retail investors (i.e. stocks and bonds). They can be very complicated financial derivatives embedded with options connecting to a single security, basket of securities or indices. Therefore, both valuation and payoff formulas are determined by their component elements. Since private investors usually have neither the financial sophistication nor the possibility to do short selling nor access option markets, structured products dramatically increase the flexibility of their financial transaction opportunities.

Among a tremendous variety of structured products, we select capital protected notes (henceforth CPNs), discount certificates (henceforth DCs) and bonus certificates (henceforth BCs) as examples, since they are fairly genetic product types. Their profiles are illustrated in Figure 3.1

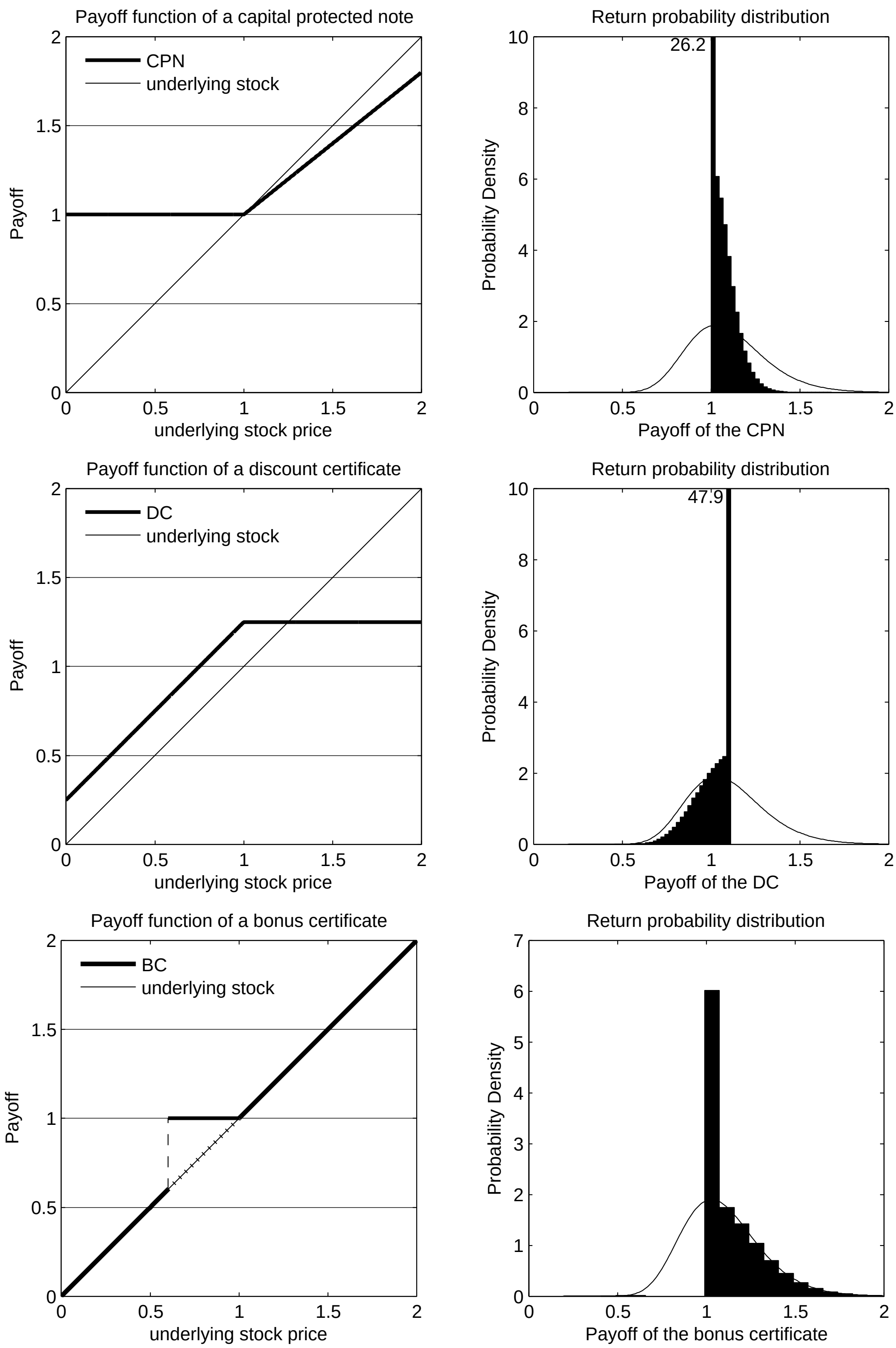
### 3.3.1 Capital Protection: Capital Protected Products

The classic CPN[4] is defined in such a way that it guarantees the redemption of the invested capital at maturity in addition to participating to a certain degree in the performance of an underlying risky asset.

In order to obtain the payoff profile described above, one needs two elements.

[4]It is no other than the uncapped capital protection(1100) according to the *SVSP Swiss Derivative Map*

Figure 3.1: Payoff function and return distributions of the structured products



First a zero-coupon bond which provides the protection in terms of safe interest and then a call option which features the participation of speculating the market and provides returns over the protected amount. Specifically, the zero-coupon bond is issued at a discount, which redeems usually at par (for 100% capital guaranteed) at the maturity of the CPN. The participation rate can vary according to the price of the embedded call option.

The payoff of the CPN $X_{CPN}$ can be replicated by the sum of the two elements, which leads to the following formula:

$$X_{CPN} = \theta B_T + \alpha \left[ \max(S_T - K, 0) \right], \tag{3.6}$$

where $\theta$ is the guaranteed level, and $\alpha$ is the participation rate. Practically $\theta$ is often equal to 1, corresponding to the 100% capital guaranteed.

### 3.3.2 Yield Enhancement Product: Discount Certificate

The yield enhancement products account for the largest number of structured products both in varieties and volume throughout Europe (Blümke (2009)). Unlike capital protected products, this category of financial products comes with capped upside potential and without downside protection. The DC is one of them. Typically, it is issued at a lower price compared to the spot price of a direct investment in the underlying stocks. The difference between the discounted price and the par value provides a security buffer as well as a cap, which also limits the potential upside performance at expiry.

DCs are normally constructed with two components: a long zero-strike call[5] and a short call option. Besides, an alternative way to frame the value of a DC is to mix a zero-coupon bond and a short put as a consequence of the put-call parity. The

[5]Sometimes it is referred to the Low Exercise Price Option(LEPO) as well, due to the feature that no dividends are paid for options compared to usual stocks.

payoff at maturity $T$ is:

$$X_{DC} = S_T - \max(S_T - K, 0) \quad (3.7)$$

$$= \min(S_T, K). \quad (3.8)$$

The value of the DC is:

$$DC_t = S_t - C_t,$$

where $C_t$ is the price of the corresponding call option embedded in the DC.

### 3.3.3 Participation Product: Bonus Certificate

Participation products are a group of products that have upside potential and no, or only conditional, protection. BCs belong to this category. It features a full upside participation and a conditional capital protection, provided that the underlying asset's price does not cross a predefined threshold $H$. If the price of the underlying equity breaches the predefined barrier during the time to maturity, an amount equal to the underlying price at maturity will be paid, nevertheless the bonus extinguishes.

BCs are established with a long position in a zero-strike call on the underlying stocks and a long position in a down-and-out put option. Thus, the payoff of BC at maturity $T$ is

$$X_{BC} = S_T + \max(K - S_T, 0) 1_{\tau > T}, \quad (3.9)$$

where $\tau = inf\{t > 0 : S_t < H\}$ is the first time of the barrier hitting, and $1_{\{.\}}$ is the indicator function. The value of the BC is given by

$$BC_t = S_t + Pdo_t,$$

where $Pdo_t$ denotes the value of the down-and-out put option with barrier $H$ and strike price $K$.

## 3.4 Simulation Setup

We consider a retail investor who can only invest in a money market account and options or structured products. As the simplest version, two stocks ($N = 2$) are employed in the initial set in the index. As a matter of fact, the number of stocks composing the index is not crucial, since increasing the number of stocks in the index will not qualitatively change the results.

To examine the theoretical properties of our results, we fix a set of base-case parameters (note that the parameters for the individual stock and each pair of stocks are identical). Specifically, the risk-free rate $r$ is set to 4%[6]. Using the conservative estimate of the equity risk premium (excess rate of return) of Dimson et al. (2006), we assume that the expected return of the individual stock equals to 9%. The initial stock price $S_0 = 1$, the volatility of each stock $\sigma = 20\%$ in the Black Scholes model, and the correlation coefficient $\rho_s$ between each pair of stocks is set to 0.4. Derivatives are set to at-the-money, i.e. the strike price $K$ is set to the spot price of the stock ($S_0$). Given the fact that many studies use one year as the investment horizon (e.g. Benartzi and Thaler (1995) and Dichtl and Drobetz (2011)), the investor also buys some of the assets above and holds them for $\tau = 1$ year. For the option pricing and the underlying stock processes under the Heston model, we apply a representative set of parameters as Eraker et al. (2003) and Vrecko and Branger (2009), with the initial variance $\upsilon_0 = 0.02$, and long-run mean $\theta = (0.2)^2$, hence coupling it with the constant volatility of the Black-Scholes model, mean-reversion speed $\kappa = 5.7960$, volatility of the volatility $\eta = 0.3528$, and the correlation coefficient between the volatility and the stock price $\rho = -0.4$.[7]

In accordance with option pricing assumptions, we distinguish the stock price

[6] This allows a comparison with Hens and Rieger (2014)

[7] The parameters properties of the Heston model have been estimated by a large number of studies, and the estimated parameters may differ from paper to paper. Our chosen parameters are in the generally agreed region as basically in line with Liu and Pan (2003).

process with the Black-Scholes model and the Heston model, respectively. Under the risk-neutral measure, we obtain individual option prices within the Black-Scholes option-pricing framework, and through the fast Fourier transform under the Heston model. In the case of path-dependent derivatives, options' prices are obtained given the stochastic volatility via the Monte Carlo simulation. There is, however, no closed-form solution for prices of index options.[8] Thus for pricing the index option, we employ the Monte Carlo simulation to compute the price of index options. More specifically, we perform $1,000,000$ simulation runs using the stochastic process in Equation 3.1 and Equation 3.2 respectively. The Monte Carlo simulation is performed through an Euler scheme with 250 time steps per holding period to generate distributions of the terminal prices of the individual stocks and the index level of these stocks. Then the expected utility of the portfolio that consists of options and risk-free assets can be determined for the two diversification strategies. As described above, we eventually transform the expected utility to certainty equivalent rates for the CRRA investor. We run Monte Carlo simulations with plain vanilla call and put options, and proceed with the selected structured products.

## 3.5 Results

This section presents the results of the numerical analysis with the Monte Carlo simulation. We divide the results into three parts: the first part gives the certainty equivalents of the two diversification portfolios with the call and the put under the base case parameters; the second part conducts a sensitivity analysis with respect to various parameters; lastly, the third part examines the optimal diversification portfolios for the selected structured products: the CPNs, the DCs and the BCs.

[8] See Branger and Schlag (2004). The analytical solution would not exist even if each stock of the index followed a geometric Brownian motion and all the stocks were independent.

### 3.5.1 Options

Since structured products are essentially investment portfolios combining long or short positions with options, we firstly consider a buy-and-hold investor who can invest in the options and the money market as a pre-experiment. Under the prevailing assumptions, we compute the expected utility of the investment of a CRRA investor, in which his portfolio is composed of the risk-free asset and plain vanilla options. Another fundamental assumption here is that an equal amount is invested in the option position as for the two methods of diversification. We compare a certainty equivalent return of portfolios consisting of $\psi_0$ options and $(1-\psi_0)$ of risk-free assets. In addition, we implement the paired-t test to assess whether differences between the two strategies are statistically significant.

Table 3.1 and Table 3.2 reveal the certainty equivalents (CEs) of the CRRA investor with various risk aversion magnitudes $\gamma = 0.5, 2, 3, 4, 5$, given the base-case parameters as described above. Note that the larger the $\gamma$, the more risk-averse the investor is. The Columns 2 to 6 represent the weights of the initial option position $\psi_0$ from 1% to 5%, which remains constant over time. The t-test results of all differences $D1-D2$ indicate the statistical significance at the 1% level. Furthermore, a common observation for both call and put is that the result is qualitatively very similar under the Black-Scholes model and the stochastic volatility of Heston model. Therefore, we restrict the discussion to the results under the Black-Scholes model (Table 3.1).

We observe that the better strategy involving call options for the CRRA investor to choose is D2 where the call option is based on the index. For an investor with $\gamma = 0.5$, for instance, the index-calls exceed the stock-calls' combination by giving the investor additional certainty equivalents from 6 basis points to 26 basis points in the Black-Scholes framework when the position in the call increases from 1% to 5%. However, for a less aggressive investor with $\gamma = 5$, the maximal certainty equivalent returns is only 6 bp as $\psi_0 = 2\%$. When $\psi_0 = 5\%$, the result even reverses, i.e. the investor gains more from the stock-based calls' combination, though the difference is quite minor (5bp).

In contrast to the results for call options, put options based on individual stocks are uniformly more appreciated by the CRRA investor for all levels of risk-aversion. The more wealth is invested into the put option, the more pronounced the utility differences. In particular, when the weight of the put is 1% ($\psi_0 = 1\%$), the difference of certainty equivalent return is only 7 bp for an investor with relative risk aversion $\gamma = 5$. When we set $\psi_0 = 5\%$, which is the fraction of the overall wealth invested into the put, the difference between the two strategies increases to 54 bp for the same investor. However, with more exposure to the put option, the utility gain of both put option strategies declines. This is intuitive, because the payoff of a put decreases against the development of the underlying stock prices, a larger position in the put thus in general deteriorates the performance of portfolios consisting of the put and the risk-free assets.

Taking everything together, the Monte Carlo simulation results show that it is more beneficial to invest in the index call rather than a portfolio of calls (as long as risk aversion is not high), while it is more beneficial to have a portfolio of stock-based put option combinations than an index-based put option.

### 3.5.2 Sensitivity Analysis

We now conduct a sensitivity analysis of the two diversification strategies to risk-aversion coefficients, correlation coefficients between stocks, expected returns, volatilities of the stock, strike prices and positions in the options. All other parameters are chosen as before.

Figure 3.2 and Figure 3.3 provide the sensitivity analysis under different scenarios for the call options and the put options in the Black-Scholes framework. Similarly, Figure 3.4 and Figure 3.5 give the sensitivity analysis under the stochastic volatility Heston model. Each of the six graphs represents a quantitative assessment of the strategy comparison, and indicates a parameter changing while the other parameters

Table 3.1: Certainty equivalents rates (CEs) for option diversifications in the **Black-Scholes model**

| | $\psi_0 = 0.01$ | | $\psi_0 = 0.02$ | | $\psi_0 = 0.03$ | | $\psi_0 = 0.04$ | | $\psi_0 = 0.05$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | D1/D2 | D1-D2 | D1 /D2 | D1-D2 | D1/D2 | D1-D2 | D1/ D2 | D1-D2 | D1/D2 | D1-D2 |
| **Call** | | | | | | | | | | |
| 0.5 | 4.35/4.41 | -0.06 | 4.70/4.81 | -0.12 | 5.03/5.20 | -0.17 | 5.36/5.58 | -0.22 | 5.68/5.94 | -0.26 |
| 2 | 4.34/4.39 | -0.06 | 4.64/4.74 | -0.10 | 4.91/5.03 | -0.12 | 5.14/5.28 | -0.14 | 5.34/5.49 | -0.15 |
| 3 | 4.33/4.38 | -0.05 | 4.60/4.69 | -0.08 | 4.83/4.92 | -0.10 | 5.00/5.09 | -0.09 | 5.13/5.21 | -0.07 |
| 4 | 4.32/4.36 | -0.05 | 4.56/4.63 | -0.07 | 4.74/4.81 | -0.06 | 4.86/4.90 | -0.04 | 4.93/4.94 | -0.01 |
| 5 | 4.31/4.36 | -0.05 | 4.53/4.59 | -0.06 | 4.67/4.72 | -0.05 | 4.74/4.75 | -0.01 | 4.75/4.70 | 0.05 |
| **Put** | | | | | | | | | | |
| 0.5 | 3.70/3.64 | 0.06 | 3.39/3.27 | 0.12 | 3.08/2.90 | 0.18 | 2.76/2.52 | 0.25 | 2.44/2.13 | 0.31 |
| 2 | 3.69/3.63 | 0.06 | 3.36/3.23 | 0.14 | 3.01/2.80 | 0.22 | 2.65/2.33 | 0.31 | 2.25/1.84 | 0.41 |
| 3 | 3.69/3.62 | 0.07 | 3.34/3.19 | 0.15 | 2.97/2.73 | 0.24 | 2.56/2.22 | 0.35 | 2.13/1.67 | 0.46 |
| 4 | 3.68/3.61 | 0.07 | 3.32/3.16 | 0.16 | 2.92/2.66 | 0.26 | 2.49/2.11 | 0.38 | 2.02/1.52 | 0.51 |
| 5 | 3.68/3.60 | 0.07 | 3.30/3.14 | 0.17 | 2.88/2.60 | 0.28 | 2.42/2.02 | 0.40 | 1.92/1.38 | 0.54 |

The table shows the numerical results of certainty equivalents with respect to the terminal wealth of the option portfolios. $\psi_0$ is the option position at time 0 and remains constant over time, while the rest $1-\psi_0$ is the amount invested into the risk-free asset. $\gamma$ is the risk aversion of the CRRA investor. $D_1$ and $D_2$ represent the CEs of the two diversification strategies: $D_1$ is the portfolio of equally weighted options each based on a single stock and the risk-free asset; $D_2$ is the portfolio of a option based on the index and the risk-free asset. $D_1 - D_2$ is the difference in certainty equivalent return between the two. We run $1,000,000$ simulations generated by the Black-Scholes model market scenarios for the underlying stocks and the index. All the numbers are percentage. All the differences of certainty equivalents between the two strategies are statistically significant at the 1% level.

Table 3.2: Certainty equivalents rates (CEs) for option diversifications in the **Heston model**

| | $\psi_0 = 0.01$ | | $\psi_0 = 0.02$ | | $\psi_0 = 0.03$ | | $\psi_0 = 0.04$ | | $\psi_0 = 0.05$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | D1/D2 | D1-D2 | D1 /D2 | D1-D2 | D1/D2 | D1-D2 | D1/ D2 | D1-D2 | D1/D2 | D1-D2 |
| **Call** | | | | | | | | | | |
| 0.5 | 4.37/4.43 | -0.07 | 4.73 /4.85 | -0.13 | 5.08/5.26 | -0.18 | 5.41/ 5.66 | -0.25 | 5.74/6.04 | -0.30 |
| 2 | 4.35/4.42 | -0.06 | 4.66 / 4.78 | -0.11 | 4.95/5.09 | -0.14 | 5.20/ 5.37 | -0.17 | 5.41/ 5.58 | -0.17 |
| 3 | 4.35/4.40 | -0.06 | 4.63 / 4.72 | -0.09 | 4.88/4.99 | -0.11 | 5.05/5.16 | -0.11 | 5.19/ 5.31 | -0.12 |
| 4 | 4.33/4.39 | -0.06 | 4.60/ 4.68 | -0.07 | 4.79/4.87 | -0.08 | 4.94/5.00 | -0.06 | 4.99/ 5.01 | -0.02 |
| 5 | 4.32/4.37 | -0.05 | 4.57/ 4.63 | -0.06 | 4.72/4.78 | -0.06 | 4.80/4.82 | -0.03 | 4.82/ 4.80 | 0.02 |
| **Put** | | | | | | | | | | |
| 0.5 | 3.68/3.62 | 0.05 | 3.35/3.24 | 0.12 | 3.03/2.85 | 0.18 | 2.70/2.46 | 0.24 | 2.35/2.03 | 0.32 |
| 2 | 3.67/3.61 | 0.06 | 3.32/3.19 | 0.14 | 2.95/2.74 | 0.22 | 2.56/2.25 | 0.30 | 2.15/1.74 | 0.41 |
| 3 | 3.67/3.60 | 0.06 | 3.30/3.16 | 0.14 | 2.91/2.66 | 0.24 | 2.48/2.14 | 0.34 | 2.03/1.58 | 0.45 |
| 4 | 3.66/3.60 | 0.07 | 3.28/3.12 | 0.15 | 2.86/2.60 | 0.25 | 2.41/2.04 | 0.37 | 1.92/1.43 | 0.50 |
| 5 | 3.66/3.59 | 0.07 | 3.26/3.10 | 0.16 | 2.82/2.54 | 0.28 | 2.34/1.94 | 0.40 | 1.81/1.28 | 0.53 |

The table shows the numerical results of certainty equivalents with respect to the terminal wealth of the option portfolios. $\psi_0$ is the option position at time 0 and remains constant over time, while the rest $1-\psi_0$ is the amount invested into the risk-free asset. $\gamma$ is the risk aversion of the CRRA investor. $D_1$ and $D_2$ represent the CEs of the two diversification strategies: $D_1$ is the portfolio of equally weighted options each based on a single stock and the risk-free asset; $D_2$ is the portfolio of a option based on the index and the risk-free asset. $D_1 - D_2$ is the difference in certainty equivalent return between the two. We run $1,000,000$ simulations generated by the Heston model market scenarios for the underlying stocks and the index. All the numbers are percentage. All the differences of certainty equivalents between the two strategies are statistically significant at the 1% level.

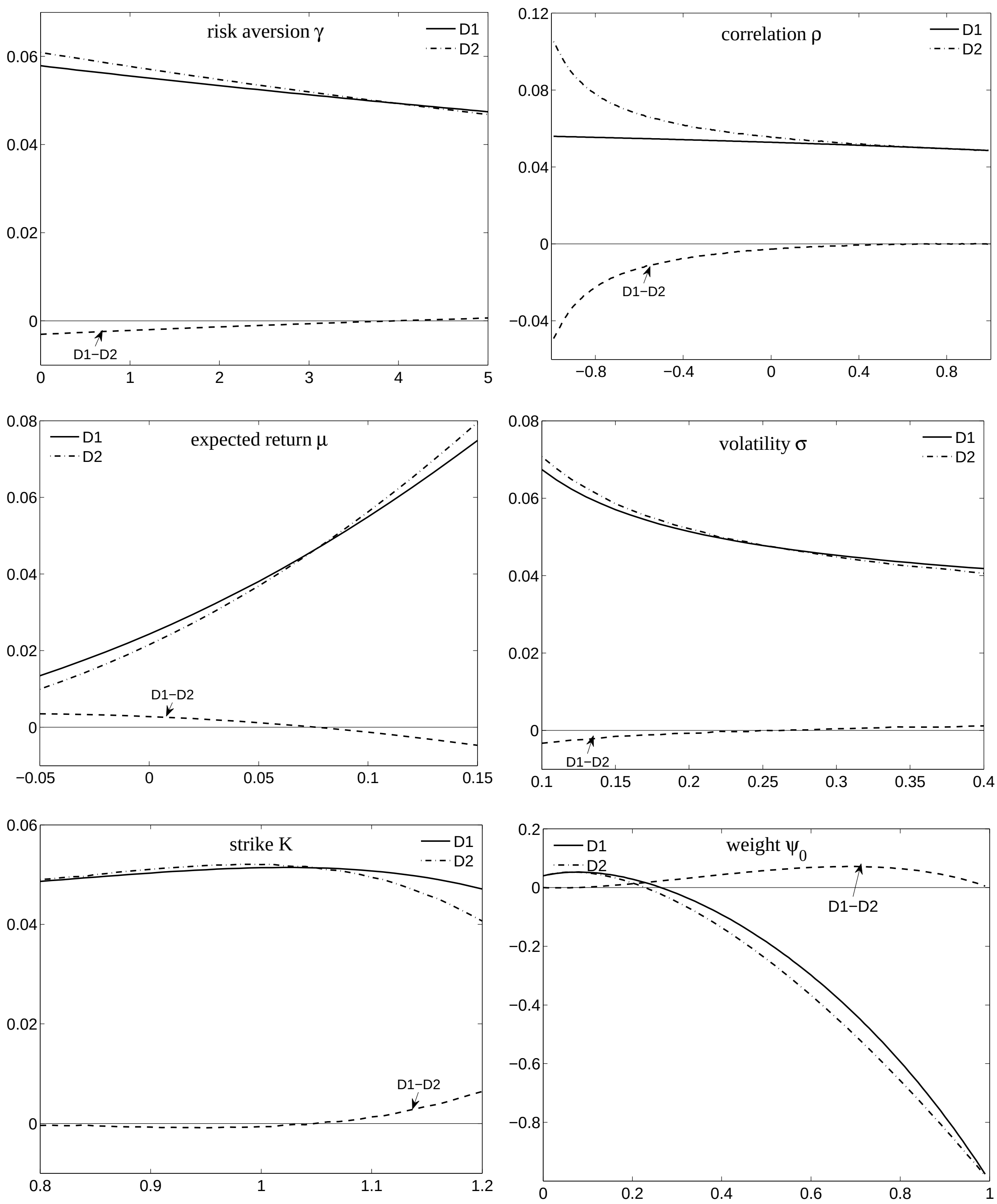
are fixed with the base-case parameters. Specifically, we assume the risk-aversion coefficient $\gamma \in [0, 5]$, the correlation between stocks $\rho \in [-0.99, 0.99]$, the stock expected return $\mu \in [-5\%, 15\%]$, the stock volatility $\sigma \in [0.1, 0.4]$, the strike of the options $K \in [0.8, 1.2]$ to the current price of the stock, and the fraction of the wealth in options $\psi_0 \in [0, 0.99]$.

In line with the observations above, the overall results of the utility comparison are qualitatively similar in both the Black-Scholes and the Heston model. Hence again we mainly focus on the results under the Black-Scholes model. In the case of the call option, we see that the superiority of the diversification strategies varies, and in fact, is very sensitive to the parameters. Under low ($\mu \leq 5\%$) or turbulent ($\sigma \geq 35\%$) market conditions, the portfolio of calls based on individual stocks is more attractive to the investor with a moderate risk-aversion coefficient($\gamma = 3$). Moreover, as the investor becomes more aggressive in investing in the call option, the utility of stock-based calls combination becomes preferable over the index-based call (bottom right panel). Similarly, the strike price also plays a role in determining the optimality of diversification strategies.

For put options, interestingly, the results are largely independent of the parameters such as risk-aversion coefficient $\gamma$, stocks' correlation $\rho$, volatility $\sigma$, strike price $K$ and the fraction in the put $\psi_0$. Despite the variation in all the parameters, the results indicate a preference for the put combinations in which each put links to an individual stock. However, the one exception is the expected return $\mu$. As it decreases to be negative ($\mu < 0$), which implies a recessive market, put based on the index becomes increasingly more attractive for the investor.

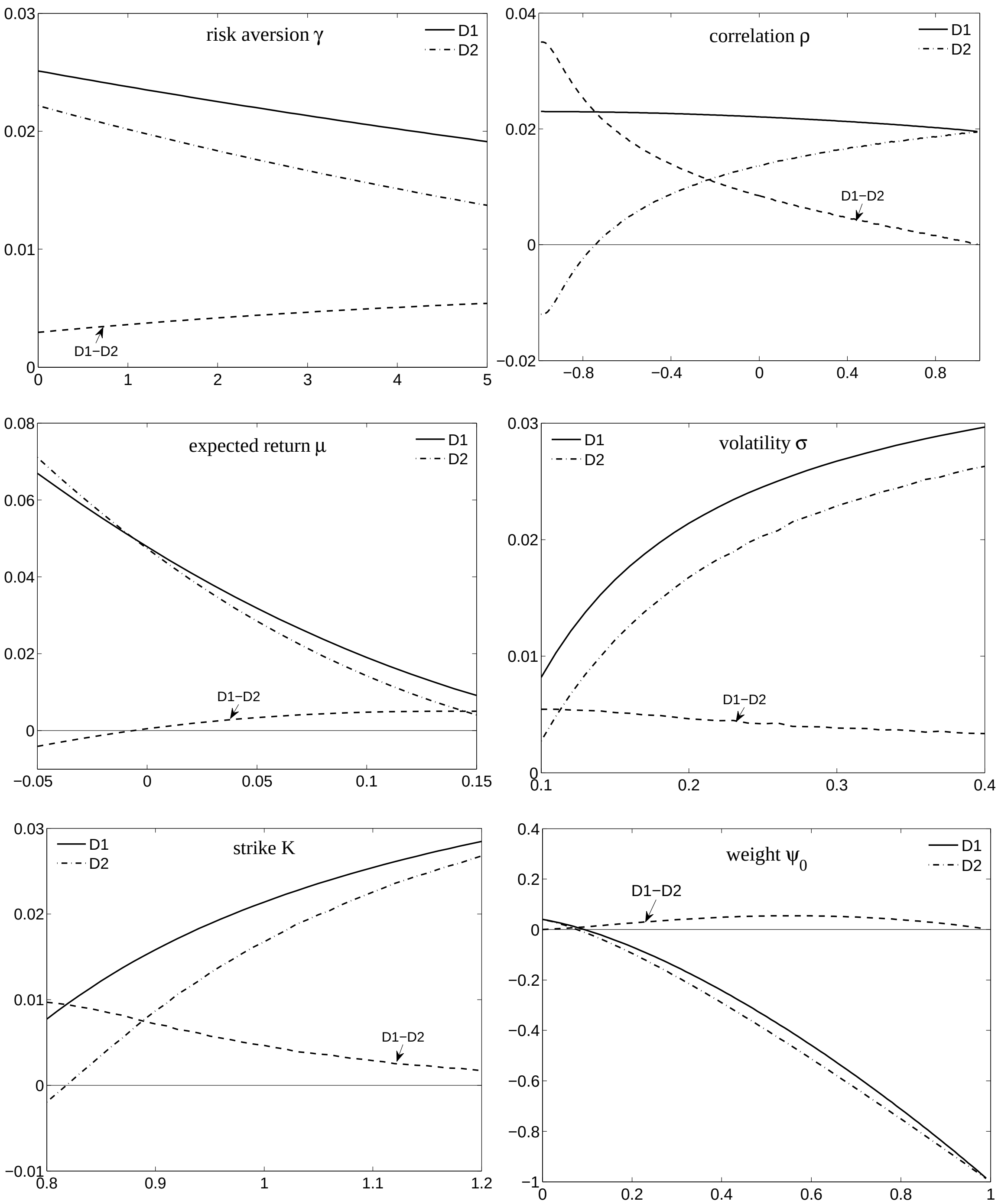
Moreover, we also run additional tests with respect to the number of stocks included in the index, by increasing the number of stocks in the index with $N = 5$ and $N = 10$. Besides, instead of employing the time to maturity of 1 year, we consider the remaining lifetime to maturity of both options with $\tau = 1/12$, $\tau = 1/4$, $\tau = 1/2$ and $\tau = 2$ years. The results are qualitatively very similar and thus not

Figure 3.2: Comparison of CEs between the two CALL diversification strategies under the **BS model**



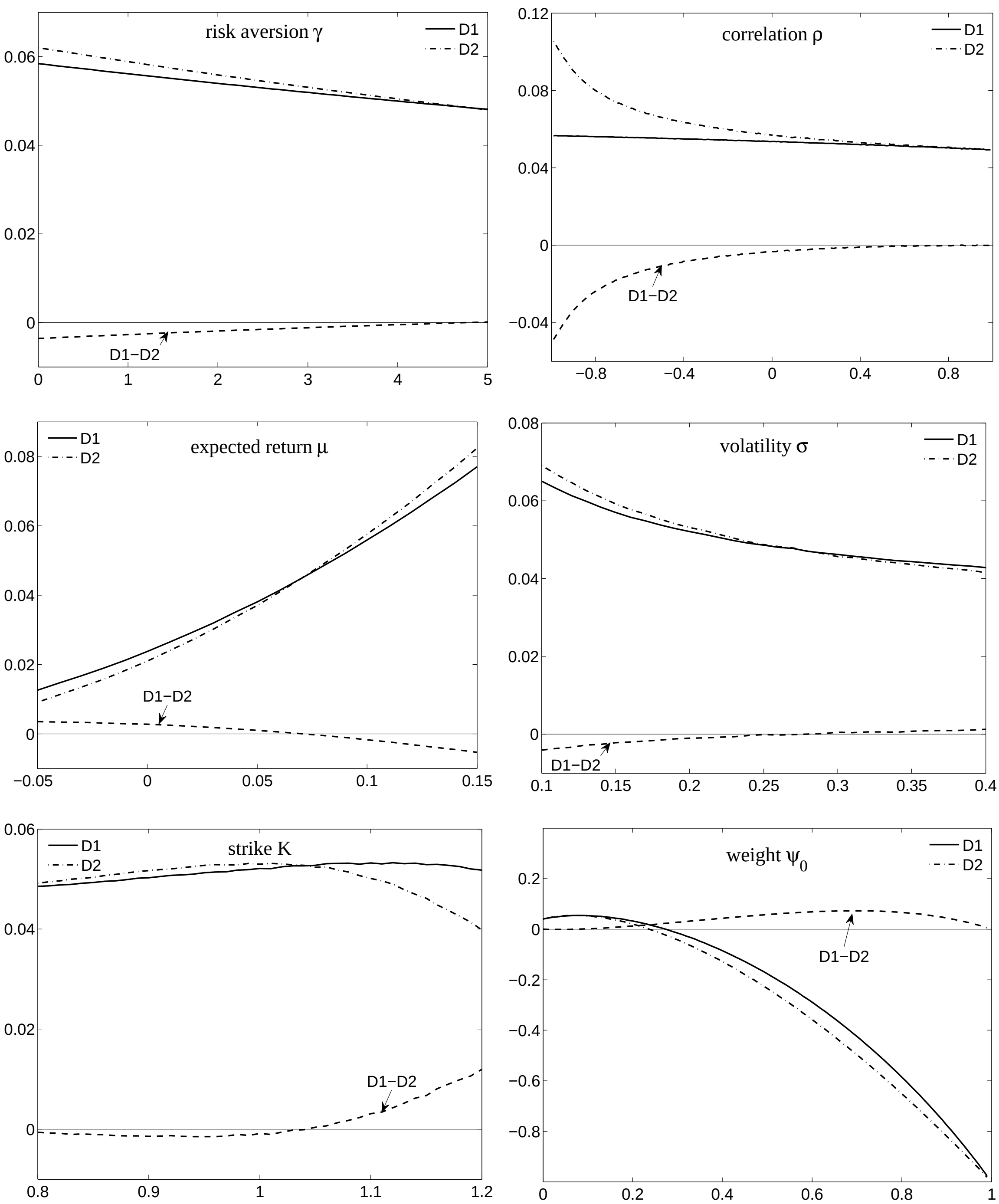

The y-axes are the certainty equivalent rates of the portfolio consisting of the stock-based calls (solid line), the index-based calls (dashed-dot line) and the difference of the two portfolios (dashed line). The base-case parameters are presented as in Section 3.4, e.g., the investor has risk aversion $\gamma = 3$; the weight of calls on both strategies are set to equivalent $\theta = 5\%$; the stock expected return $\mu = 9\%$, and the volatility is $\sigma = 20\%$.

Figure 3.3: Comparison of CEs between the two PUT diversification strategies under **the BS model**



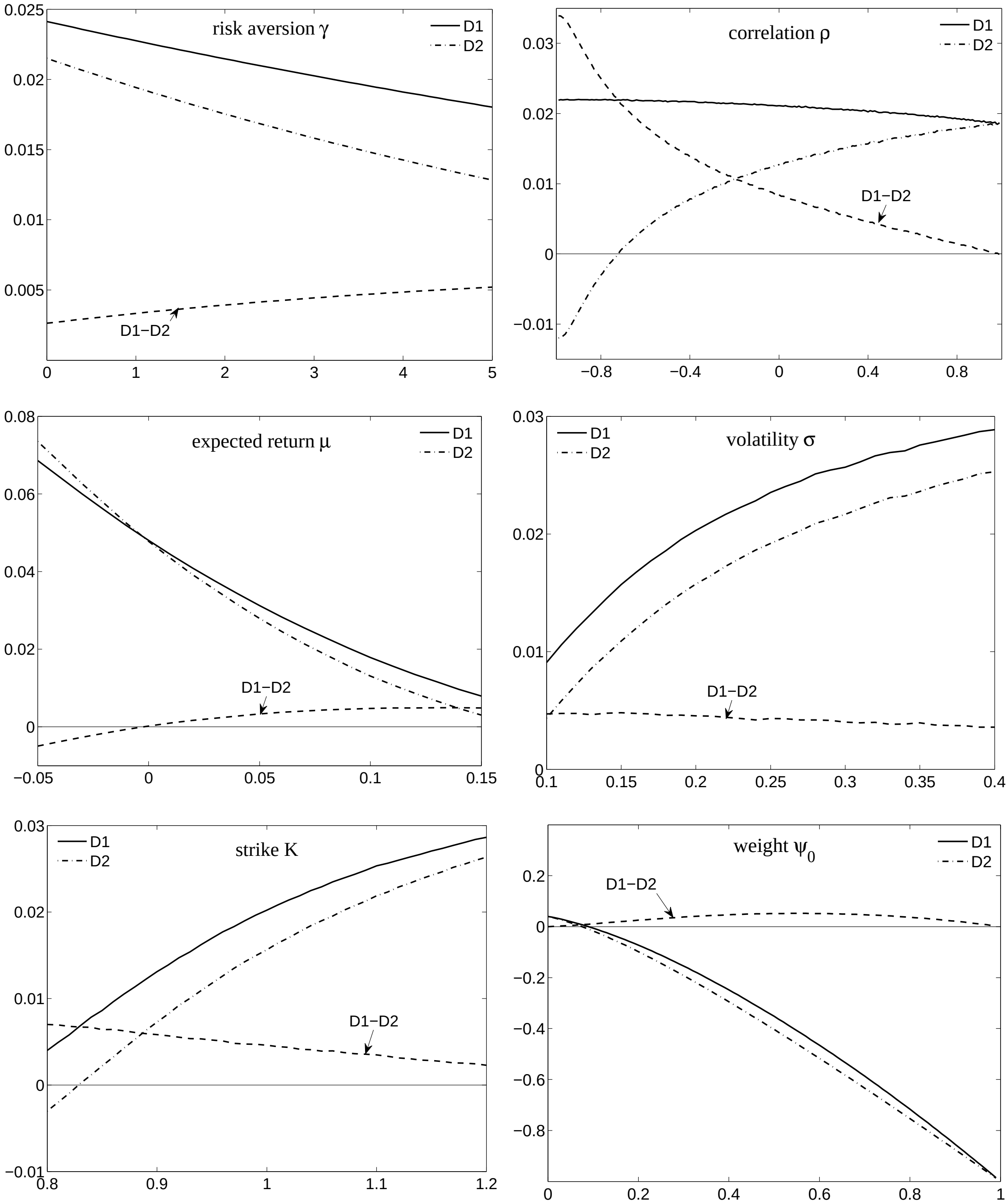

The y-axes are the certainty equivalent rates of the portfolio consisting of the stock-based puts (solid line), the index-based puts (dashed-dot line) and the difference of the two portfolios (dashed line). The base-case parameters are presented as in Section 3.4, e.g., the investor has risk aversion $\gamma = 3$; the weight of calls on both strategies are set to equivalent $\theta = 5\%$; the stock expected return $\mu = 9\%$, and the volatility is $\sigma = 20\%$.

Figure 3.4: Comparison of CEs between the two CALL diversification strategies under **the SV model**




The y-axes are the certainty equivalent rates of the portfolio consisting of the stock-based calls (solid line), the index-based calls (dashed-dot line) and the difference of the two portfolios (dashed line). The base-case parameters are presented as in Section 3.4, e.g., the investor has risk aversion $\gamma = 3$; the weight of calls on both strategies are set to equivalent $\theta = 5\%$; the stock expected return $\mu = 9\%$, and the volatility is $\sigma = 20\%$.

Figure 3.5: Comparison of CEs between the two PUT diversification strategies under **the SV model**




The y-axes are the certainty equivalent rates of the portfolio consisting of stock-based puts (solid line), index-based puts (dashed-dot line) and the difference of the two portfolios (dashed line). The base-case parameters are presented as in Section 3.4, e.g., the investor has risk aversion $\gamma = 3$; the weight of calls on both strategies are set to equivalent $\theta = 5\%$; the stock expected return $\mu = 9\%$, and the volatility is $\sigma = 20\%$.

reported[9].

### 3.5.3 Financial Applications: Structured Products

Previous sections have examined the differences in diversification strategies on the options. In this section, we extend our investigation on diversification to structured products, which are often embedded with options as well as bond or underlying securities as packages.

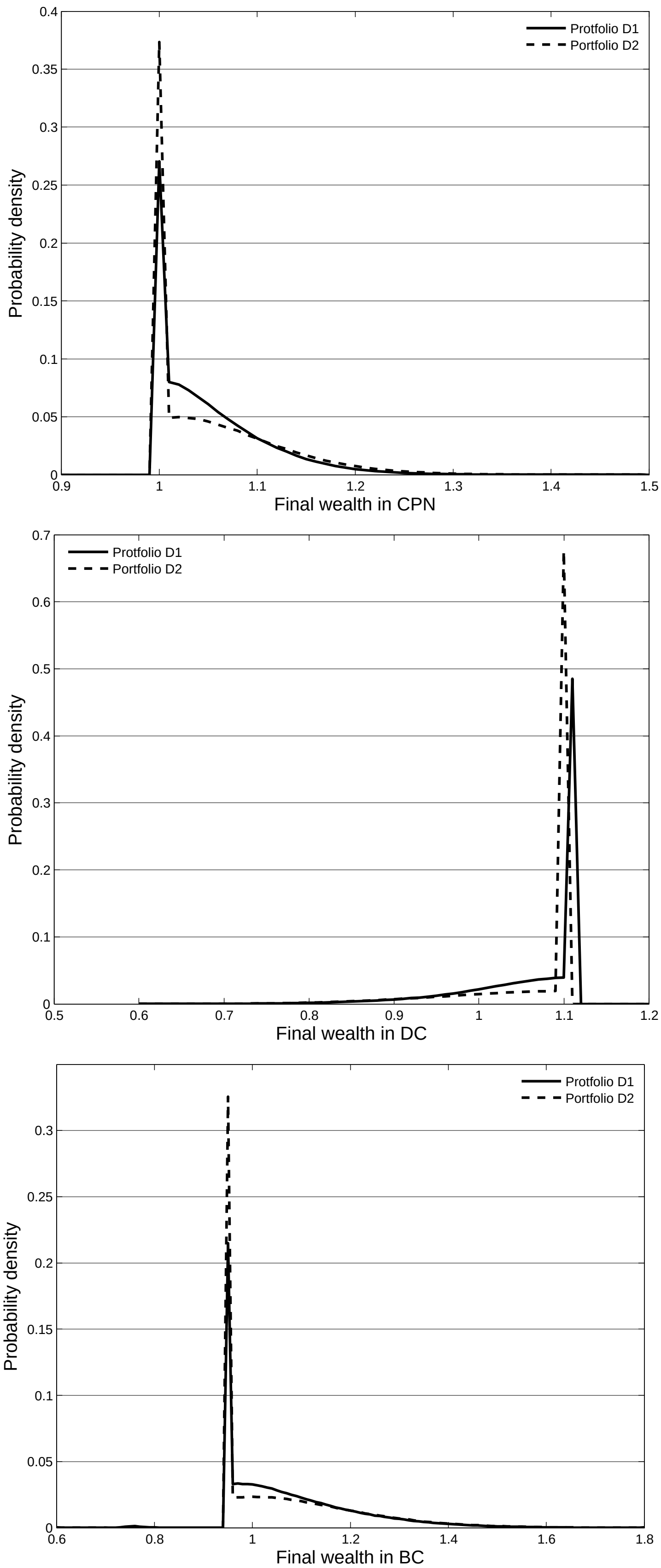
We repeatedly implement a series of strategy comparisons between the pair of portfolios among CPNs, DCs and BCs. Different from the analysis with options, here we assume that the total wealth is invested into the structured products, because the investor holds essentially stocks or bonds positions that are already embedded into the products. Figure 3.6 depicts the density of the terminal wealth from the investment in the structured products. It depicts that there is a substantially larger probability that the index product ends up with the terminal wealth being equal to the predefined level[10] than the addition of the two products. On the other hand, the CPNs and the BCs display positive skewness, whereas the DCs result in negatively skewed return distributions.

Panel A of Table 3.3 shows the certainty equivalent rates for the structured products under the Black-Scholes framework. For the CPNs, index linked products are superior to the stock products combination ($D2 > D1$), and the difference declines monotonically in magnitude from 21 basis points to 1 basis point as the risk-aversion increases proportionally from 0.5 to 5. This reflects the fact that the CRRA investor with a lower risk-aversion level ($\gamma = 0.5$, for example), would benefit more from investing in CPNs on the index than in CPNs on individual stocks. This result is consistent with our previous results on call options. Recall that for the

[9] Summary of results is available upon request.

[10] The predefined level refers to the nominal protection level for the capital protected note, the limited profit potential (cap) for the discount certificate, and the conditional protection level for the bonus certificate if the barrier is not breached.

Figure 3.6: Density function of terminal wealth distributions for the diversification strategies with structured products



call options considered in Table 3.1, a portfolio consisting of a small amount of calls (e.g. $\psi_0 = 4\%$) on the index and the risk-free assets is always better than the other alternative (ceteris paribus). As discussed earlier, a CPN is basically a combination of a call and a bond. Given the base-case parameters' setting in our analysis, the CPNs can be translated equivalently to a fraction of 3.92% of the wealth in calls when considering a portfolio of calls and risk-free assets. However, as we can see from Figure 3.2 as well as Figure 3.4, the exposure to calls substantially affects the attractiveness of the strategy, which implies that the higher the participation rate (or the lower the protection level) of a CPN, the higher the likelihood that the stock-based CPN portfolio wins against the index-based CPN.

In the case of the DCs, in contrast, we see single stock-based products exceed their counterparts on the index, and there is no clear trend among the risk aversion level with the differences around 11 basis points. The optimal strategy for the BCs is the same as the DCs, though the difference in performance is quite marginal. In addition, all the differences are statistically significant at the 1% level.

Panel B displays the results conducted under the stochastic volatility of the Heston model. The results exhibit very similar patterns to those in Panel A and the differences between the two strategies are of high statistical significance. A CRRA investor profits more from the index CPNs with the gain in the certainty equivalent rates of 8 basis points ($\gamma = 5$) to 28 basis points ($\gamma = 0.5$). Portfolios of stock-based products deliver explicitly higher utility improvement than the index-based products for both DCs and BCs, and thus should be more attractive to the investor. Compared to the Black-Scholes model, the magnitude of both strategies' certainty equivalent rates of the strategies and their differences in the Heston model are remarkably larger.

Above all, the results of structured products are essentially consistent with the empirical analysis discussed in Henderson and Pearson (2007), which reveals that products based on the prices of individual equities predominantly have concave pay-

Table 3.3: Certainty equivalent rates for diversifications of structured products

| $\gamma$ | CPN | | | DC | | | BC | | |
|---|---|---|---|---|---|---|---|---|---|
| | $D1$ | $D2$ | $D1-D2$ | $D1$ | $D2$ | $D1-D2$ | $D1$ | $D2$ | $D1-D2$ |
| Panel A: Black-Scholes Model | | | | | | | | | |
| 0.5 | 5.41 | 5.63 | -0.21 | 5.95 | 5.83 | 0.11 | 7.20 | 7.16 | 0.04 |
| 2 | 5.20 | 5.34 | -0.13 | 5.52 | 5.41 | 0.11 | 6.05 | 5.97 | 0.08 |
| 3 | 5.07 | 5.16 | -0.09 | 5.20 | 5.09 | 0.11 | 5.35 | 5.26 | 0.08 |
| 4 | 4.95 | 4.99 | -0.04 | 4.84 | 4.72 | 0.12 | 4.68 | 4.61 | 0.07 |
| 5 | 4.82 | 4.83 | -0.01 | 4.44 | 4.31 | 0.13 | 4.05 | 4.00 | 0.05 |
| Panel B: SV Heston Model | | | | | | | | | |
| 0.5 | 5.50 | 5.84 | -0.34 | 5.93 | 5.75 | 0.19 | 7.43 | 7.22 | 0.22 |
| 2 | 5.31 | 5.56 | -0.24 | 5.54 | 5.37 | 0.17 | 6.36 | 6.10 | 0.25 |
| 3 | 5.20 | 5.38 | -0.18 | 5.24 | 5.07 | 0.17 | 5.69 | 5.43 | 0.26 |
| 4 | 5.09 | 5.21 | -0.13 | 4.91 | 4.73 | 0.18 | 5.06 | 4.81 | 0.25 |
| 5 | 4.98 | 5.05 | -0.07 | 4.53 | 4.34 | 0.19 | 4.45 | 4.22 | 0.24 |

The table shows the numerical results of certainty equivalents from the investment in CPNs, DCs and BCs, respectively, in the Black-Scholes and the Heston model. The overall wealth is invested into the structured products at time 0 and is not adjusted during the term. $\gamma$ is the risk aversion of the CRRA investor. $D_1$ and $D_2$ represent two means of diversification strategies: $D_1$ is the portfolio of equally weighted structured product each based on a single stock; $D_2$ is the structured product based on the index. $D_1 - D_2$ is the difference in certainty equivalent return between the two strategies. We run $1,000,000$ simulations for the underlying stocks and the index. All the numbers are percentage. All the differences of certainty equivalents between two strategies are statistically significant at the 1% level.

off profiles, while those based on equity indices predominantly have convex payoffs. Figure 3.1 displays this clear pattern for the CPNs with convex payoffs and the DCs with concave payoffs.

On the other hand, the patterns from the simulation results are also applicable to investors' preferences for categories of structured products proposed by Blümke (2009), in which CPNs are categorized as fairly low-risk instruments, whereas DCs and BCs are categorized as average-to-high risk instruments. As shown in Table 3.3, the BC is the optimal choice in general for a relatively aggressive investor (e.g. $\gamma = 0.5$), in which he can realize up to 7.43% certainty equivalent return, whereas for an investor with higher risk-aversion (e.g. $\gamma = 5$), the CPN dominates among others by generating up to 5.05% profit. Investors who tend to buy CPNs are more likely to be conservative, and they are prone to picking the index products passively. Investors who tolerate higher risks may choose to actively manage their own portfolios by for example constructing portfolios with individual stock-linked DCs and BCs.

## 3.6 Conclusion

Among the many purposes that options and structured products serve, diversification is one of the most important goals for investors. Different from rules of constructing a diversified portfolio with common stocks, there are two possibilities to diversify financial derivatives: one is to combine individual stock-based products, and the other one is index-linked products. Both readily diversified investment strategies seem to be reasonable and also feasible to retail investors. This chapter studies the optimal investment portfolios with respect to options as well as different structured products by comparing the expected utility benefits for a CRRA investor.

Specifically, applying the numerical Monte Carlo procedure through the widely used Black-Scholes model as well as the Heston stochastic volatility model, we firstly assess the optimal strategy for the plain vanilla call and put, then extend the investigation to structured products including the capital protected notes, the discount certificates and the bonus certificates in the analysis. Our results on options show that the attractiveness of a diversification strategy related to different underlying assets largely depends on parameters with respect to the market condition, the risk attitude of investors and the position of options in the portfolio. As financial applications under the base-case parameters' setting, having the opportunity to invest in structured products, the CRRA investor obtains a higher utility from the investment of index capital protected notes, while he profits more from a combination of stock-linked products for discount certificates and bonus certificates.

In the analysis, we omit transaction costs in the model. Taking transaction costs into account would be valuable and more realistic, since diversification for retail investors is costly, hence they should invest only in products where diversification benefits exceed construction costs. It would therefore be interesting to extend our study in this direction. Another limitation is that we only considered a static buy-and-hold strategy in this article. Further research could examine the problem in a dynamic environment.

# Chapter 4

# Does underlying really matter for structured products?

## 4.1 Introduction

Structured products are practically financial synthetic investment strategies that combine an underlying, typically a stock or a stock index with at least a derivative on that underlying. Consider the design of a structured product from the perspective of an issuing financial institution. While the valuation of the structured product is no doubt of predominant importance, we discuss the issue from a slightly different angle that is by no means straightforward: the choice of an appropriate reference stock, on which equity performance the structured product is based. In this chapter we study the optimal design of structured products between alternative choices: an individual stock and an index composed of those stocks. Among the vast majority of structured products, we concentrate on capital protected notes (CPNs) and discount certificates (DCs) for purpose of illustration in our analysis. Ultimately, we evaluate and compare the utility gain of the CPNs and the DCs on single stocks and those on indices, respectively, for investors in a behavioral finance context. The aim is to explore the dependence of structured products on their underlying assets, especially on the attractiveness of the structured products between an index and its component stock for investors with loss aversion and mental accounting.

It is well-known from an academic perspective that structured products are generally overpriced (e.g. Stoimenov and Wilkens (2005), Benet et al. (2006) and Henderson and Pearson (2011)), which is natural, since Carlin (2009) demonstrates that financial firms strategically enhance the complexity of their financial products, which tends to induce investors away from rational behavior but to end up with a rather irrational choice. The financial firms, on the other hand, preserve the market power even in the face of competitive pressures and are able to charge a higher price on their products. Rieger (2012) and Das and Statman (2013) provide evidence that the irrationality of investors for structured products can be to some extent explained within the behavioral portfolios theory. As a matter of fact, the asymmetric nature of the payoffs of the structured products also necessitates nonexpected utility preferences to explain the demand.

While conventional expected utility framework is not plausible explaining the desire for structured products under numerous stances, we focus on the cumulative prospect theory (CPT) as proposed by Tversky and Kahneman (1992) to measure the utility gain of CPNs and DCs. The CPT is widely viewed as the best descriptive theory of decision making under risk in experimental settings, which features a S-shaped utility (value) function as divided by a reference point with loss-aversion and probability weighting, so that it reflects a binary pattern of people's different risk behaviors in gains and losses. We consider an individual stock's price being determined under a complete and frictionless market corresponding to the Black and Scholes (1973) model, then the index level can be derived as the weighted sum of the individual component stock prices. Given the structured products being fairly priced according to the Black and Scholes (1973) model, we calculate the final payoff of each product hence the utility gain of a CPT investor via Monte Carlo simulations. Moreover, to control the impact of investor's risk preferences on their choices, we analyze the results on a step-by-step procedure, in which different aspects of the CPT are added cumulatively. In particular, we explore the robustness of the results by dynamically combining parameters of the drift and the volatility of the return generating process. Moreover, with empirical daily returns of the Deutsche Bank (DBK) and the index that contains the DBK: the German Stock Index (Deutscher Aktien IndeX: DAX), we implement historical simulations from January 1980 to July 2013. Based on a one-day moving window, we use 250 subsequently daily stock returns as proxy for yearly performance. Then we are able to produce 8207 overlapping yearly scenarios for the structured products. So that certain features of stock return distributions that are missed out in the Monte Carlo simulations, like fat tails and autocorrelation, are compensated in the historical simulations.

Our main results are as follows. Index-based CPNs generally outperform stock-based CPNs given low to moderate volatility levels as well as relatively high return expectations, whereas stock-based CPNs beat index-based CPNs for the opposite scenarios. As loss aversion has virtually no impact on investors' decisions owing to the CPNs' property of '100% guaranteed principal', the competing factors affecting investors' decisions are the participation rate on the underlying stocks and investors' overweighting of small probability events. Since in general, the index is less volatile than its component stocks, thus this gives the index-based CPNs more opportunity to take participation in the performance of the market index as long as the product being fairly priced. As a result, a higher participation rate contributes to the superiority of the index-based CPNs when the return expectation is high, and correspondingly, it deteriorates the performance of the index-based CPNs when the expected return of the stock is low. When volatility of the individual stock increases, ceteris paribus, the individual stock generates a more right-skewed return distribution compared to the index. The subjective probability weighting scheme affects the CPT investor by making him overestimate the tail events, which induces the investor in favor of the stock-based CPNs. On the other hand, a qualitatively stable result with respect to the DCs shows that for various expected return and volatility combinations, DCs on the index is preferable. We can attribute this finding to the combination effect of loss aversion and probability weighting. The CPT investor overweights probabilities in both tails of distribution, in combination with loss aversion in the value function, the power of negative events magnifies more heavily. In that respect, even though the stock-based DC attains a higher maximum possible payment (cap), more extreme negative returns significantly damage its attractiveness perceived by the CPT investor, leaving the index-based DC the better choice.

A number of studies have contributed in the literature in various aspects. For instance, Branger et al. (2008) analyze the benefit of rational investors from trading structured products under the Expected Utility Theory framework. Breuer and Perst (2007) evaluate the discount reverse convertibles and reverse convertible bonds as examples of structured products under the cumulative prospect theory. Ameur (2010) illustrates the optimal design of structured products within the rank dependent utility theory framework. More related to our topic, patterns of reference stock classes to which structured products are linked have also been investigated. Benet et al. (2006) argue that stocks with “large float” are used for structured products, which may avoid the possibility that managers of the structured products manipulate underlying stock prices. Henderson and Pearson (2007) provide evidence that issuers of structured products in aggregate choose underlying stocks not randomly, but follow stylized rules. More explicitly, they document striking patterns in the underlying securities of structured products. For example, products based on the prices of individual equities predominantly have concave payoff profiles, while those based on a diversified stock index possess convex payoff profiles.

To the best of our knowledge, this article is the first one that conducts the analysis on a theoretical background to investigate the interplay of underlying stocks and the optimal design of structured products in the CPT framework. In particular, under the assumptions with an investor being a median decision maker subject to Tversky and Kahneman (1992) and Lattimore et al. (1992), our results generally predict that DCs on the index are more appealing, which seems to be contrary to what is observed in practice as pointed out by Henderson and Pearson (2007). We argue that this contradiction could occur because investors might deviate from the standard CPT when they make decisions of purchasing a DC or other more complex products without downside protection in reality, as Erner et al. (2013) show that it is too much of a leap of the CPT parameters elicited via simple lotteries to be applied in real world situations.

The remainder of the article is organized in the following. In section 4.2 we introduce the fundamentals of the cumulative prospect theory. Section 4.3 presents the design and the main results from the Monte Carlo simulations and the historical simulations. Section 4.4 contains a short summary.

## 4.2 Cumulative Prospect Theory

We consider an investor with preferences obeying the cumulative prospect theory (CPT) as proposed by Tversky and Kahneman (1992), which takes two central properties, framing effect and probability overweighting, into account, as compared to the classic expected utility theory (EUT). Instead of the final absolute wealth that an EUT investor values, a CPT investor sets a reference point and makes decisions according to relative gains and losses. The investor is risk-averse in the domain of gains and risk-seeking in the domain of losses, which implies a S-shaped value function being concave over gains and convex over losses. Additionally, he is more sensitive to losses in that losses generate a more pronounced negative utility than the equal gains. Moreover, the CPT investor distorts statistical probabilities in a sense that he is highly likely to overweight events with extremely low probability, while underweight normal or average events.

To apply the CPT, we interpret the terminal outcome $x_i$ as the difference to the reference point $x_0$ for each product, i.e. $\Delta x_i = x_i - x_0$. With probability $p_i$ assigned to each outcome $\Delta x_i$, a prospect is a vector of pairs as $((\Delta x_1, p_1), (\Delta x_2, p_2), \ldots, (\Delta x_N, p_N))$. Defining the decision weights $\pi_i$ and the value function $v(\Delta x_i)$, the CPT investor's subjective evaluation for an investment strategy is given by

$$CPT(Strategy) = \sum_{i=1}^{N} \pi_i \cdot v(\Delta x_i) \tag{4.1}$$

Following Tversky and Kahneman (1992) we assume a two-part value function $v(\Delta x)$:

$$v(\Delta x) = \begin{cases} \Delta x^{\alpha} & \Delta x \geq 0 \\ -\lambda \cdot (-\Delta x)^{\beta} & \Delta x < 0, \end{cases} \tag{4.2}$$

based on the relative deviation from the reference point. So that the S-shaped value function is concave in the range of gains and convex in the range of losses, resulting in a kink at $\Delta x = 0$. The parameters $\alpha \approx 0.88$ and $\beta \approx 0.88$ reflect the investor's sensitivity towards gains and losses, while $\lambda \approx 2.25$ stands for the loss aversion coefficient, which captures the investor's propensity of valuing losses more than twice as important as gains.

The objective statistical probabilities $p_i$ are not multiplied in equation 4.1, but the transformed probability weighting functions $\pi_i$ are used. Specifically, decision weights $\pi$ are computed slightly different for losses $w^-$ and gains $w^+$, as shown below

$$\pi_i = \begin{cases} \pi_i^- &= w^-(p_1 + \cdots + p_i) - w^-(p_1 + \cdots + p_{i-1}) \\ \pi_i^+ &= w^+(p_i + \cdots + p_N) - w^+(p_{i+1} + \cdots + p_N), \end{cases} \tag{4.3}$$

where $i$ denotes the outcomes $\Delta x_i$ ranking in an ascending order, with $\Delta x_i < 0$ for $i = 1, \ldots, k$ and $\Delta x_i \geq 0$ for $i = k+1, \ldots, N$. The key idea of the CPT that deviates the original version of the prospect theory is that it replaces the probabilities with differences of cumulative weighted probabilities, so that the problem in the original prospect theory such as the violation of stochastic dominance can be avoided.[1] We follow the probability weighting function in line with Lattimore et al. (1992) and Gonzalez and Wu (1999) assuming $w_p$ to be of a two-parametric form

$$w_{\delta,\gamma} = \frac{\delta \cdot p^\gamma}{\delta \cdot p^\gamma + (1-p)^\gamma} := \begin{cases} w^+(p) := \frac{\delta^+ \cdot p^{\gamma^+}}{\delta^+ \cdot p^{\gamma^+} + (1-p)^{\gamma^+}} \\ w^-(p) := \frac{\delta^- \cdot p^{\gamma^-}}{\delta^- \cdot p^{\gamma^-} + (1-p)^{\gamma^-}} \end{cases}. \tag{4.4}$$

The parameter $\gamma$ determines the curvature in a sense that it measures how the decision maker discriminates the probabilities of each outcome, while $\delta$ controls the elevation, signifying how attractive the decision maker perceives different outcomes. Alternative probability weighting specifications with only one parameter are not considered, like Tversky and Kahneman (1992) and Prelec (1998), since they do not permit an independent variation of elevation and curvature. In this article we consider a median CPT investor elaborated by the empirical estimation according

[1]The violation of stochastic dominance is based on the fact that a large number of small probability outcomes added up to a "subjective" probabilities larger than one, which implies that one prospect might be preferred even if it yields a worse outcome.

to Abdellaoui (2000), in which the parameters are setting as following: $\delta^+ = 0.65$, $\delta^- = 0.84$, $\gamma^+ = 0.60$ and $\gamma^- = 0.65$.

## 4.3 Simulation Results

### 4.3.1 Monte Carlo Simulation

#### Base Case

We firstly investigate the question under plausible economic assumptions as a base case. We run 1,000,000 Monte Carlo simulations to approximate $N = 20$ individual stocks, hence the index return distributions. The drift parameter is set to $\mu = 0.09$, and the volatility is constant and equal to $\sigma = 0.2$. Moreover, we assume a risk-free rate $r = 0.04$, the investment horizon $T = 1$ year and the correlation between each pair of stocks to be identical with $\rho = 0.4$. Given the fundamental settings of the underlying stocks, parameters of the CPNs and the DCs can be determined. Table 4.1 summarizes the descriptive statistics of the underlying stocks and the structured products through the Monte Carlo simulation. In particular, as key parameters often appear in products' fact-sheets and prospectuses, at first glance the CPN on the index looks intuitively better than that on the individual stock, in that the index-based CPN participates more (with participation rate 53.73% vs 39.51%) in the performance of the underlyings given both the principal being equally 100% protected. Conversely, the DC on the individual stock seems more attractive, followed by the fact that it sells at a cheaper price (with discount rate 9.93% vs 7.32%), thus would potentially be able to offer a higher maximum return.

Based on the above assumptions, Figure 4.1 depicts the return distributions of the individual stock, the index, the CPNs and the DCs with the Monte Carlo simulation. Different from payoff function which only gives the magnitude of potential payoffs, probability distribution provides information on the possibilities that events occur[2]. It is noteworthy here that both of the two classes of structured products have truncated return distributions, but in an opposite way. While the CPNs' return is truncated above a "floor", the DCs' return is limited by a "cap".

Table 4.1: Statistical summary of the alternative underlyings and the structured products generated by Monte Carlo simulations

| | Underlying | | CPN | | DC | |
|---|---|---|---|---|---|---|
| Statistics/Parameters | Stock | Index | Stock | Index | Stock | Index |
| mean return | 0.094 | 0.094 | 0.055 | 0.061 | 0.061 | 0.058 |
| standard deviation | 0.221 | 0.144 | 0.069 | 0.064 | 0.089 | 0.048 |
| skewness | 0.621 | 0.401 | 1.557 | 1.134 | -1.993 | -2.682 |
| kurtosis | 3.719 | 3.279 | 5.742 | 4.102 | 6.543 | 10.515 |
| protected level | | | 100% | 100% | | |
| participation rate | | | 39.51% | 53.73% | | |
| discount | | | | | 9.93% | 7.32% |

This table describes the statistical characteristics of the underlying assets and the structured products for the time horizon of 1 year simulated by Monte Carlo simulations. Basic parameter values are $\mu = 9\%$, $\sigma = 20\%$, $r = 4\%$, $\rho = 0.4$.

Now we assess an investor's utility given the cumulative prospect theory. As a CPT investor distinguishes between gains and losses relative to a neutral reference point, which is usually set to the status quo[3], we use the strategy returns of zero as the reference point. We implement the simulation procedure on a step-by-step basis. Specifically, in the first step, we neither account for loss aversion (setting the parameter in the value function $\lambda = 1$) nor probability weighting. Next, we include

[2]This is important because unsophisticated investors are highly likely to misestimate the objective probability and overoptimistic about an unrealistic favorable payoff, see for example Bernard et al. (2011) and Rieger (2012).

[3]See for example Benartzi and Thaler (1995).

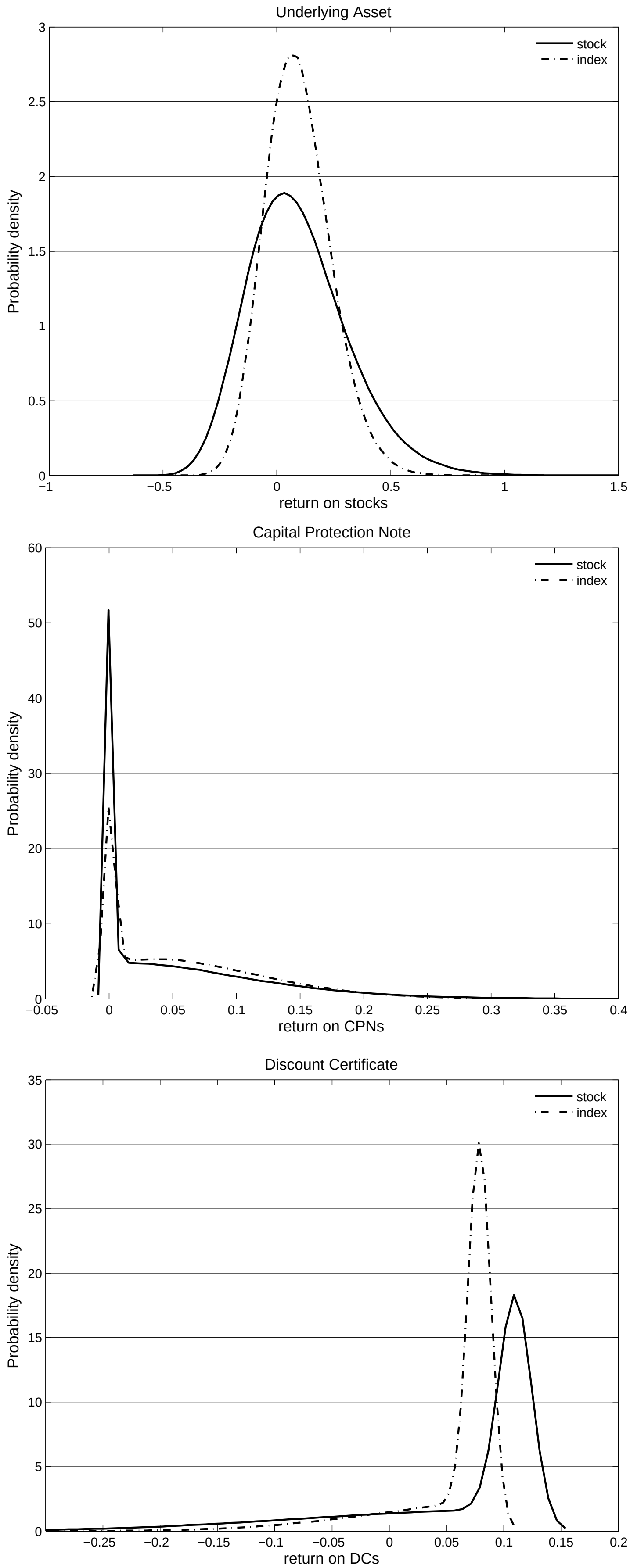



Figure 4.1: Density estimate of return distributions with MonteCarlo Simulation

the loss aversion in the value function with $\lambda = 2.25$, but still ignore the probability weighting. In the end, we complete the whole cumulative prospect theory by incorporating the loss aversion and the probability weighting. Controlling parameters step-by-step, we capture the results' deviation attributed to different aspects of the cumulative prospect theory. In particular, to evaluate the utility improvement more directly, we transform the final CPT value into certainty equivalent rates (CE), which can be interpreted as equally desirable risk-free payment that generates the same CPT value resulting from other risky investment strategies.

Table 4.2 presents the results of the two structured products for the base case. Firstly, we find that the optimal CPN is based on the index (5.679% in CEs), whereas the best DC is derived from the single stock (5.701% in CEs) in Panel A, where neither loss aversion nor probability weighting is considered. Including loss-aversion coefficient $\lambda = 2.25$, as shown in Panel B, does not influence the outcome of the CPNs[4], however, the opposite results are observed for the DC in comparison to the outcome in Panel A. Incorporating the probability weighting in Panel C evidently mitigates the magnitude of the CPT value difference between the CPN on the single stock and that on the index, whereas it dramatically sharpens that difference for the DCs. Additionally, all differences of the CPT values are statistically significant at the 1% level as measured by the paired t-test. In a nutshell, the results above suggest that for a given moderate market with stock expected return being 9% and volatility being 20%, both the CPN and the DC linked to the index deliver a higher utility for the investor with CPT preferences relative to those linked to the single stock. A closer look at the Figure 4.1 can help with understanding the results. For both CPNs and DCs, stock-based products have obviously longer tails than their counterparts on the index. In spite of the longer positive 'tail' of the CPN on the single stock, its inferiority results from the reduced upside participation of the underlying stocks

[4]Since the investor is fully protected in the investment of the CPNs, thus there is no negative returns.

(39.5% vs 53.7%)[5]. As for the DCs, extremely adverse events of the stock-based-DC are weighted more by the CPT investor, whose negative prospect value in addition to the loss-aversion effects substantially dominate over the higher maximum return (the "cap") it achieves. As a result, the DC on the index with less adverse events exceeds, hence is optimal to the investor.

### Robustness Check

Furthermore, we conduct a number of robustness tests by modifying parameters to check the dependence of our simulation results on the choice of parameter specifications. Most importantly, we compare the CPT values by changing either the expected return $\mu$ or the volatility $\sigma$ of the individual stock while the other parameter being fixed. We allow for $\mu$ changing between -0.05 and 0.15, and $\sigma$ varying between 0.1 and 0.4, as shown in Figure 4.2 and Figure 4.3, where the sensitivity analyses of different market scenarios for the two structured products are depicted.

Interestingly, Figure 4.2 illustrates that the optimality of CPNs shifts, largely depending on the parameters $\mu$ and $\sigma$. Partially inconsistent with the result shown in Table 4.2, the stock-based CPNs exceed the index-based CPNs given relatively lower values of $\mu$ (e.g. $\mu < 0.06$) or higher values of $\sigma$ (e.g. $\sigma > 0.35$). On the other hand, as displayed in Figure 4.3, the CPT value of the index-based DCs outperforms that of the stock-based DCs for all expected returns and volatility levels. This suggests that the CPT investor should always choose the index-based DCs rather than the stock-based DCs.

In Figure 4.4 and Figure 4.5 we sketch the 3-dimension diagrams of the differences of CPT values by dynamically changing $\mu$ and $\sigma$ at the same time. The preference structure reveals that the index-based CPNs are mainly desired relative to the stock-based CPNs for $\mu$ being relatively high and $\sigma$ being relatively low. There exists a "fuzzy" ($\mu$, $\sigma$) threshold at which the difference is close to 0, implying

[5] See the participation rate in the Table 4.1

Table 4.2: Base case with Monte Carlo simulation results

| | CPN | | | DC | | |
|---|---|---|---|---|---|---|
| | Stock | Index | S-I | Stock | Index | S-I |
| *Panel A: without loss aversion $\lambda = 1$* | | | | | | |
| CPT | 4.098 | 4.610 | -0.512*** | 4.626 | 4.499 | 0.128*** |
| CE (%) | 4.968 | 5.679 | -0.711 | 5.701 | 5.523 | 0.178 |
| *Panel B: with loss aversion $\lambda = 2.25$* | | | | | | |
| CPT | 4.098 | 4.610 | -0.512*** | 2.842 | 3.837 | -0.996*** |
| CE (%) | 4.968 | 5.679 | -0.711 | 3.277 | 4.610 | -1.333 |
| *Panel C: with loss aversion and probability weighting* | | | | | | |
| CPT | 4.159 | 4.328 | -0.170*** | -1.215 | 1.085 | -2.300*** |
| CE (%) | 5.051 | 5.285 | -0.235 | -0.497 | 1.097 | -1.594 |

This table describes the numerical results of our base case with Monte Carlo simulations for the capital protected notes and the discount certificates on the two alternative underlying assets. While the individual stock returns are generated using the Geometric Brownian motion, the index is the weighted sum of individual stock prices. Investors are characterized with different preferences that can be interpreted as following: the investor in Panel A contains the fundamental setup of the CPT with risk-aversion in the domain of gains and risk-seeking in the domain of losses, but his preference does not account for loss aversion and probability weighting; in Panel B loss aversion ($\lambda = 2.25$) is added into the value function, but decision weighting is omitted; in Panel C all elements of the CPT are incorporated, with loss aversion and probability weights. The paired t-test is conducted to test the prospect value of a structured product based on the single stock is statistically different from that based on the index. *** indicates that the test statistic is significant at the 1% level.

Figure 4.2: CPT value comparison of the **CPNs**

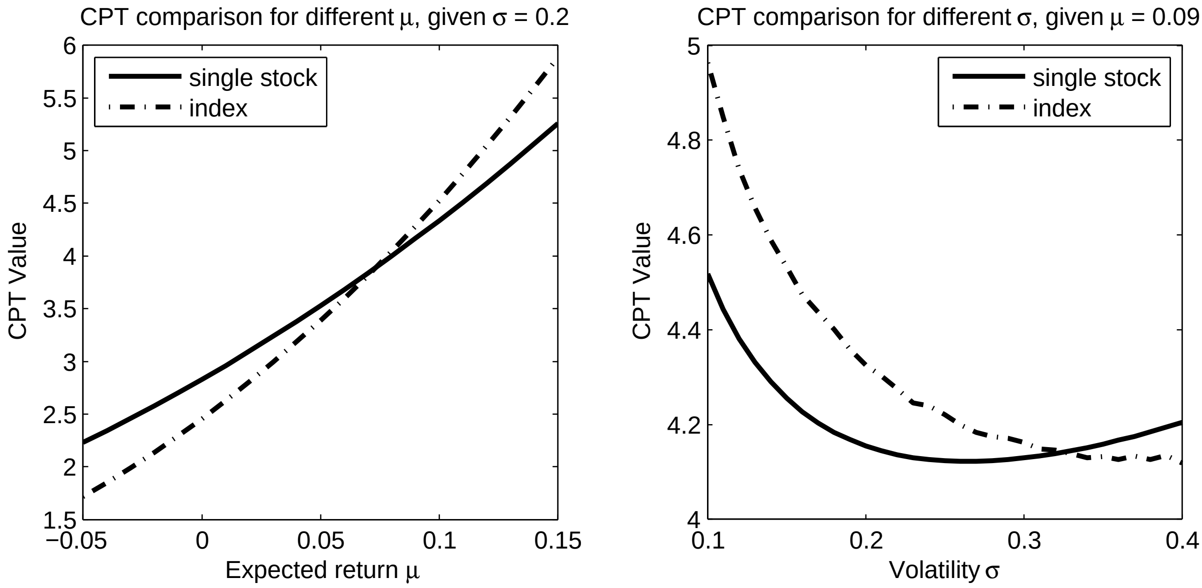
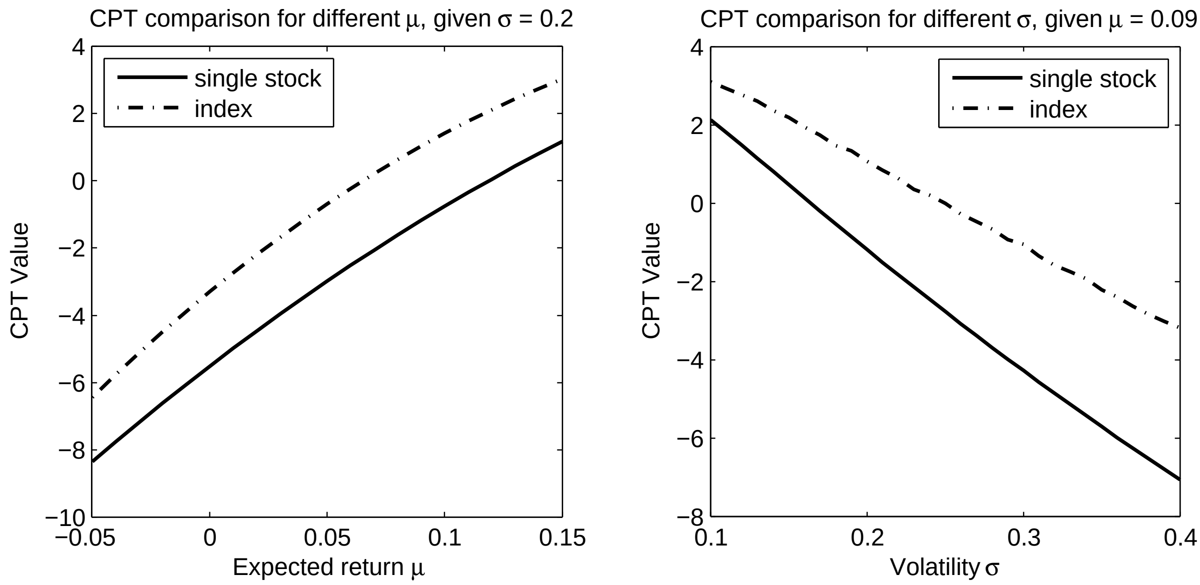



Figure 4.3: CPT value comparison of the **DCs**



that it is difficult to determine the superiority of CPNs between the two alternative underlyings. As the differences of CPT values of the DCs ($CPT_{Index} - CPT_{Stock}$) are always positive, meaning that the index-based DCs benefits more to the CPT investor whatever ($\mu$, $\sigma$) parameter combination is assumed. It is also worth noting that the magnitude of differences tends to become larger for increasing the volatility of individual stocks.

Besides, while all the others being equal, we rerun the Monte Carlo simulations with different risk-free rate ($r = 3\%, 5\%$), correlation coefficient ($\rho = 0.2, 0.6$), time to maturity ($\tau = 1/12, 1/2, 2, 5$ year(s)), and the number of stocks in the index ($N = 5, 10, 30$) respectively. However, the qualitative findings remain valid for all of these variations.

Our results imply that given a normal state of the market with a moderate $\mu$ (e.g. 9%) and $\sigma$ (e.g. 20%), an index-based CPN is more attractive to the CPT investor. As a component stock is likely more volatile than its index, the CPT investor with probability overweighting overestimates probabilities of rare positive events, thus would opt for CPNs linked to single stocks. However, the marginal utility of stock-based CPNs earned by probability overweighting is offset by its insufficient participation of the upward market movements compared to the index-based CPNs, which reduces the desirability of the stock-based CPN, especially when the expected return of the underlying stocks is high. Ultimately the index-based CPNs with much higher participation in the rising market deliver higher utility than its stock-based counterparts, thus are more attractive to the investor. The situation with the DCs can be interpreted as the impact of the downside risk and probability overweighting perceived by the investor. Specifically, an asymmetric perception to losses over gains in addition with an excessive probability weighting of negative returns leads the investor to be preferable to the index-based DC.

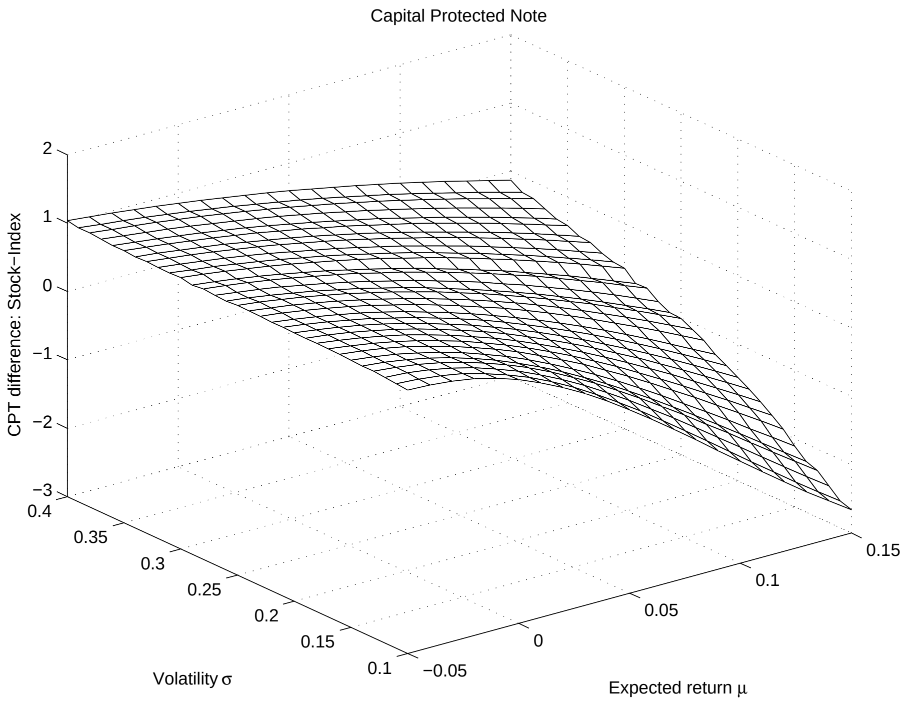



Figure 4.4: CPT value differences between stock-based CPNs and index-based CPNs as a function of expected return $\mu$ and volatility $\sigma$

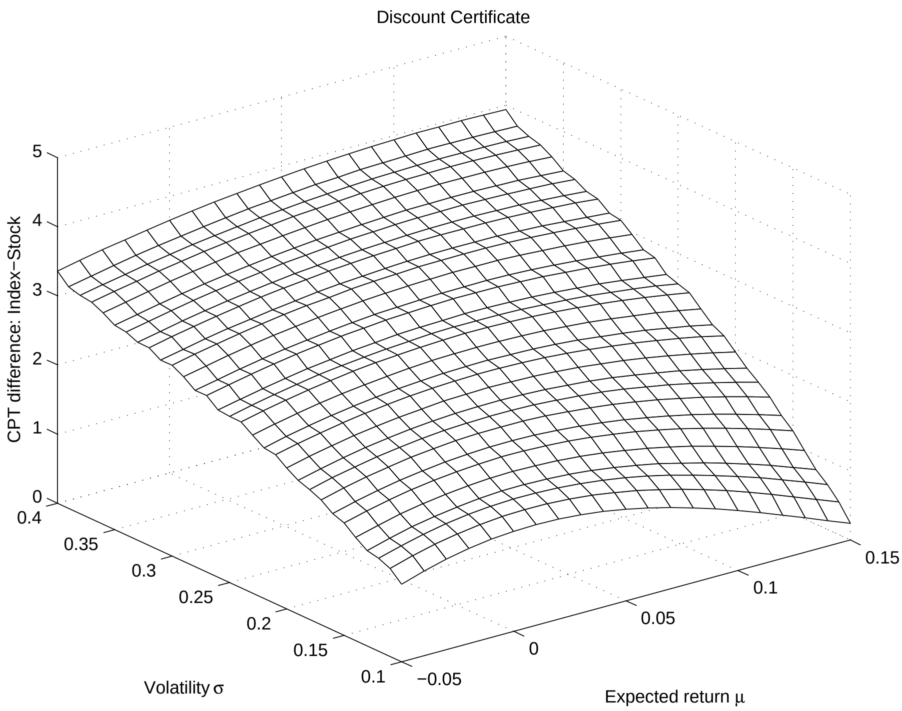



Figure 4.5: CPT value differences between index-based DCs and stock-based DCs as a function of expected return $\mu$ and volatility $\sigma$

### 4.3.2 Historical Simulation

Obviously, the results of Monte Carlo simulation are highly dependent on the input parameters. Nevertheless, the scenario sets should match the market observations and financial fundamentals as close as possible. In the simulation analysis above under the perfect market condition, it is inevitable that our generated scenarios of the underlying stock prices miss certain real data properties, such as return distributions with significant asymmetries, excess skewness and kurtosis as well as heavy tails. We therefore employ the historical simulation approach in order to capture those missing characteristics in the Monte Carlo simulations.

We select the German stock index (Deutscher Aktien IndeX: DAX) and one of its component stocks: the Deutsche Bank (DBK) in the analysis as underlying representatives. The stock prices of the DAX and the DBK are the most frequently used reference index and individual stock, respectively, for structured products (see Bergstresser (2009)) in Germany. Therefore, we believe that the DAX and the DBK can be the good proxy and suitable choice for underlying stocks. Specifically, we focus on daily returns of the DAX and the DBK from January 1980 through July 2013. For the risk-free rate we employ the three-month Frankfurt interbank rate. All data in the analysis is from the Thomson Reuters Datastream.

For historical simulation we use 250 subsequently daily stock returns as proxy for yearly performance by moving window of one day. With this method, we obtain 8207 overlapping yearly data, in a sense that the available data is most efficient utilized, and more importantly, crucial properties of the historical returns, such as autocorrelation and heteroscedasticity[6], are preserved.

Table 4.3 gives descriptive statistics of the continuously compounded returns scaled by the beginning-of-period price on the Deutsche Bank, the DAX and the

[6]We test for autocorrelation up to 5 days and heteroscedasticity at lag up to 3 for the DBK and the DAX, respectively. The results that for brevity are not reported indicate statistically significant autocorrelations and heteroscedasticity effects for both the DBK and the DAX.

money market rates. We see that the annual average return of the Deutsche Bank is 11.54%, which is close to that of the DAX with 11.26%. But returns of the DBK is notably more volatile in comparison to the DAX (32% vs 22%). This is intuitive: the index provides more diversification than the individual stocks, hence is less volatile. The observation also falls corresponding to our estimation in the Monte Carlo simulations, where the stock expected return $\mu$ ranges from $-5\%$ to 15%, and the volatility changes from 10% to 40%. Additionally, the average risk-free rate is 4.77% lying between 3% and 5% in the scenarios checked in the robustness of Monte Carlo simulations. For a more detailed insight of return distributions, Figure 4.6 illustrates the estimate of return distributions for 1 year with historical simulations.

Table 4.3: Descriptive statistics

| | DBK | DAX | fixed interest rate |
|---|---|---|---|
| Mean return p.a. | 11.54% | 11.26% | 4.77% |
| Volatility p.a. | 31.95% | 21.71% | 2.88% |

This table reports descriptive statistics of returns of the Deutsche Bank (DBK), German stock index (DAX) and the money market rate as represented by the three-month Frankfurt interbank rate over the period of January 1980 to July 2013.

Parallel to the table 4.2 that obtained from the Monte Carlo simulations, table 4.4 contains the results derived from the historical simulations. It can be seen that the general conclusion from the Monte Carlo simulation is strengthened by the historical simulation using data of the DBK and the DAX. The only exception is in Panel C, where dominance between the DBK-based CPN and the DAX-based CPN cannot be determined, as the paired t-test shows no statistical significance. This inconsistence is not surprising, as seen from the Figure 4.4 presented before, it might arguably fall in the "fuzzy" area in which there is no clear sign indicating the superiority of the CPNs between the two alternative underlyings.

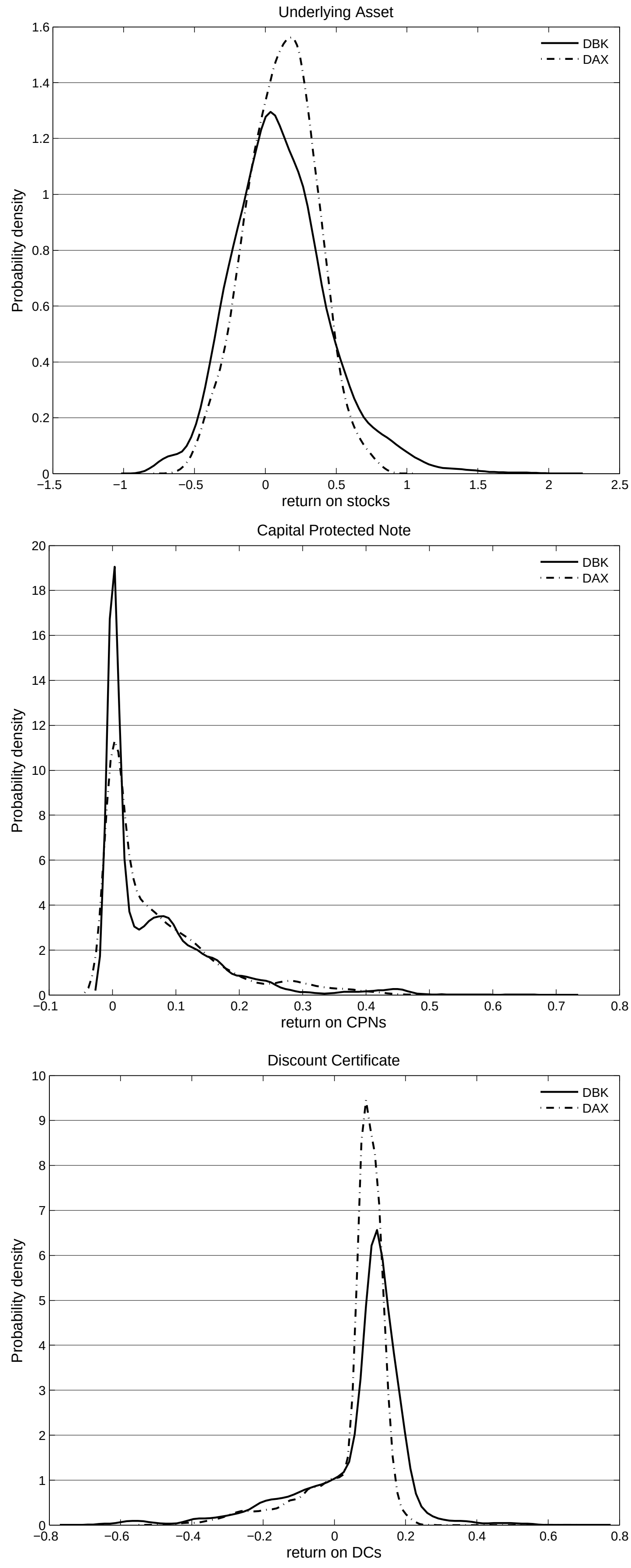



Figure 4.6: Density estimate of return distributions with historical simulations

Table 4.4: Historical simulation results

| | CPN | | | DC | | |
|---|---|---|---|---|---|---|
| | DBK | DAX | DBK-DAX | DBK | DAX | DBK-DAX |
| *Panel A: without loss aversion* $\lambda = 1$ | | | | | | |
| CPT | 4.781 | 5.188 | -0.408*** | 5.004 | 4.579 | 0.425*** |
| CE (%) | 5.917 | 6.494 | -0.577 | 6.233 | 5.635 | 0.598 |
| *Panel B: with loss aversion* $\lambda = 2.25$ | | | | | | |
| CPT | 4.781 | 5.188 | -0.408*** | 1.999 | 2.608 | -0.608*** |
| CE (%) | 5.917 | 6.494 | -0.577 | 2.197 | 2.972 | -0.774 |
| *Panel C: with loss aversion and probability weighting* | | | | | | |
| CPT | 5.239 | 5.231 | 0.008 | -0.053 | -0.027 | -0.026*** |
| CE (%) | 6.566 | 6.555 | 0.011 | -0.014 | -0.007 | -0.007 |

The table describes the numerical results for the capital protected notes and the discount certificates with historical simulation based on daily returns of the Deutsche Bank (DBK) and the German stock index (DAX) over the period of January 1980 to July 2013. We use 250 subsequently daily returns by moving the rolling window forward by one day and obtain 8207 overlapping yearly data. Investors are characterized with different preferences that can be interpreted as following: the investor in Panel A contains the fundamental setup of the CPT with risk-aversion in the domain of gains and risk-seeking in the domain of losses, but does not account for loss aversion nor probability weighting; in Panel B loss aversion ($\lambda = 2.25$) is added into the value function, but decision weighting is omitted; in Panel C all elements of the CPT are incorporated, with loss aversion and probability weighting. The paired t-test is conducted to test the prospect value of a structured product based on the single stock is statistically different from that on the index. *** indicates that the test statistic is significant at the 1% level.

## 4.4 Conclusion

Structured products are combined financial-engineered securities that reward investors based on the performance of a reference asset. Taken capital protected notes (CPNs) and discount certificates (DCs) as typical examples of structured products, in this article we systematically compare the utility gain of a CPN and a DC linked to a single stock with that to an index composed of these stocks, respectively, in the behavioral finance context according to the cumulative prospect theory. To this end, we conduct extensive Monte Carlo simulations as well as historical simulations to examine which alternative underlying asset is more attractive to a median CPT investor.

First of all, we find that in a moderate or optimistic market with rising expected returns and a relatively low volatility of the underlying stocks, the CPNs linked to the index are more of interest to the CPT investor, while in the situation of a declining and turbulent market, the CPNs linked to the single stock are prone to be more appealing. Furthermore, we obtain that the DCs based on the index perform predominantly better than those based on the individual stocks for all combinations of expected returns and volatilities. Parameter variations, like time to maturity, correlation coefficient between pairs of stocks, and risk-free rate, are of minor importance to the results. Moreover, our qualitative findings carry on to the results using real market data by historical simulations.

In particular, it is notable that the result on DCs is contrast with the empirical findings of structured products issued in the U.S. market documented by Henderson and Pearson (2007), where they predict that "if a structured product has a concave payoff, it is highly likely to be based on an individual common stock". Given the concave payoff function of DCs, our findings might be surprising, which results in the empirical observation that individual stock-based DCs dominate over index-based DCs challenging to be explained under the CPT in our settings. It is nevertheless not difficult to trace that the extent of loss aversion and probability weighting is critical in determining the attractiveness of the DCs. As CPT investors are more sensitive toward losses than gains, index-based DCs with less skewed return distributions yields higher utility improvement than stock-based DCs. Therefore, it would be worthwhile broadening the investigation on some modifications of the assumptions on the inherent preferences of investors.

Though we only consider two specific types of structured products, they are adequately simple and representative to justify theoretically. It is hoped that this article provides insights to our knowledge on how a structured product should be designed with respect to its reference stocks in order to meet investor's preferences. Further research could also look at properties of underlying assets of other more complex structured products.

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# Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Trier

Shuonan Yuan