Konstruktion universeller Funktionen mit zusätzlichen Eigenschaften

Construction of universal functions with additional properties

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Abstract

An entire function φ is called T-universal with respect to a given sequence $b := \{b_n\}_{n\in\mathbb{N}}$ of complex numbers with $b_n \to \infty$, if a suitable sequence $\{\varphi(z + b_{n_k})\}$ of additive translates of φ converges to any preassigned entire function locally uniformly in $\mathbb{C}$. Moreover, if a suitable sequence $\{\varphi^{(n_k)}\}$ of derivatives of φ converges to any preassigned entire function locally uniformly in $\mathbb{C}$, we call such a function universal under derivations.
The existence of such universal functions is shown in theorems of Birkhoff (1929), MacLane (1952) and generalisations of them.

In this thesis, we study the construction of such universal functions, which are in addition bounded on each line or have zeros at certain prescribed points.

In particular, we show that the set of all the entire functions, that are bounded on each line and which are T-universal with respect to a given sequence b, where b satisfies a certain condition, is a dense, but not a residual subset of all the entire functions endowed with the compact-open topology.
The existence of T-universal functions, whose asymptotic distribution of zeros is regular in a sense, is also surprising.